EPJ Web of Conferences **340**, 02001 (2025)
Powders & Grains 2025 https://doi.org/10.1051/epjconf/202534002001

Rheology of dense suspensions of non-Brownian soft particles across flow regimes

Franco Tapia[1,2,*], *Chong-Wei* Hong[1,**], *Pascale* Aussillous[1,***], and *Élisabeth* Guazzelli[3,****]

[1]Aix-Marseille Université, CNRS, IUSTI, Marseille, France
[2]Institute of Urban and Industrial Water Management, TU Dresden, Dresden, Germany
[3]Université Paris Cité, CNRS, MSC, Paris, France

Abstract. This study investigates the rheological behavior of soft spherical particles across the jamming transition and within both viscous and inertial flow regimes. We extend the established granular rheology for hard sphere suspensions to develop a soft granular rheology by adjusting the critical volume fraction and friction coefficient to account for pressure variations, and by incorporating the combined effects of viscous and inertial stress scales. This framework effectively characterizes dense suspensions across a broad spectrum of sizes and flow conditions, from colloidal to granular. The implications for suspension flow dynamics and sediment transport processes are also explored.

1 Introduction

Understanding dense particulate systems [1–5], particularly near the jamming transition from fluid-like to solid-like states, presents significant challenges. Most research has concentrated on hard grains, highlighting the necessity to investigate soft particle systems, including colloids, microgels, emulsions, micelles, foams, and soft granular materials. These systems exhibit weak elasticity at rest and low stress, transitioning to liquid-like behavior upon surpassing a yield stress [6–8]. Unlike hard grains, soft particles can jam at high concentrations but also unjam and flow under shear. Previous studies have examined the rheology near the jamming transition, revealing power-law scalings indicative of critical behavior [9–16].

This paper surveys rheological studies of soft spherical particles across the jamming transition and within viscous and inertial regimes. Utilizing hydrogel spheres (with small but non-zero interparticle friction) suspended in fluids of varying viscosity and a custom rheometer [17, 18], we demonstrate that the soft particulate system can be described by a soft granular rheology. This approach involves renormalizing the critical volume fraction and friction coefficient to pressure-dependent values [15] and employing stress additivity across flow regimes [19–21]. The soft granular rheology effectively captures the behavior on both sides of the jamming transition, leading to an approximate data collapse into two branches when scaled with the distance to jamming [22]. Finally, we explore the potential application of this rheology to novel flow dynamics, such as bedload sediment transport.

2 The soft granular rheology

The rheological response of a sheared particulate system under confining pressure P is characterized by the effective friction coefficient $\mu = \tau/P$ (where τ is the shear stress) and the packing fraction ϕ. In the inertial regime, these quantities depend on $I^2 = \rho_p d^2 \dot{\gamma}^2 / P$ [1], i.e. the ratio of the inertial stress scale to the confining pressure, while in the viscous regime, the relevant control parameter is $J = \eta_f \dot{\gamma}/P$ [17], i.e. the ratio of the viscous stress scale to the confining pressure (where $\dot{\gamma}$ is the shear rate, d and ρ_p are the particle size and density, and η_f is the fluid viscosity). Unifying these regimes through stress additivity yields a single dimensionless form $J + \alpha I^2$, where α is related to the Stokes number at the viscous-inertial transition [19–21]. Experimental findings indicate different values for α related to ϕ and μ, reflecting the slower viscous-inertial transition of shear stress compared to pressure [21].

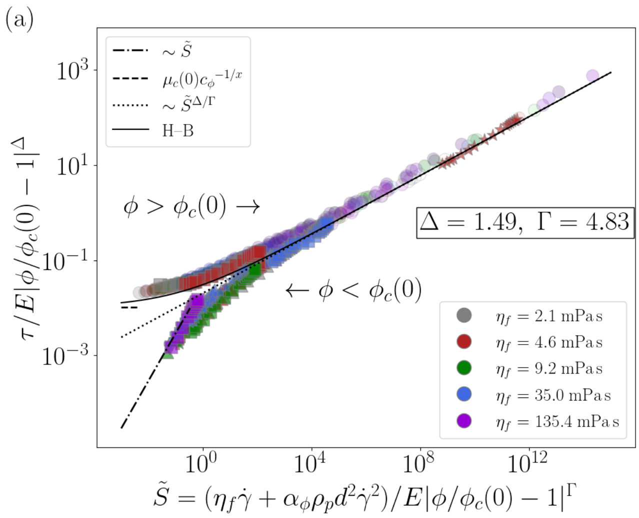
Insights from hard-sphere suspensions [21] provide a framework for analyzing the nonlinear rheology of soft-sphere suspensions across the jamming transition [22]. The effects of particle softness resemble those of finite pressures, leading to shifts in the jamming packing fraction, ϕ_c, and minor adjustments in the effective friction coefficient, μ_c. These corrections can be integrated into a generalized soft granular rheology, maintaining the rheological function for ϕ and μ while retaining the values of α from hard spheres. The pressure dependence of the critical volume fraction and friction coefficient can be described by power laws of the pressure (more precisely by P that has been scaled by the Young's modulus E of the hydrogels) [15, 22].

This soft granular rheology encompasses the behavior of non-Brownian soft particles across the jamming transition, accounting for yield stress above jamming, divergent

---

[*] e-mail: franco.tapia@tu-dresden.de
[**] e-mail: chong-wei.hong@univ-amu.fr
[***] e-mail: pascale.aussillous@univ-amu.fr
[****] e-mail: elisabeth.guazzelli@u-paris.fr

https://doi.org/10.1051/epjconf/202534002001



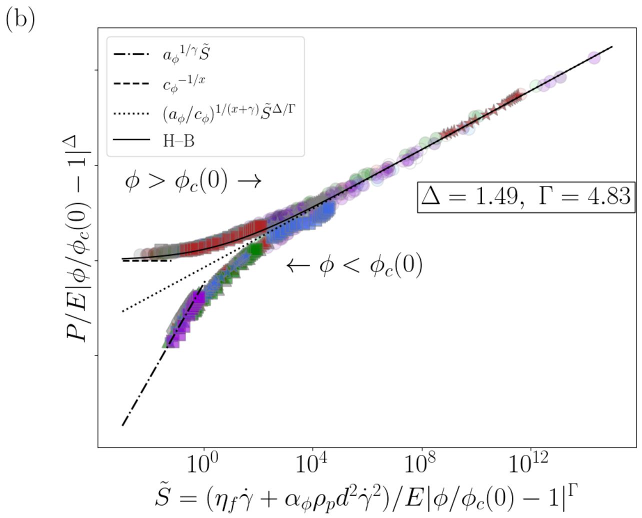


Figure 1. Collapse of the scaled (a) τ and (b) P against the scaled addition of stress scales with $\tilde{S} = (\eta_f\dot{\gamma} + \alpha_\phi\rho_p d^2\dot{\gamma}^2)/E|1 - \phi/\phi_c(0)|^\Gamma$ using the critical exponents Δ and Γ, for data collected in pressure- and volume-imposed rheometry. The values for $\phi_c(0) = \phi_c(\frac{P}{E} = 0)$ are close to ≈ 0.64 within $\pm 5\%$.

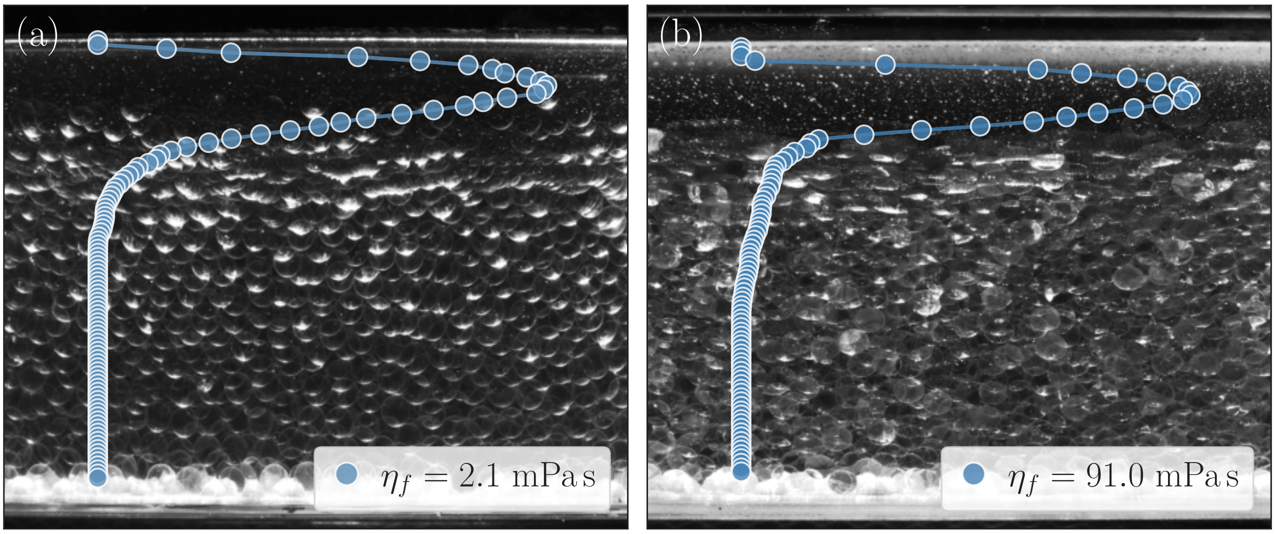


Figure 2. Velocity profile over a long-exposure image of bedload transport experiment with hydrogels particles. (a) $\eta_f = 2.1$ mPa s with a flow input $Q = 59.9$ cm s^{-1} at $t = 10$ s and (b) $\eta_f = 91.0$ mPa s with a flow input $Q = 13.3$ cm s^{-1} at $t = 36$ s.

viscosity below, and shear thinning near the transition. The data collapse into two curves (above and below jamming) when stresses and shear rates are scaled as power laws of the distance to jamming as illustrated in figure 1. The critical exponents are derived from the soft granular rheology, with the upper branch approximating a generalized Herschel-Bulkley law and the lower branch behaving as a power law, merging with the upper branch near the jamming transition.

3 Application to bedload transport of soft suspensions and concluding remarks

This work demonstrates that the granular rheology for hard sphere suspensions can be extended to a soft granular rheology by renormalizing the critical volume fraction and friction coefficient to pressure-dependent values and utilizing the additivity of viscous and inertial stress scales. The near-jamming collapse observed has been previously noted in the viscous regime and is now shown to apply to dense suspensions across the entire size and flow range.

These findings have practical implications for various industries that utilize soft matter systems and for understanding geophysical flows involving complex materials. To further investigate the potential applications of soft granular rheology in different flow contexts, a novel flow configuration is examined. This configuration involves a sheared sediment bed of particles within a pressure-driven, linear channel flow, as illustrated in figure 2. This question has been examined for hard spherical particles in the viscous regime [23–25] and is now extended to soft hydrogel spheres in both the viscous and inertial flow regimes. The rheology of the mobile layer can be described by the soft granular rheology using the principle of the additivity of the viscous and inertial stress scales.

References

[1] Y. Forterre and O. Pouliquen, Flows of dense granular media. Annu. Rev. Fluid Mech. **40**, 1 (2008).

[2] É. Guazzelli and J. F. Morris, A Physical Introduction to Suspension Dynamics (Cambridge University Press, Cambridge, 2012).

[3] B. Andreotti, Y. Forterre, O. Pouliquen, Granular Media: Between Fluid and Solid (Cambridge University Press, Cambridge 2013).

[4] É. Guazzelli and O. Pouliquen, Rheology of dense granular suspensions. J. Fluid Mech. **852**, P1 (2018).

[5] É. Guazzelli, Rheology of dense granular suspensions across flow regimes. Phys. Rev. Fluids **9**, 090501 (2024).

[6] J. Seth, L. Mohan, C. Locatelli-Champagne, M. Cloitre, and R. Bonnecaze, A micromechanical model to predict the flow of soft particle glasses. Nat. Mater. **10**, 838 (2011).

[7] T. Liu, F. Khabaz, R. T. Bonnecaze, and M. Cloitre, On the universality of the flow properties of softparticle glasses. Soft Matter **14**, 7064 (2018).

[8] F. Khabaz, M. Cloitre, and R. T. Bonnecaze, Particle dynamics predicts shear rheology of soft particle glasses. J. Rheol. **64**, 459 (2020).

[9] P. Olsson and S. Teitel, Critical scaling of shear viscosity at the jamming transition. Phys. Rev. Lett. **99**,178001 (2007).

[10] T. Hatano, Scaling properties of granular rheology near the jamming transition. J. Phys. Soc. Jpn. **77**,123002 (2008).

[11] M. Otsuki and H. Hayakawa, Critical behaviors of sheared frictionless granular materials near the jamming transition. Phys. Rev. E **80**, 011308 (2009).

[12] B. P. Tighe, E. Woldhuis, J. J. C. Remmers, W. van Saarloos, and M. van Hecke, Model for the scaling of stresses and fluctuations in flows near jamming. Phys. Rev. Lett. **105**, 088303 (2010).

[13] K. N. Nordstrom, E. Verneuil, P. E. Arratia, A. Basu, Z. Zhang, A. G. Yodh, J. P. Gollub, and D. J. Durian, Microfluidic rheology of soft colloids above and below jamming. Phys. Rev. Lett. **105**, 175701 (2010).

[14] J. Paredes, M. A. J. Michels, and D. Bonn, Rheology across the zero-temperature jamming transition. Phys. Rev. Lett. **111**, 015701 (2013).

[15] T. Kawasaki, D. Coslovich, A. Ikeda, and L. Berthier, Diverging viscosity and soft granular rheology in non-Brownian suspensions. Phys. Rev. E **91**, 012203 (2015).

[16] S. Chialvo, J. Sun, and S. Sundaresan, Bridging the rheology of granular flows in three regimes. Phys. Rev. E **85**, 021305 (2012).

[17] F. Boyer, É. Guazzelli, and O. Pouliquen, Unifying suspension and granular rheology, Phys. Rev. Lett. **107**, 188301 (2011).

[18] F. Tapia, O. Pouliquen, and É. Guazzelli, Influence of surface roughness on the rheology of immersed and dry frictional spheres. Phys. Rev. Fluids **4**, 104302 (2019).

[19] M. Trulsson, B. Andreotti, and P. Claudin, Transition from the viscous to inertial regime in dense suspensions. Phys. Rev. Lett. **109**, 118305 (2012).

[20] L. Amarsid, J.-Y. Delenne, P. Mutabaruka, Y. Monerie, F. Perales, and F. Radjaï, Viscoinertial regime of immersed granular flows. Phys. Rev. E **96**, 012901 (2017).

[21] F. Tapia, M. Ichihara, O. Pouliquen, and É. Guazzelli, Viscous to inertial transition in dense granular suspension. Phys. Rev. Lett. **129**, 078001 (2022).

[22] F. Tapia, C.-W. Hong, P. Aussillous, and É. Guazzelli, Rheology of suspensions of non-Brownian soft spheres across the jamming and viscous-to-inertial transitions. Phys. Rev. Lett. **133**, 088201 (2024).

[23] M. Ouriemi, P. Aussillous, and É. Guazzelli, Sediment dynamics. part 1. bed-load transport by laminar shearing flows. J. Fluid Mech. **636** , 295-319 (2009).

[24] P. Aussillous, J. Chauchat, M. Pailha, M. Médale, M., and É. Guazzelli, Investigation of the mobile granular layer in bedload transport by laminar shearing flows. J. Fluid Mech. **736**, 594-615 (2013).

[25] B. Vowinckel, E. Biegert, E. Meiburg, P., Aussillous, P., and É. Guazzelli, Rheology of mobile sediment beds sheared by viscous, pressure-driven flows. J. Fluid Mech. **921** , A20 (2021).