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Study of astrophysical S-factor, S (E) and thermonuclear fusion reaction rate, TNFRR for some α-trapped interactions

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Abstract:

Diverse types of reactions among nuclei occurring at the interior of stars which are at the accreting stage are tremendously affected by plasma environment of high density. Nuclear fusions at very low energy end (~ keV), much below the mutual Coulomb potential barrier, play a pivotal role in the Big-Bang nucleosynthesis of nuclei of lower masses. Mechanism of nuclear fusion reactions can be successfully described by quantum mechanical tunnelling through the effective mutual Coulomb barrier of interacting nuclei. Computation of some astro-nuclear observables, such as the astrophysical S-function, S(E), and thermonuclear fusion reaction rate, TNFRR (<σv>), at such an extremely low energy, is very challenging for the nuclear astrophysicist community. In the present work, our interest is focused on the study of the variation of S(E) and TNFRR (<σv>) with energy in the cases of few α-trapped fusions. For computations, we adopt the selective resonant tunnelling model (SRTM) technique by following the Khan et al. approach. As an improvement over our previous work, here we invoke the idea of a double-folding potential model. The findings of our computation are compared with some of those available in the literature. Our computed data agrees fairly with the experimentally observed data.

Keywords: Nucleosynthesis, double folding potential

1. Introduction:

In nuclear astrophysics, the intersection of the physics of nuclei and astrophysics, is crucial for explaining nucleosynthesis and energy production in stars [1-2]. Big bang Nucleosynthesis (BBN) provides very important information about the origin of the universe. It has successfully explained the creation of the very light nuclei [3-5]. In 1939, H.A. Bethe explained the various cycles of stellar evolution. In 1957, Burbidge et. al. explained the different types of stellar synthesis [2]. The creation of elements beyond the iron can't proceed in primordial nucleosynthesis. Chemical elements having their masses higher than those of iron (Fe) isotopes are created through various electrically neutral nucleon (neutron) captured reactions [6-10] and explosive events in supernovae [11]. The reaction network is significant for analyzing the nucleosynthesis processes. The transfer and capture processes play a vital role for the charge particle interactions. Any standard analytical model for computation of the reaction rate necessitates the astrophysical S-factor which is also an integral component of the expression for cross-section of nuclear fusion reaction.

A direct measurement of the fusion reaction cross section in the laboratories at astrophysical energy ranges (E~ 1 eV to a few keV) is very difficult. Evaluation of the astrophysical S-factor is model-dependent and energy-dependent and uncertainties are very significant for the computation in nuclear astrophysics. There are few experimental methods to determine the cross section of fusion reactions [12] and a bold theoretical model is needed for the justification of the experimental findings. Many indirect techniques are proposed for the determination of nuclear fusion cross section, like the Trojan-Horse Method (THM) of Baur et al., Spitalery et al. [14] and Tumino et al. [15]. In 2001, Mukhamedzhanov et al. [16] discussed the Asymptotic Normalization Coefficient method as an indirect approach for study of fusion cross section. All the above discussed methods need a bold theoretical justification for the captured data of cross section.

In this work, we invoked the double folding model of nucleus-nucleus interaction for the computations. In 2002, X. Z. Li explained the cross section of the D+T fusion reaction by the use of a selective resonant tunneling model [17]. X. Z. Li et al. had explained the fusion cross section by using complex potential and made comparisons of the computed data with the NRL Memorandum Report [18-19]. In 2008, Li, Wei and Liu calculated the fusion cross section for some light nuclei by using a simple new formula [20]. In 2015, Liang, Dong and Li explained the fusion cross section of p+D, p+ ^{6}Li, p+ ^{6}Li, D+T, D+D, D+^{3}He, T+T and T+^{3}He reactions by using a selective resonant tunneling model [21]. Khan et al. [21] studied the astrophysical S-function and cross section of fusion for some light elements adopting the SRTM at energies of astrophysical relevance [22].

Main purpose of the present work is to investigate the astrophysical S-function and the rate of thermonuclear fusion of ^{3}He + ^{3}He and ^{3}He + ^{4}He reactions using the double folding potential model for the pair of interacting nuclei. We have explored how the thermonuclear reaction rate depends on the temperature for the above reactions. The astrophysical S-factor which is an integral component of in the expression of the thermonuclear reaction rate is also studied here. The updated values will give a better picture for the nucleosynthesis network.

In Section 2, we have introduced the overview of the theoretical framework for the cross-section of fusion of some alpha-induced nuclear reactions at low energy. Next, in Section 3, we have compared our computed data with some of the observed data available in the literature. In particular, we have compared the experimentally observed results and theoretically estimated results for the cross-section of fusion at low energy. The corresponding graphs have been drawn. At last, in section 4, we close with a brief summary and conclusions.

2. Theoretical framework:

A good knowledge and understanding of σ(E), the cross section of nuclear fusion reactions is essential for a broad picture of the nucleosynthesis network [23]. In astrophysical aspects, the Coulomb barrier plays a crucial role for the fusion reaction and sub-barrier tunneling has occurred in the stellar energy regime [24]. Though, the temperature is not sufficient for triggering the fusion reaction, by tunneling fusion reaction has been accomplished. For the calculation of Gamow factor [25], an effective potential has been obtained that includes Coulomb, nuclear (DFP) and centrifugal potential terms. The astrophysical S-function, S(E)

[27] which constitute an integral part in the expression of cross section σ(E) of fusion and it is a slowly changing function of the energy (E) and important parameter.

The reaction rate is estimated as [28]

$$\langle\sigma v\rangle = N_A \frac{(\frac{8}{\Pi})^{1/2}}{\mu^{1/2}(K_B T)^{(\frac{3}{2})}} \int_0^\infty \sigma(E) E exp(-\frac{E}{(K_B T)})\, dE \qquad (1)$$

Where <σv> is the reaction rate in cm^3 $mole^{-1}$ sec^{-1} , $N_A = 6.0221 x 10^{23}$ $mole^{-1}$ is the Avogadro number, σ(E) is in μb, E in keV, T_9 is temperature in 10^9 K. The effective mass of the interacting nuclei pair has been given by

$$\langle\sigma v\rangle = 3.7313 x 10^{10}\, \mu^{-1/2}\, T_9^{-3/2} \int_0^\infty \sigma(E) E\; exp(-\frac{E}{(K_B T)})\, dE \qquad (2)$$

In the integral equation (2), the fusion cross section σ(E) far below the Coulomb barrier is been given by equation (3) [28]

$$\sigma(E) = \frac{S(E)}{E}\ \ exp(-\int_{r_1}^{r_2} \sqrt{\frac{2\mu(V_{eff}(r)-E)}{\hbar^2}}\, dr) \qquad (3)$$

where $\hbar$ is the reduced Planck's constant and E is the energy in keV. The astrophysical S-factor has been given by [30]

$$S(E) = S_0 + S_1.E + S_2.E^2 \qquad (4)$$

where S_0, S_1, S_2 are constants. The term, V_{eff}, in equation 3, is the nucleus-nucleus effective potential given in equation 5[31].

$$V_{eff}(r) = V_C(r) + V_N(r) + V_{Centi}(r) \qquad (5)$$

The mutual Coulomb potential (V_C) of the colliding projectile and target system [32] is been defined as

$$V_C(r) = 1.44\,\frac{Z_1Z_2}{r} \qquad \text{for } r > R \quad [\text{MeV}]$$

$$= 1.44\left(\frac{Z_1Z_2}{2R}\right)\left(3-\frac{r^2}{R^2}\right) \qquad \text{for } r \le R \quad [\text{MeV}] \qquad (6)$$

In equation (6) are indicates R is the touching separation between the centres of the colliding spherical nuclei and Z_1 and Z_2 are the charge number of colliding nuclei. The centrifugal barrier i.e. V_{Centi} (r) of the colliding system is defined as

$$V_{Centi}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2} \qquad (7)$$

where l indicates the orbital angular momentum and r is the radial variable.

The double folding potential has been defined as [33]

$$V_F(r) = \int\int \rho_1(r_1)\,\rho_2(r_2)\,V_{NN}(s)d^3r_1d^3r_2 \qquad (8)$$

Where r is the distance between the projectile and target nuclei and r_1 and r_2 are the distances from the centre of nuclei respectively. The equation (8) involves a six-dimensional integral.

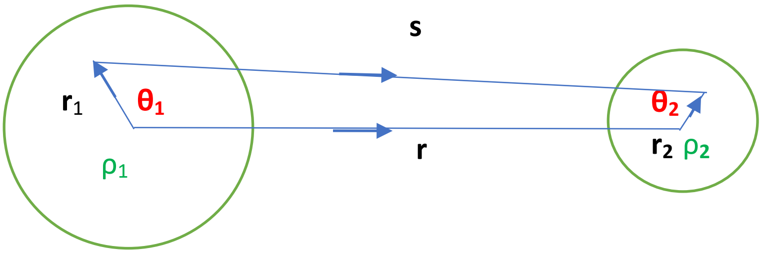

Figure 1: Coordinates used in double-folding potential calculations for the nuclei of target and projectile

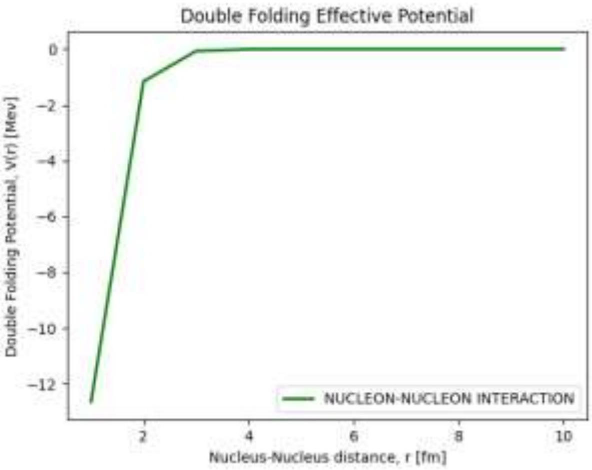

Figure 2: Graphical representation of the double-folding nucleus-nucleus potential given by equation (8)

The nucleon-nucleon distance (s) can be written in terms of vector form

$$\vec{s} \;=\; \vec{r} - \vec{r_1} + \vec{r_2} \tag{9}$$

The Fermi distribution of nuclear matter is given by

$$\rho(r) = \rho_o/[1+\exp(r-c)/a\,] \tag{10}$$

Where $\rho_o = 2.607$ (fm^{-3}), $c = 2.0$ (fm), $a = 0.486$ (fm)

The nuclear interaction term $V_N(r)$ in equation (5), has two explicit forms like DDM3Y-Reid nucleon-nucleon interaction and the DDM3Y-Paris effective nucleon-nucleon interaction [33].

DDM3Y-Reid: $v(r) = 7999(\exp(-4r)/4r) – 2134(\exp(-2.5r)/2.5r)$ (11)

DDM3Y-Paris: $v(r) = 11062(\exp(-4r)/4r) – 2538(\exp(-2.5r)/2.5r)$ (12)

3. Results and discussion:

At first, we have presented the double folding potential in Figure 2. For the computation of the $^3He(^3He,2p)\,^4He$ and $^3He(\alpha,\gamma)^7Be$ reactions [34], we have used that space varying potential function in which lies the novelty of the present work because earlier such calculations have been performed using square well type nuclear potentials [17, 20-21]. The quantum tunneling plays a crucial role for the fusion reaction in the astrophysical energy regime [35]. We have studied S(E) for the $^3He(^3He,2p)\,^4He$ reaction at the astrophysical relevant energies lying in the sub-barrier range. Then, we have made comparisons of experimental results and computed data in figure 3. In this study, the theoretical calculation has been carried out at low energies of the $^3He(^3He,2p)\,^4He$ reaction. The values of the parameters used for the input channel $^3He+^3He$ are shown in the first row of Table 1. In this context, Figure 3 indicates that the computed data are in fair agreement with the experimentally observed data.

Table 1: Parameters used in the expression of the S-function given in equation (4)

Input channel	S_0 (KeV mb)	S_1 (mb)	S_2 (mb/KeV)
$^3He + ^3He$	0.2	$0.2\text{x}10^{-2}$	$0.88\text{x}10^{-4}$
$^3He + ^4He$	0.8	$0.14\text{x}10^{-2}$	$5.8\text{x}10^{-4}$

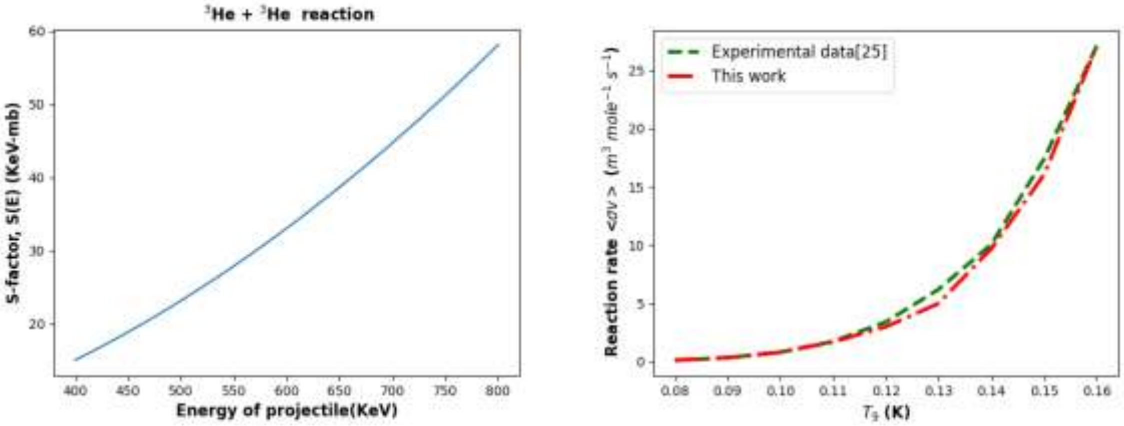

Figure 3: Left panel: Fitting of Astrophysical S-factor by using equation-1 and table-1. Astrophysical S-factor in KeV mb and energy in KeV in lab system. Right panel: Comparison between experimental data points and computed data points for ^{3}He + ^{3}He reaction. The experimentally observed data points are taken from ref. [26]. Cross section is in mb and energy in KeV in the laboratory system.

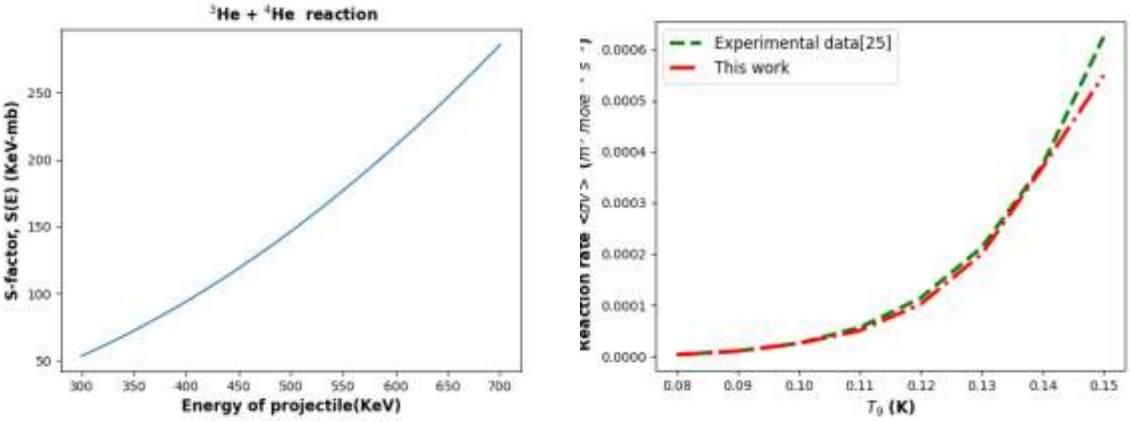

Figure 4: Left panel: Fitting of Astrophysical S-factor by using equation-1 and table-1. Astrophysical S-factor in KeV mb and energy in KeV in lab system. Right panel: Comparison between experimentally observed data points and computed data points for ^{3}He + ^{4}He reaction. The experimentally observed data points are taken from ref. [26].

Secondly, we have calculated the astrophysical S function for the ^{3}He+^{4}He nuclear system at the astrophysical relevant energies [36]. We have made a comparison between the computed data and experimental results of the ^{3}He+^{4}He system and presented the result in figure 4 and we have a good agreement between them. The values of parameters used for the input channel ^{3}He+^{4}He are depicted in the second row of Table 1.

4. Summary and conclusion:

Here we have explored the thermonuclear reaction rate in the low-temperature energy regime. At astrophysical relevant energy, which lies in the sub-barrier region S-factor is extensively used in nuclear astrophysics for extrapolation. The ^{3}He(^{3}He,2p)^{4}He and ^{3}He(α,γ)^{7}Be reactions are very important for the nuclear astrophysics studies specially for the primordial nucleosynthesis. In this work, we have theoretically estimated the thermonuclear fusion reaction rate of ^{3}He(^{3}He,2p) ^{4}He and ^{3}He(α,γ)^{7}Be reactions by using density dependent double folding potential. We have outlined the double folding model in figure 4. For the computations and graphs, we have utilized the Python-3 programming language. The Matplotlib is very useful for creating the graphs.

As a conclusion, we have made a comparison of our computed results and experimentally observed data for the thermonuclear reaction rate. The computed results are in good agreement with the experimentally observed data. The double folding model indicates the applicability of explaining the light nuclei reaction rate by using quantum mechanical tunnelling phenomena. Novelty of the present work lie in the choice of the space varying potential function, because in the earlier theoretical works a square well type complex potential has been employed [17, 20-21]. The present piece of original work could be a fruitful method to estimate the light-nuclei fusion cross-section and corresponding thermonuclear rate in the sub-barrier energy range where those quantities are found to be too small to measure.

Competing Interests:

Authors hereby declared that there is no competing interest.

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