

Microstructural anisotropy as the origin of dilatancy

Ravi Gautam^{1,*} and Prabhu R. Nott^{1,**}

¹Department of Chemical Engineering, Indian Institute of Science, Bangalore 560012, India

Abstract. Dilation in sheared granular materials has been widely observed, yet its micromechanical origins remain debated. Using discrete element method (DEM) simulations of slow granular flows, we find that grain rearrangements occur intermittently, alternating between jammed phases, where the contact network remains stable, and yielding phases, characterized by rapid plastic rearrangements. Our analysis reveals that, under constant confining stress, granular materials dilate during jammed phases and compact during yielding phases. We show that as the material is sheared, the contact network develops anisotropic features. By incorporating these microstructural details into a macroscopic linear elastic framework, we derive a constitutive equation that explains the volumetric expansion in sheared granular materials. These findings highlight the essential role of elastic grain deformation and microstructural anisotropy in stress transmission and dilation, providing new insights into the mechanics of static and quasi-static granular materials.

1 Introduction

Granular materials are ubiquitous in nature and industry, examples being soil, sand, pharmaceuticals powders, and foodgrains. They exhibit diverse flow behaviors depending on deformation rate, ranging from quasi-static regime where inter-particle contacts dominate stress transmission, to rapid flow where stress is transmitted primarily through fleeting collisions [1]. The flow is classified into these regimes by the Savage number, a dimensionless parameter defined as

$$Sa = \rho_p d_p^2 \dot{\gamma}^2 / N,$$

where ρ_p is particle density, d_p is particle diameter, $\dot{\gamma}$ is nominal shear rate, and N is the characteristic stress. In the slow, quasi-static regime ($Sa \ll 1$), granular flows display several distinctive features, including rate-independent stress [2], large stress fluctuations [3], and dilatancy – the tendency of dense granular assemblies to expand in volume when sheared [4].

First observed by Reynolds in 1885 [4], granular dilatancy has been extensively documented in experimental studies [5]. The interplay between dilatancy and gravity has been shown to generate large-scale secondary vortices, demonstrating the crucial role of dilatancy in macroscopic flow patterns [6]. Despite decades of research, however, the precise micromechanical origins of dilation remain a subject of ongoing debate.

The classical explanation of dilatancy, proposed by Reynolds [4], relies on purely geometric considerations. According to this view, densely packed particles must climb over one another during shear, necessarily leading to volume expansion. While this geometric interpretation is intuitive for regular lattice arrangements where

the necessity of particles climbing over each other is easily visualized, its extension to random, disordered arrangements—which better represent real granular materials—proves significantly more complex. More fundamentally, the geometric interpretation fails to account for the dynamic evolution of disordered granular assemblies during continuous shear. This traditional view also overlooks key features of slow granular flows, such as elastic deformation at grain contacts and the characteristic intermittency observed in granular rearrangements. These limitations underscore the need for a more comprehensive micromechanical framework to understand dilatancy.

In this study, we investigate the micromechanical origins of dilatancy through discrete element method (DEM) simulations of slow granular flows. As reported previously [7], granular packing rearranges intermittently, with distinct “jammed” phases (predominantly elastic deformation) and “yielding” phases (plastic rearrangements). The jamming-yielding dynamics are referred to as stick-slip motion in the literature. Crucially, we reveal the previously unrecognized pattern that, when the confining stress is held constant, granular materials dilate during jammed phases and compact during yielding phases. By focusing on the jammed phases and incorporating the anisotropic nature of the contact network into a linear elastic framework, we develop a constitutive relationship that successfully explains both volumetric expansion under constant confining stress and stress evolution under constant volume conditions. Our findings highlight that microstructural anisotropy, combined with elastic grain deformation, plays a critical role in granular dilatancy – offering a perspective that extends beyond the traditional geometric interpretation and provides new insights into the mechanical behavior of granular materials.

*e-mail: ravigautam@iisc.ac.in

**e-mail: prnott@iisc.ac.in

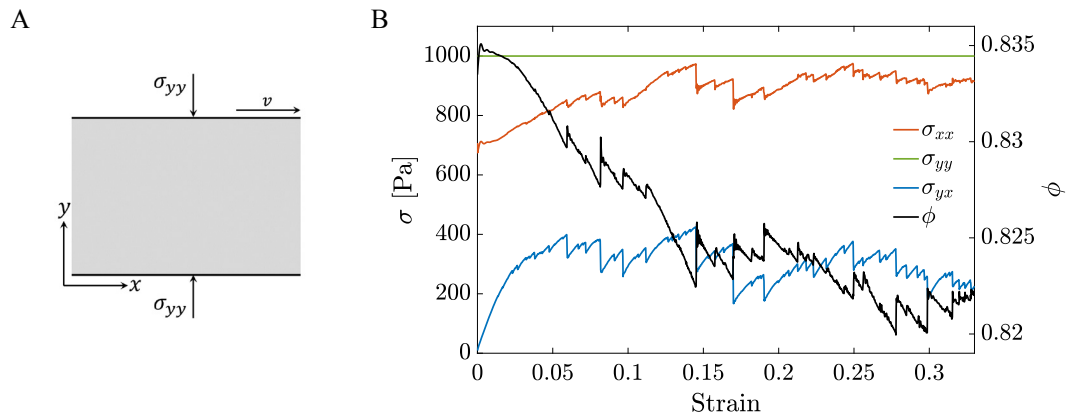


Figure 1. (A) Schematic of the simulation setup, where a granular assembly is subjected to a normal stress σ_{yy} and shear is applied by moving the top wall with a constant velocity v . (B) Evolution of stress components (σ_{xx} , σ_{yy} , σ_{yx}) and packing density ϕ as a function of shear strain. The fluctuations highlight the jamming-yielding dynamics, with dilation (a drop in ϕ) occurring during jammed phases.

2 Results and Discussion

We simulated a two-dimensional granular system under shear between parallel walls using the discrete element method (DEM). A schematic is presented in Figure 1A. The granular material consists of polydisperse disks with mean diameter $d_p = 1$ cm and uniform size distribution between $0.8d_p$ and $1.2d_p$ to ensure structural disorder. The walls are constructed from particles of diameter d_p arranged in linear arrays. The system extends $50d_p$ along the periodic flow direction, with the gap between two walls is approximately $40d_p$. Particle interactions were modeled using a linear spring-dashpot system in the normal direction and a spring-dashpot-Coulomb slider arrangement in the tangential direction to account for friction. The normal spring stiffness was set to $k_n = 10^5$ N/m, with tangential stiffness $k_t = 2k_n/7$. Damping coefficients were chosen to yield a coefficient of restitution of 0.7, with tangential damping coefficient $\gamma_t = \gamma_n/2$. Shear was applied by moving the top wall at a constant velocity v while maintaining a constant confining stress σ_{yy} , allowing the packing density ϕ to evolve freely. The corresponding Savage number for this set of parameters is 10^{-9} .

2.1 Dilation and jamming-yielding dynamics

As shear commences, the stress components (σ_{xx} and σ_{yx}) and packing density ϕ evolve toward statistically steady-state values. The temporal fluctuations in these parameters, illustrated in Figure 1B, capture the hallmark jamming-yielding dynamics of slow granular flows. The pattern of steady rises followed by abrupt drops in the stress components reveals alternating phases of elastic loading and sudden plastic yielding. During the jammed phases, the stress gradually increases as the contact network deforms elastically while maintaining its topological structure. Conversely, the abrupt stress drops indicate rapid rearrangements of grains as the material yields. The

jamming-yielding dynamics and corresponding stress fluctuations are inherent features of quasi-static granular flow and underpin the rate independence of stress [8].

The evolution of ϕ provides crucial insight into the volumetric response of the material. Notably, ϕ decreases steadily during jammed phases—indicating systematic dilation—while exhibiting sharp increases during yielding phases as grains rearrange into more compact configurations. This clear temporal correlation between elastic deformation and dilation challenges conventional interpretations that attribute volumetric expansion primarily to grain rearrangements. Instead, our observations reveal that dilation occurs precisely when the contact network remains topologically unchanged and undergoes predominantly elastic deformation. This finding suggests that an elastic framework is appropriate for accurately capturing shear-induced dilation, where the effective elastic moduli depend on the microstructural details of the contact network.

In loosely packed granular materials, the contact network is initially unstable, leading to rapid rearrangement of grains that compact the material as soon as shear is applied. Following this initial compaction, the stress components and packing density evolve in a manner similar to Figure 1B.

2.2 Anisotropy in the microstructure

Unlike crystalline solids, granular materials lack long-range translational order due to polydispersity and their athermal nature. However, by representing the granular assembly as a contact network – with particle centers as nodes and interparticle contacts as edges – we can quantify the microstructural anisotropy. From our simulation data, we extracted the orientation θ of each contact relative to the horizontal axis and calculated the probability distribution $P(\theta)$ of contact orientations.

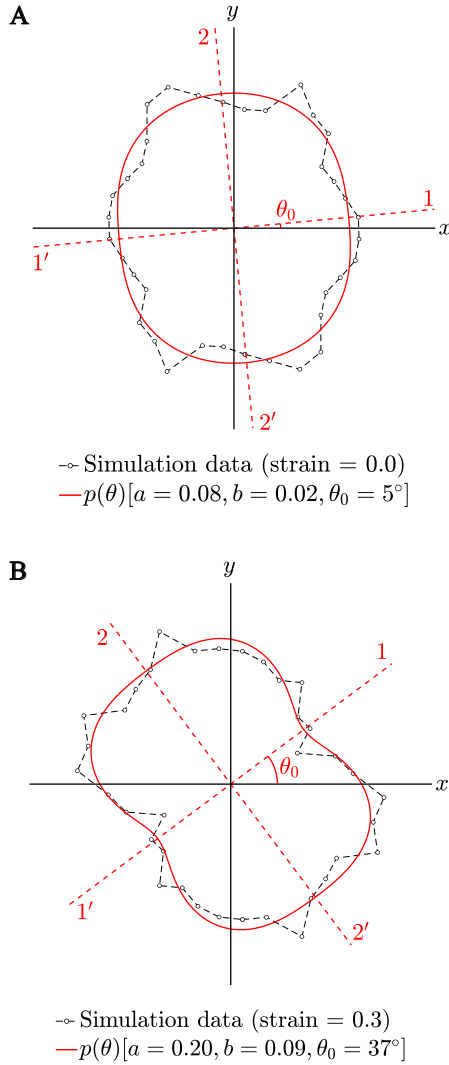


Figure 2. The contact angle probability distributions illustrating anisotropy in the contact network. (A) Distribution in the initial, pre-shear configuration. (B) A representative distribution during shear. The black dashed lines represent simulation data, while the red solid lines show fits using Equation 1. The red dashed lines (1 – 1' and 2 – 2') indicate the two planes of symmetry in the microstructure.

To characterize the anisotropy statistically, we fitted this distribution to a truncated Fourier series [9]:

$$p(\theta) = \frac{1}{2\pi} [1 + a \cos(2(\theta - \theta_0)) - b \cos(4(\theta - \theta_0))] \quad (1)$$

The coefficients a and b quantify the degree of anisotropy, while θ_0 indicates the principal direction of the contact network. This formulation inherently captures the two planes of symmetry in the distribution: one at θ_0 and another perpendicular to it. Figure 2A presents the polar representation of the contact orientation before the commencement of shear. Upon shearing, the contact network evolves, as depicted in Figure 2B, exhibiting both a reorientation of the principal direction and an increase in the anisotropy coefficients. This evolution highlights the development of anisotropy in the contact network as a result of shear.

3 Linear Elastic Model

The observation that shear strain in the jammed phase is accommodated elastically by the contact network suggests that the granular packing can be effectively described as a macroscopic elastic body. We propose adopting linear elasticity theory—the simplest possible approach—to model the deformation of granular material during these jammed phases. In linear elastic theory, stress σ varies linearly with strain ϵ , with the material stiffness tensor \mathbb{C} serving as the proportionality constant:

$$\sigma = \mathbb{C}\epsilon \quad (2)$$

In general, \mathbb{C} is a fourth-order tensor with 81 elements for a three-dimensional system. However, due to the symmetric nature of stress and strain tensors, along with the requirement for path independence during loading and unloading, the number of independent elements in \mathbb{C} reduces to 21 [10].

For a two-dimensional anisotropic material with two perpendicular planes of symmetry in the fabric (denoted by 1 – 1' and 2 – 2' in Figure 2A), the constitutive relation can be expressed in Voigt notation as:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ 2\epsilon_{12} \end{bmatrix} \quad (3)$$

The elements of the stiffness matrix C depend on the anisotropic coefficients a and b from Equation 1.

It is important to note that the stress and strain tensors in Equation 3 are expressed in the principal 1 – 2 coordinate frame, which is rotated by angle θ_0 from the laboratory $x - y$ coordinate frame. To apply our model to the simulated system, we must transform the constitutive equation to the $x - y$ frame:

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{yx} \end{bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} \\ C'_{12} & C'_{22} & C'_{23} \\ C'_{13} & C'_{23} & C'_{33} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ 2\epsilon_{xy} \end{bmatrix} \quad (4)$$

Now, consider a granular material confined between two parallel walls by a constant normal stress σ_{yy} , and sheared by moving one wall relative to another. During a jammed phase, an imposed shear strain increment $\Delta\epsilon_{yx}$ induces changes in stress components $\Delta\sigma_{xx}$ and $\Delta\sigma_{yx}$, as well as a change in normal strain by $\Delta\epsilon_{yy}$. Due to the periodic boundary conditions along the flow direction $\Delta\epsilon_{xx} = 0$, and since the normal stress is held constant $\Delta\sigma_{yy} = 0$. With these conditions, the stress-strain relation in Equation 4 takes the form

$$\begin{bmatrix} \Delta\sigma_{xx} \\ 0 \\ \Delta\sigma_{yx} \end{bmatrix} = \begin{bmatrix} C'_{11} & C'_{12} & C'_{13} \\ C'_{12} & C'_{22} & C'_{23} \\ C'_{13} & C'_{23} & C'_{33} \end{bmatrix} \begin{bmatrix} 0 \\ \Delta\epsilon_{yy} \\ 2\Delta\epsilon_{xy} \end{bmatrix}$$

Solving this system of equations yields

$$\Delta\epsilon_{yy} = \frac{\Delta\epsilon_{xy} \sin 2\theta_0 (\beta_1 \sin^2 \theta_0 - \beta_2 \cos^2 \theta_0)}{C_{22} - \sin^2 \theta_0 [2\beta_2 - (\beta_1 + \beta_2) \sin^2 \theta_0]} \quad (5)$$

where,

$$\begin{aligned}\beta_1 &= C_{11} - C_{12} - 2C_{33} \\ \beta_2 &= C_{22} - C_{12} - 2C_{33}\end{aligned}\quad (6)$$

The normal strain increment $\Delta\varepsilon_{yy}$ in Equation 5 is a measure of dilation in the system. For small deformations, where $\Delta\varepsilon_{yy} \ll 1$, the fractional change in packing density is given by $\Delta\phi \approx \Delta\varepsilon_{yy}$. The model also predicts an increase in σ_{xx} during the jammed phase, consistent with the evolution of stresses observed in simulations. These results establish that microstructural anisotropy in the jammed phases is the key feature that explains shear-induced dilatancy.

This calculation can be extended to predict the normal stress variations observed in the jammed phases of our simulations (see Figure 1B). A more detailed development of the anisotropic elastic model, along with its application to other systems, will be presented in our forthcoming work [11].

4 Conclusion

Our findings highlight that dilation in dense granular packings is intrinsically linked to the jamming-yielding dynamics with volumetric expansion occurring during the jammed phases. Starting from an approximately isotropic initial configuration, the contact network develops anisotropy as elastic shear strain accumulates due to shear imposed from the boundaries. This elastic shear strain is accompanied by volumetric deformation. In the statistically steady state, the buildup of elastic strain, anisotropy, and volumetric deformation during jammed phases is offset by their partial release during yielding phases. These insights refine our understanding of the micromechanical origins of dilatancy. We have presented an anisotropic linear elastic framework to model the elastic response of granular assemblies in the jammed phases. By integrating anisotropy into the stiffness matrix, the model captures key features of granular mechanics, including shear-induced dilation and normal stress variations.

The model is limited to elastic deformations in jammed phases and does not account for plastic rearrangements

during yielding events. Future work should focus on understanding the evolution of microstructure during plastic deformation to develop a more comprehensive description of granular flows. The proposed framework provides important insights into the role of anisotropy in granular mechanics and lays the groundwork for improved constitutive models of static and slow-flowing granular materials.

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