New insight into the lens design landscape

Florian Bociort*

Dept. of Imaging Physics, Faculty of Applied Sciences, TU Delft, 2628CJ Delft, Netherlands

Abstract. Novel formulas have been derived for the primary spherical aberration, coma and axial color of systems of thin lenses in contact. Even in complex optical systems, groups of lenses can be modelled as thin lenses in contact. The new mathematical formalism helps explaining significant qualitative properties of the lens design landscape.

1 Introduction

The assertion that the venerable theory of primary aberrations still has potential for significant new insights may surprise lens designers. However, we show that the novel formalism described below provides a simple explanation both for established (but insufficiently understood) and for recent findings. Consider a system of *L* thin lenses, all with the same refractive index *n*, in air and in contact with each other (i.e. all axial thicknesses are set to zero). Below, all *N=2L* lens surfaces are considered spherical and have variable curvatures c_k , but aspheres can be included. The system has the fixed total power *K* and has the fixed marginal ray angles α in the object space and $\alpha - hK$ in the image space, where h is the common marginal ray height at all surfaces. Because groups of lenses in a larger system can also be modelled as thin lenses in contact, this new formalism can explain properties of the design landscape even for highly complex systems such as lithographic objectives.

2 New variables

We introduce new surface variables z_k that, as shown below, reveal a more straightforward relationship between the primary aberrations and the constructional data than in the standard formalism. The new variables satisfy the constraint

$$\sum_{k=1}^{N} z_k = 1. \quad (1)$$

When all *N* *z*-values of the system are found, the powers P_{2m-1} for the first and P_{2m} for the second surface of each lens *m* turn out to be

$$P_{2m-1} = K\left[z_{2m-1} + \frac{2(n^2-1)}{(n+2)}\left(\sum_i^{2m-1} z_i - \frac{\alpha}{hK}\right)\right] \quad (2)$$

$$P_{2m} = K\left[z_{2m} - \frac{2(n^2-1)}{(n+2)}\left(\sum_i^{2m-1} z_i - \frac{\alpha}{hK}\right)\right] \quad (3)$$

respectively, and the corresponding surface curvatures are then $c_k = P_k/\Delta n$. Because the power of lens m, $\bar{P}_m = P_{2m-1} + P_{2m} = K(z_{2m-1} + z_{2m})$ is proportional to the sum of the two corresponding *z*-values, the variables z_k are called quasi-powers.

* f.bociort@tudelft.nl

3 New primary aberration formulas and why they are useful

The Seidel sum for the 3rd-order spherical aberration is derived from well-known aberration formulas [1] as

$$S = \frac{h^4 K^3 n}{3(n+2)}\left[\frac{(2n+1)^2}{(n-1)^2}\sum_{k=1}^{N} {z_k}^3 - 3\left(\frac{\alpha}{hK}\right)^2 + 3\frac{\alpha}{hK} - 1\right]. \quad (4)$$

(However, the derivation is non-trivial.) As an example, S has a minimum when we have equal values $z_k = 1/N$ for all N surfaces. For *N=2* we recover the well-known singlet with optimal bending that has minimal spherical aberration [1]. For a quartet (*N=8*) with object at infinity (i.e., $\alpha = 0$) and all $z_k = 1/8$**,** the square bracket for the minimal S value becomes zero for *n=1.5*. This is the remarkable Fulcher quartet [2] in which zero spherical aberration was achieved with four spherical lenses having equal powers (but different bendings that with the present approach result immediately from Eqs. (2) and (3)). If this system is chosen as starting point for further design, then the resulting design is arguably the most relaxed system (or lens group) encountered in lens design [3]. Such Fulcher groups can also be observed in a slightly modified form in e.g. both bulges of dioptric lithographic objectives.

As shown in detail in the special case of doublets and triplets when spherical aberration S is the most significant aberration present, the new formalism helps answering a fundamental question in lens design: why do we have so many local minima in the design landscape? Because in Eq. (4) the quasi-powers appear as a sum of cubes, if a certain set of variables z_k corresponds to a local minimum, then any permutation of these variables will have the same value of S and will correspond to a different minimum. This permutation symmetry increases the number of existing minima significantly. This symmetry was not observed earlier, because of the sequential character of ray propagation (first through surface 1, then through surface 2 etc.). However, the present formalism reveals this symmetry because the sequential character of ray propagation is completely absorbed in Eqs. (2) and (3).

When the aperture stop is at the lens, then the Seidel sum for the 3rd-order coma given by

$$C = \frac{-\bar{A} h^3 K^2}{2(n+2)}\left(\frac{(n+1)(2n+1)}{n-1}\sum_{k=1}^{N}(-1)^k {z_k}^2 - 2\frac{\alpha}{hK} + 1\right). \quad (5)$$

where $\bar{A} = n\bar{\imath}$ is the (constant) refraction invariant for the chief ray. In the example above, because N is even for all systems with $z_k = 1/N$, including the Fulcher quartet, the sum over k in Eq. (5) is zero. As expected, coma is zero for $\alpha = hK/2$, which corresponds to the case of equal conjugates. For an arbitrary stop position, the stop-shift formulas can be used [1].

Using the Abbe number V_m for lens m, the axial color is given by

$$A = h^2 K \sum_{m=1}^{L} V_m^{-1}(z_{2m-1} + z_{2m}). \quad (6)$$

The remaining primary aberrations can be handled as usually [1]. We also show how the new formalism enables a simpler explanation of the four well-known achromatic doublet configurations (Fraunhofer, Gauss, Reversed Gauss and Steinheil).

References

1. W.T. Welford, Aberrations of optical systems. Adam Hilger, Bristol, (1986)
2. G. S. Fulcher, Telescope objective without spherical aberration for large apertures, consisting of four crown glass lenses. JOSA **37**(1), 47-54 (1947)
3. D. Shafer, Optical Design and the Relaxation Response. Proc. SPIE **0766**, 2-9 (1987)