Absolute electromagnetic transition rates in semi-magic N = 50 and 126 isotones as a test for $(\pi 9/2)^n$ single particle calculations.

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Abstract. Assuming a seniority-conserving interaction, single-*j* calculations for $(j)^n$ configurations with $n = 1,..,2j+1$ can be performed using a semi-empirical approach, provided that the energies and absolute electromagnetic transition rates are known for the two-particle (hole) nucleus. Using those and the coefficients of fractional parentage, all needed matrix elements for the $(j)^n$ configurations can be predicted. At the Cologne Tandem Accelerator of the Institute for Nuclear Physics these relations were tested by measuring lifetimes of excited states in $(\pi 9/2)^n$ isotones with N = 50 and N = 126 over the last years. The studies started in the two-proton nucleus ^{210}Po where the abnormal $B(E2:2_1^+ \rightarrow 0_1^+)$ value was remeasured, providing important input for the other N = 126 configurations. Then lifetimes of excited states in ^{211}At were measured using the electronic γ-γ fast timing technique, the Recoil Distance Doppler Shift (RDDS) method, and the Doppler Shift Attenuation (DSA) method. Very good agreement with the analytical single-j calculation is obtained. For N = 50 isotones, we recently started by measuring the previously unknown $B(E2:4_1^+ \rightarrow 2_1^+)$ value in ^{92}Mo needed for the prediction of other N = 50 isotones. We also report on experiments on ^{93}Tc and ^{94}Ru.

1. Motivation

While the energies, spins, and parities of the lowest yrast states of $N = 126$ isotones were well described with realistic interactions in a large model space [1] and even in a single-*j* shell [2], there was a large discrepancy concerning the electric quadrupole transition rates. Specifically, in ^{210}Po the seniority-changing $B(E2:2_1^+ \rightarrow 0_1^+)$ value was observed to be six times smaller than the theoretically predicted one while the transitions between the 8_1^+, 6_1^+, 4_1^+, and 2_1^+ levels, all with seniority $v = 2$, are almost perfectly reproduced in the shell model [1]. Because the B(E2) value in question was based on a single inelastic-scattering experiment performed in the early 1970s, an experiment aiming at the measurement of the lifetime of the 2_1^+ level was performed at the Cologne 10 MV FN Tandem accelerator using the $^{208}Pb(^{12}C,^{10}Be)^{210}Po$ two-proton-transfer reaction. The light fragment was identified with an array of solar cells, and the Doppler shift attenuation method was used to determine the lifetime of the 2_1^+ level as $\tau = 2.6(4)$ ps [3], yielding a $B(E2:2_1^+ \rightarrow 0_1^+)$ value three times larger than previously obtained. This reduced the discrepancy with theory, but did not eliminate it, as a factor two is still missing.

2. Semi-empirical single-*j* calculations

Observed discrepancies of this kind can be treated in a semi-empirical way, which is intermediate between a purely analytical and a numerical approach. Assuming only a two-body seniority-conserving interaction relations exist between the energy spectrum of two nucleons in a *j* shell, j^2, and that of three nucleons j^3 [4]. Two particles are first coupled to an anti-symmetrized wavefunction with angular momentum *I* and then coupled with *j* to total angular momentum *J*, yielding $|j^2(I)j;J>$. After anti-symmetrization the total analytical wavefunction can be expanded as [4]:

$$|j^3 \alpha J> = \Sigma_R [j^2(R)jJ|\}j^3 \alpha J]\,|j^2(R)j;J>, \qquad (1)$$

with $[j^2(R)jJ|\}j^3\alpha J]$ the $3 \rightarrow 2$ coefficient of fractional parentage and α an additional quantum number which is necessary when there are several states with the same angular momentum *J*. Similarly, the matrix elements of a scalar two-body interaction between anti-symmetric *n*-particle states can be expressed in terms of those between anti-symmetric $(n-1)$-particle states. In a single-*j* -shell approximation, the two- and three-particle diagonal matrix elements may be identified with the excitation energies of states,

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to arrive at the following relation between the spectra of nuclei with two and three valence nucleons:

$$E(j^3\alpha J) = 3\ \Sigma_R\,[j^2(R)jJ\,|\}j^3\alpha J]^2\ E(j^2(R)). \qquad (2)$$

The availability of two different approaches, realistic multi-j and semi-empirical single-j shell calculations, has motivated the study of ^{211}At, which we present below. A one-body electric quadrupole operator can be determined empirically from the E2 matrix elements in the one- and two-nucleon configurations. This requires a complete knowledge of E2 transitions and electric quadrupole moments, which is seldom available. Therefore, in Appendix A of Ref. [5], a method was developed that only requires the knowledge of E2 transitions in the j^2 nucleus, leading to the following relations:

$$B(\mathrm{E2}; j^3\,\alpha J \rightarrow j^3\,\alpha' J') = (\Sigma_R g_j(J, \alpha, J', \alpha', R)\sqrt{B_R})^2, \quad (3)$$

with $B_R \equiv B(E2; j^2\,R \rightarrow j^2\,R-2)$. Explicit values for the g_j coefficients for yrast states are given in Ref. [5] for the case $j = 9/2$. The relations (3) are analogous to those in Eq. (2) for energies and they allow the prediction of B(E2) values in the three-nucleon system based on the knowledge of those in the two-nucleon system. Similarly, the matrix elements of the E2 operator between anti-symmetric n-particle states can be expressed in terms of those between anti-symmetric $(n-1)$-particle states.

3. Studies of ^{211}At

With only three valence protons outside ^{208}Pb, ^{211}At is an ideal candidate for testing the nuclear shell model since calculations in a large basis without any truncation are feasible. In addition, the low-energy structure of ^{211}At conceivably can be described in a single-j shell approximation, with three protons in the $1h_{9/2}$ shell. The availability of these two different approaches, realistic and single-j shell, has motivated the spectroscopic study of ^{211}At.

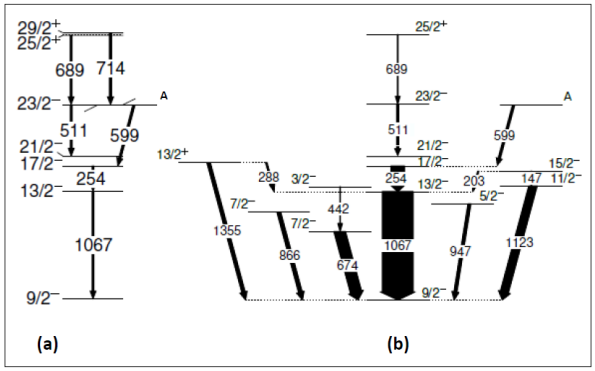

Fig. 1. Observed transitions in ^{211}At using the $^{208}Pb(^6Li,3n)$ (a) and the $^{209}Bi(^{16}O, ^{14}C)$ (b) reactions [5,6].

Essential for the study has been the availability of two different excitation mechanism to measure lifetimes in ^{211}At (see Fig.1). The $^{208}Pb(^6Li,3n)$ fusion evaporation reaction allowed the measurement of lifetimes for high-spin states using the fast timing technique and the standard HORUS fast timing setup with 8 HPGe detectors and 9 $La_3Br(Ce)$ scintillators of which six with BGO shield [5]. The $^{209}Bi(^{16}O, ^{14}C)^{211}At$ two-proton transfer reaction with 84 MeV ^{16}O beam and ^{14}C detection with solar cells mounted at backwards angles in the Cologne plunger and eleven HPGe detectors mounted in two rings at 45 and 142 degrees, then allowed the determination of low-lying low-spin states using Doppler Shift techniques [6]. Details can be found in Refs. [5,6]. Fig. 2 illustrates the result for the $5/2_1^-$ state obtained using the Recoil Distance Doppler Shift (RDDS) method [6].

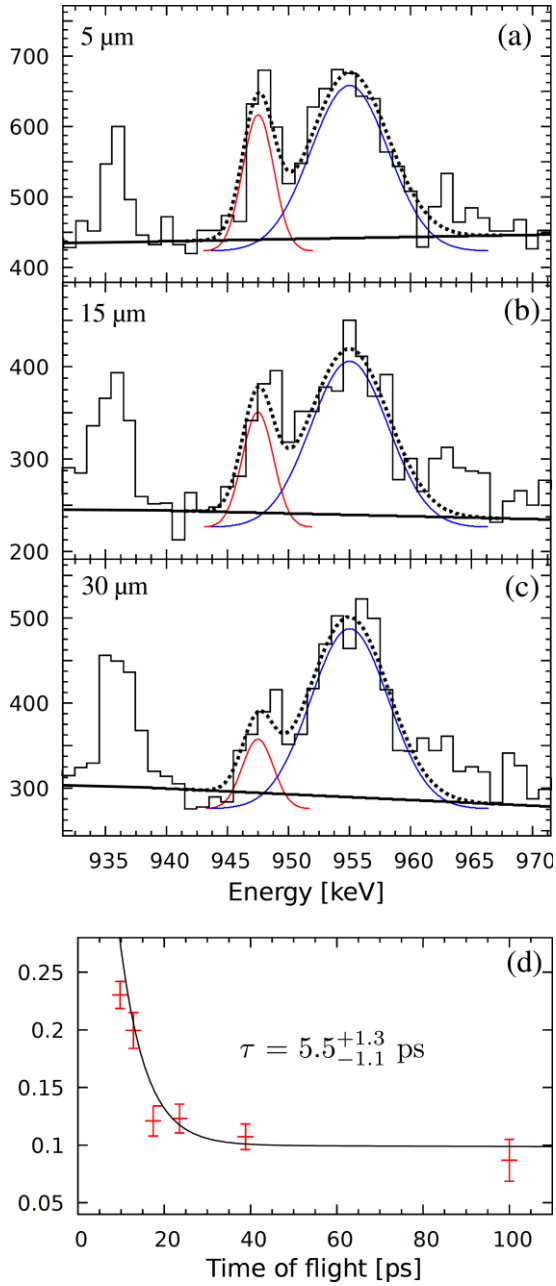

Fig. 2. (a-c): Fits to the $5/2_1^- \rightarrow 9/2_1^-$ transition of the forward-detector ring (dotted black line) for three distances used to determine the intensity of the shifted (blue solid line) and the unshifted components (red solid line). (d) The Bateman fit to the obtained decay curve R(t) [6].

Lifetimes could be determined for the $5/2_1^-$, $7/2_1^-$, $7/2_2^-$, $11/2_1^-$, $13/2_1^-$ and $17/2_1^-$ states [5,6]. Note that the $7/2_1^-$ state does not belong to the $(1h_{9/2})^3$ states. Table 1 compares the experimental *B(E2)* values between the $(1h_{9/2})^2$ and $(1h_{9/2})^3$ states in ^{210}Po and ^{211}At with the empirical $(1h_{9/2})^n$ calculations and the shell model calculation, as described in Ref. [5], based on the Kuo-Herling interaction in a basis that includes the full $82 \leq Z \leq 126$ proton shell above ^{208}Pb. One notices the remarkably good agreement of the single-*j* shell predictions both with the experimental data and with the full shell-model calculation in ^{211}At.

Table 1. Experimental and calculated (see text) *B(E2)* values in [e^{2}fm^4] for transitions in ^{210}Po and ^{211}At.

$J_i^\pi \rightarrow J_f^\pi$	Exp.[a]	Single-*j*	SM[b]
$2_1^+ \rightarrow 0_1^+$	136(21)	136(21)	260
$4_1^+ \rightarrow 2_1^+$	331(13)	331(13)	331
$6_1^+ \rightarrow 4_1^+$	227(5)	227(5)	227
$8_1^+ \rightarrow 6_1^+$	83(3)	83(3)	90
$3/2_1^- \rightarrow 5/2_1^-$	955(104)	678(9)	740
$3/2_1^- \rightarrow 7/2_2^-$	133(13)	94(4)	130
$5/2_1^- \rightarrow 7/2_2^-$	-	83(10)	107
$5/2_1^- \rightarrow 9/2_1^-$	195^{+49}_{-37}	195(12)	279
$7/2_2^- \rightarrow 9/2_1^-$	400^{+43}_{-36}	419(13)	456
$11/2_1^- \rightarrow 7/2_2^-$	-	95(6)	102
$11/2_1^- \rightarrow 9/2_1^-$	141^{+9}_{-8}	149(7)	154
$11/2_1^- \rightarrow 13/2_1^-$	-	266(6)	252
$13/2_1^- \rightarrow 9/2_1^-$	213^{+18}_{-15}	226(11)	291
$15/2_1^- \rightarrow 11/2_1^-$	127(22)	167(4)	169
$15/2_1^- \rightarrow 13/2_1^-$	28(6)	48(2)	50
$17/2_1^- \rightarrow 13/2_1^-$	300(20)	306(6)	332
$17/2_1^- \rightarrow 15/2_1^-$	-	86(4)	82
$21/2_1^- \rightarrow 17/2_1^-$	198(7)	173(3)	191

a: from Ref. [6] and references therein
b: Shell model calculations as described in Ref. [5]

It is worth to make some reflections on this result. First of all, the approach is strongly based on symmetries, which provide perhaps the most important tools to study atomic nuclei [7]. Secondly, the semi-empirical approach gives a large weight to the experimental observations, whether they are understood or not. That this combination works is apparent because the results for ^{211}At are very similar to the full shell model calculation, which incorporates many more degrees of freedom. One inherent advantage of the method is that the errors of the used two-particle (hole) experimental data can be transferred via error propagation into the predictions for higher particle numbers. This is very similar to what is done in a very different approach of nuclear structure: Effective Field Theory (EFT). There one starts from experimental data on the nucleon-nucleon force from scattering experiments such as phase shifts and polarization properties in order to determine by symmetry dictated interaction parameters. Those are then used to predict further properties with theoretical errors.

4. Ongoing experiments on N = 50 isotones

Motivated by the success of the description of ^{211}At a further candidate for the semi-empirical calculations are the $(\pi g_{9/2})^n$ configurations for the N = 50 isotones above ^{90}Zr. Compared to the N = 126 isotones the situation is less favourable due to the low-lying $\pi 2p_{1/2}$ single-particle state in ^{91}Nb. Nevertheless, it is of interest to investigate whether the semi-empirical approach could be valid here.

Two independent experiments using different nuclear reactions were used to populate excited states in ^{92}Mo at the Cologne 10 MV FN Tandem accelerator. In the first experiment the states were populated using a ^{90}*Zr(α,2n)* fusion-evaporation reaction at a beam energy of 27 MeV and the second experiment used a ^{93}*Nb(p,2n)* reaction at an energy of 18 MeV. Both experiments were measured using the HORUS spectrometer fast-timing configuration (details can be found in Ref. [8]).

Table 2. Experimental and calculated *B*(E2) values in [e^{2}fm^4] for transitions in ^{92}Mo , ^{93}Tc , ^{94}Ru and ^{95}Rh.

	$J_i^\pi \rightarrow J_f^\pi$	Exp.[a]	Single-*j*
^{92}*Mo*	$2_1^+ \rightarrow 0_1^+$	207(12)	207(12)
	$4_1^+ \rightarrow 2_1^+$	132^{+7}_{-6}	132^{+7}_{-6}
	$6_1^+ \rightarrow 4_1^+$	81(2)	81(2)
	$8_1^+ \rightarrow 6_1^+$	28.6(3)	28.6(3)
^{93}*Tc*	$17/2_1^+ \rightarrow 13/2_1^+$	88(18)	114(3)
	$21/2_1^+ \rightarrow 17/2_2^+$	73(5)	61(1)
^{94}*Ru*	$2_1^+ \rightarrow 0_1^+$	165(80)	186(4)
	$4_1^+ \rightarrow 2_1^+$	38(3)[b],103(24)[c]	7.8(7)
	$6_1^+ \rightarrow 4_1^+$	3.0(2)	3.9^{+5}_{-4}
	$8_1^+ \rightarrow 6_1^+$	0.09(1)	1.0(2)
^{95}*Rh*	$21/2_1^+ \rightarrow 17/2_1^+$	24(2)	1.0(2)
	$21/2_1^+ \rightarrow 17/2_2^+$	113(13)	176(3)

a: from Ref. [10] and references therein
b: Ref. [11]
c: Ref. [9]

The previously unknown value of the $B(E2{:}4_1^+ \rightarrow 2_1^+)$ in ^{92}Mo could be measured to be 132^{+7}_{-6} e^{2}fm^4 in contrast to the recent value of 84.3(14) e^{2}fm^4 from Ref. [9] using radioactive-ion beams. This value together with other improved values for yrast *B(E2)* values allowed then the prediction for all other N = 50 isotones to be made [8]. Table 2 compares these with experimental values when available. The comparison shows a reasonable agreement with the known *B(E2)* values, except for the $B(E2{:}4_1^+ \rightarrow 2_1^+)$ value in ^{94}Ru, where the theory underpredicts the two contradictory experimental values, and the $21/2_1^+ \rightarrow 17/2_1^+$ value in ^{95}Rh.

Because only eight *B(E2)* values of interest are known in ^{93}Tc, ^{94}Ru and ^{95}Rh new experiments using the fast-timing setup in Cologne were started using the $^{90}Zr(^{6}Li, 3n)^{93}Tc$ @ 31MeV and $^{92}Mo(^{4}He, 2n)^{94}Ru$ @ 28MeV reactions. Improved values for the $17/2^{+}_{1}$ and 4^{+}_{1} state lifetimes in ^{93}Tc and ^{94}Ru could be obtained [10] solving in the latter case the strong discrepancy between two radioactive-ion beam experiments [9,11]. Fig. 2 illustrates the excellent statistics that could be obtained in the very clean triple coincidence spectra. Due to this and the reliability of fast-timing experiments the deduced $B(E2:4_1^+ \rightarrow 2_1^+)$ of 50(2) e^{2}fm^4 is to be preferred over the values in Table 2. The still remaining discrepancy with the theoretical predictions can be related to seniority mixing between $v = 2$ and $v = 4$ states as discussed in Ref. [8].

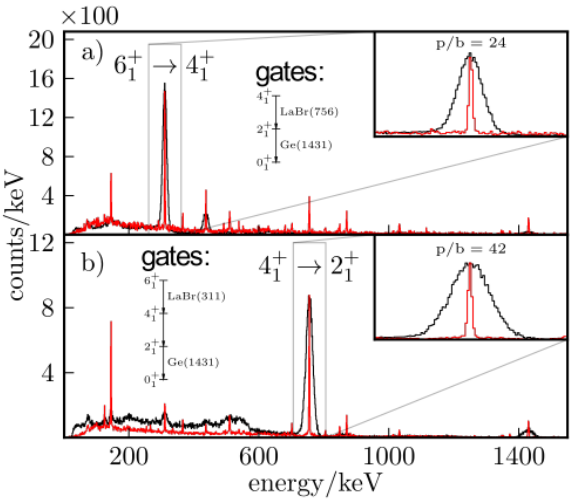

Fig. 3. Triple coincidence spectra with Ge-LaBr-LaBr coincidences shown in black and Ge-LaBr-Ge coincidences shown in red, showing the very clean transitions used to determine the lifetime of the 4_1^+ state in ^{94}Ru using the $^{92}Mo(^{4}He, 2n)^{94}Ru$ reaction[10].

5. Conclusions and outlook

Nine lifetimes in ^{211}At were measured in three different experiments. Excellent agreement with the semi-empirical single-*j* predictions for the *B(E2)* values in ^{211}At was obtained when using the ones in ^{210}Po as input. In order to perform the same for the N = 50 isotones, precise lifetimes in ^{92}Mo were determined to serve as input for the calculations with more than two protons. A fast-timing experiment in ^{93}Tc yields promising results but more *B(E2)* values are still needed. The $B(E2:4_1^+ \rightarrow 2_1^+)$ in ^{94}Ru was measured to solve contradictory results from radioactive-ion beam experiments, but it still disagrees with the single-*j* predictions. Much more stable and radioactive-ion beam experiments are needed to understand the role of seniority in the structure of the proton-rich N = 50 isotones.

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