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# Homogenization and dispersion in granular flows

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**Abstract.** Granular mixing in continuous flow conditions remains poorly understood. We investigate mixing in a 2D bidisperse granular system stirred by a rod in a chaotic flow; by tracking individual particles, we characterize exponential stretching, a hallmark of chaotic advection. Dispersion analysis reveals that the concentration field variance follows an exponential decay, confirming efficient mixing. Our results establish a link between granular mixing and fluid chaotic advection, suggesting that multidirectional shear enhances dispersion.

## 1 Introduction

Mixing is crucial in many industrial processes. In industries such as glass, concrete, or abrasive manufacturing, mixing steps can lead to local inhomogeneities and defects, ultimately reducing the quality of the final product.

Fluid mixing has been extensively studied as an interplay between advection and diffusion. In particular, chaotic advection is highly effective, as Lagrangian dynamics promote the rapid dispersion of fluid particles, allowing molecular diffusion to act across relevant length and time scales [1]. The contributions of advection and diffusion are characterized by the Péclet number, a dimensionless quantity derived from the advection-diffusion equation [2].

A useful framework for analyzing fluid mixing involves modeling it as the stretching of lamellae affected by diffusion, which can be quantitatively assessed through the concentration field [3]. Additionally, the evolution of the concentration field variance serves as a metric of mixing states to characterize different chaotic flow protocols [4]. While fluid mixing has been widely explored, research on granular mixing remains limited, with most studies focusing on mixing and segregation in high-energy systems such as rotating drums [5].

Studies on granular mixing in smooth flow conditions are still scarce. Dense granular flows can be described by friction and dilatancy laws [6]. In such systems, the movement of the grains with respect to the mean flow is primarily driven by shear-induced diffusion, where the diffusion coefficient is not constant but depends on characteristic length and time scales, such as particle diameter and shear rate [7]. Scaling laws further account for the influence of the inertial number on diffusion [8]. However, most of these studies focus on linear shear conditions, leaving other types of smooth granular flows largely unexplored [9].

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## 2 Experiments

In this work, we investigate a different type of flow. We designed a 2D horizontal system with a smooth circular boundary of diameter $H = 250$ mm, in which we placed disks of two different sizes: $d_s = 5$ mm and $d_l = 6$ mm, to prevent crystallization. The system is stirred using a rod of diameter $d_r = 16$ mm (Figure 1). The number of small and large particles was chosen so that they cover equal surface areas. The global packing fraction is given by $\phi = \frac{Q_s \cdot d_s^2 + Q_l \cdot d_l^2}{H^2 - d_r^2} = 0.82$, where $Q_s = 1067$ and $Q_l = 681$ represent the number of small and large grains, respectively.

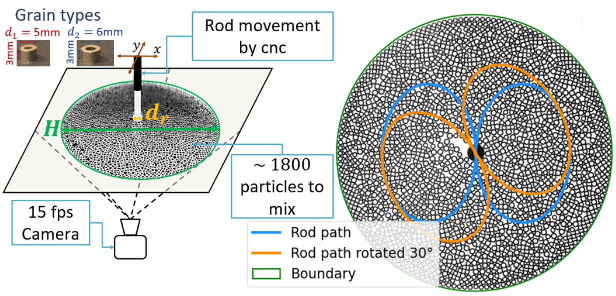


**Figure 1.** Experimental setup consisting of a 2D horizontal granular system stirred by a rod following a figure-eight trajectory. The system contains bidisperse disks confined within a circular boundary.

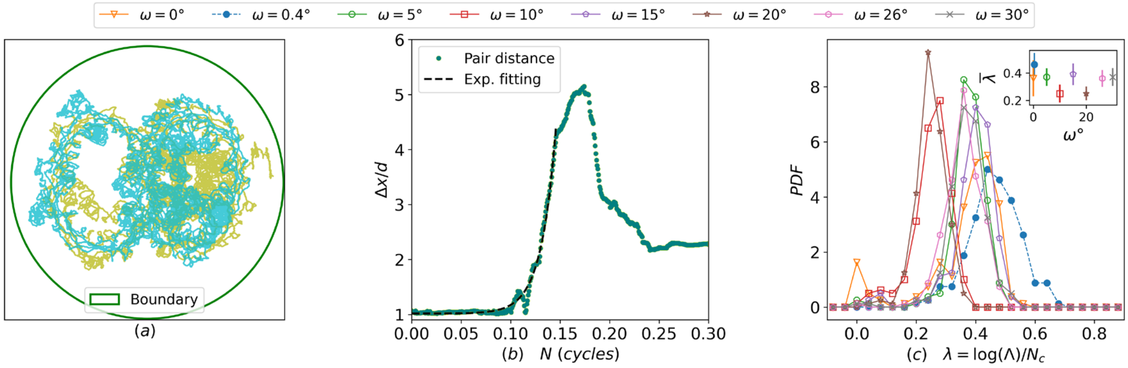


**Figure 2.** (a) Particle trajectory visualization for a single pair of grains initially close together. (b) Exponential stretching of the distance between particles, normalized by the mean diameter $d$, during the phase of the cycle when the intruder passes close to the pair. (c) Distribution of Lyapunov exponents for different stirring protocols corresponding to various rotation angles $\omega$. Inset: Average Lyapunov exponent as a function of $\omega$

The dispersion of particles is induced by a figure-eight stirring protocol executed by a CNC-controlled system, which moves the rod at a constant velocity of $1.2cm/s$. In addition, at the end of each cycle the axis of the figure-eigth protocol is rotated by an angle $\omega$. Each stirring cycle takes approximately one minute. A grayscale camera records the experiment at 15 fps.

For normalization purposes, we define a mean particle diameter as $d = \frac{Q_s \cdot d_s + Q_l \cdot d_l}{d_s + d_l}$.Time is represented in terms of the number of periodic cycles, denoted as $N$.

## 3 Results and Discussions

Using image analysis, we track the positions of the particles over approximately 300 mixing cycles. Taking a Lagrangian approach, we track individual grains in the system. By analyzing the distance between a single pair of particles that start in close proximity (Figure 2a), we measure how the distance between the particles evolved. We can measure the stretching during the phase of the cycle when the intruder moves near them. Figure 2b shows an example with a clear exponential increase of the distance following $\Delta x(N) = \Delta x(N = 0) \cdot \exp(\lambda N)$, where $\lambda$ is the stretching rate, also called Lyapunov exponent. When the distance is large enough to let a new disk insert in the middle of the segment, we redefine the pair, keeping only one of the previous pair and the new inserted disk, to ensure a precise measurement of the stretching.

By repeating this analysis for 200 particle pairs over many cycles, we obtain a distribution of the accumulated stretching rates, or Lyapunov exponents, for different protocols (Figure 2c). This distribution follows a bell curve, indicating that the system exhibits ergodicity over long timescales. The measured Lyapunov exponent remains largely unchanged with increasing rotation angle, except at specific points where resonance effects emerge due to grain recirculation around the moving intruder.

Since exponential stretching is a hallmark of effective mixing, we extend the fluid approach of chaotic advection [4] to our granular system. To characterize mixing, we artificially assign black and white colors to a subset of particles and analyze the dispersion of black particles over time (Figure 3a). A key challenge is that granular media are discontinuous. To define a local concentration, we usually can use a Voronoï tesselation based on the position of the disks. We chose here another approach: with a coarse graining involving a Gaussian filter of size $0.5d$, we create a smooth concentration field composed of gray levels.

Initially, the intensity distribution exhibits two peaks arround zero and close to the unity, which gradually decrease as particles disperse under the figure-eight stirring protocol. After 300 cycles, the system reaches a homogeneous state, as evidenced by the distribution of gray levels (Figure 3b). Indeed, the final distribution of concentration corresponds to a spatial random distribution of Gaussian of size $d/2$. The variance of gray levels is noted $\sigma_l$ in the steady state and is removed from the variance. The evolution of the concentration field variance, when normalized, follows an exponential decay, another signature of chaotic advection. By fitting an exponential function, we obtain the characteristic mixing time (Figure 3c). The mixing time $N_i = 11$, represents the exponential decay in the central region of the experiment, where the influence of non-sheared borders is negligible at low cycle counts. The inset of Figure 3c shows the variation of mixing time as a function of the rotation angle $\omega$ imposed to the stirring motion, i.e., the global rotation of the system. These results suggest that shearing in multiple directions enhances diffusion and accelerates mixing. However, comparing with results in Newtonian fluid [4], the effect of the rotation is less clear here in the inset of Figure 3c. This could be due to the slip velocity induced by the smooth boundary of the system. In addition, the curve is not monotonous, suggesting resonances between the frequency of the rotation and the dispersion by the rod.

Since shear-induced diffusion enhances mixing, we explored a simple description for the mixing time in regions of high stretching. Based on PIV analysis of the particle trajectories inspired by [10], we examined the velocity

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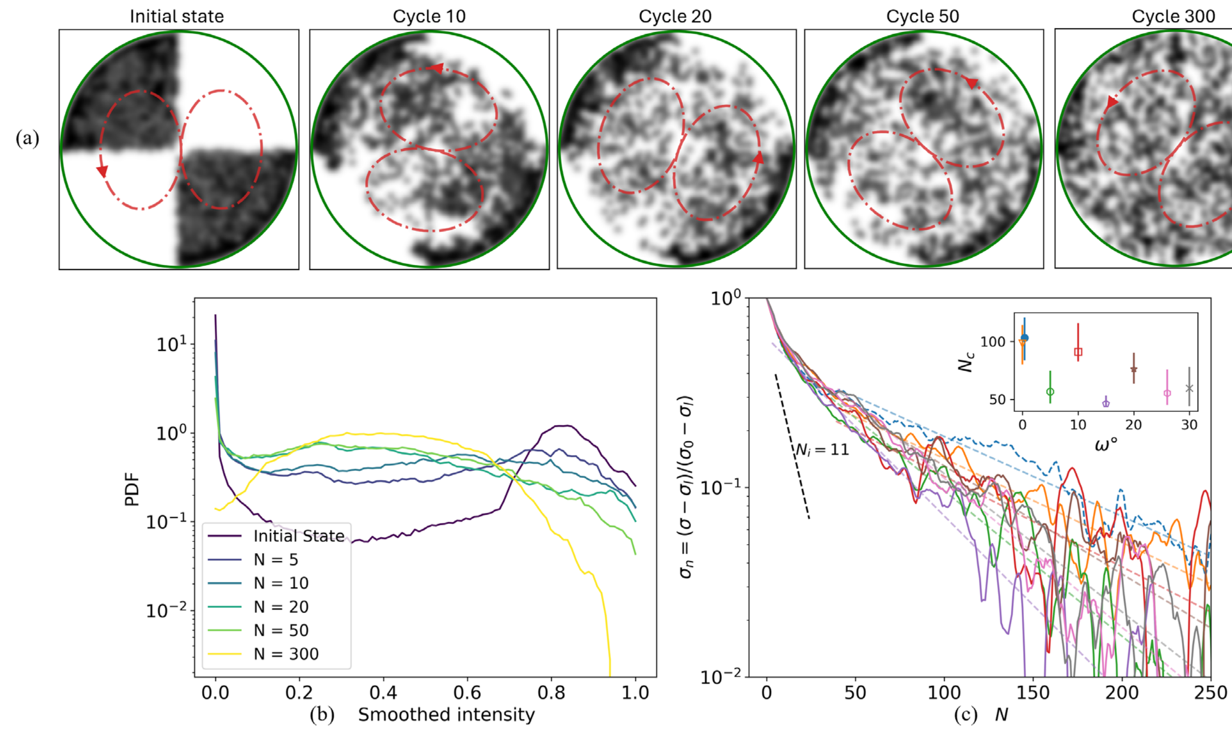


**Figure 3.** (a) Concentration field with artificially assigned black and white particles after gaussian filter application for $\omega = 10°$. (b) Evolution of the intensity distribution over time, leading to homogenization. (c) Exponential decay of the normalized concentration field variance. Inset: Variation of mixing time as a function of the rotational angle of the stirring motion.

profile in the direction perpendicular to the motion of the rod (see Figure 4). We observed that the rod effectively drags particles over a distance of approximately 3 particle diameters.

We estimate the local Péclet number as $Pe = \dfrac{U \cdot L}{D}$ where $U$ is a characteristic velocity, $L$ a characteristic length scale, and $D$ the diffusion coefficient. In our case, we take $U = v$, the velocity of the rod, and $L = 3d$, corresponding to the typical mixing length scale, and the diffusion coefficient $D = f(\phi) \cdot \dot{\gamma} \cdot d^2$ modeled from [9], where, assuming linear shear close to the rod, $\dot{\gamma} = v \cdot (3d)^{-1}$ Substituting this into the expression for $D$, we obtain $D = f(\phi) \cdot v \cdot (3d)^{-1} \cdot d^2$. Therefore, the Péclet number becomes $Pe = \dfrac{v \cdot 3d}{f(\phi) \cdot v \cdot (3d)^{-1} \cdot d^2} = \dfrac{9}{f(\phi)}$.

This result shows that $Pe$ is independent of the intruder velocity and depends only on the number of grains dragged by the rod, which is proportional to $\dfrac{d_r}{d}$.

Using the expression from [11], $f(\phi) = 0.027 \cdot (\phi_c - \phi)^{-1/2}$, with $\phi_c \approx 0.84$, we estimate $Pe \approx 47$. From the fluid mixing framework, the mixing time can be approximated as $t_m \approx \dfrac{1}{\lambda} \ln(Pe)$ where $\lambda$ is the finite-time Lyapunov exponent [12]. Based on our measurements, $\bar{\lambda} = 0.36$ cycles$^{-1}$, so $t_m \approx \frac{1}{0.36} \ln(47) \approx 10.7$ cycles. This prediction matches well with the observed value of $N_i = 11$ cycles measured in the central region of the experiment, where the influence of non-sheared borders is neglected. It suggests that in regions of high shear, the mixing process is governed by shear-induced diffusion, and that the mixing time can be predicted from local properties. To generalize this approach to the entire system, a more complete description is still needed. Preliminary simulations changing the intruder's velocity and size show that the mixing time remains constant with varying velocity, but decreases as the rod size increases.

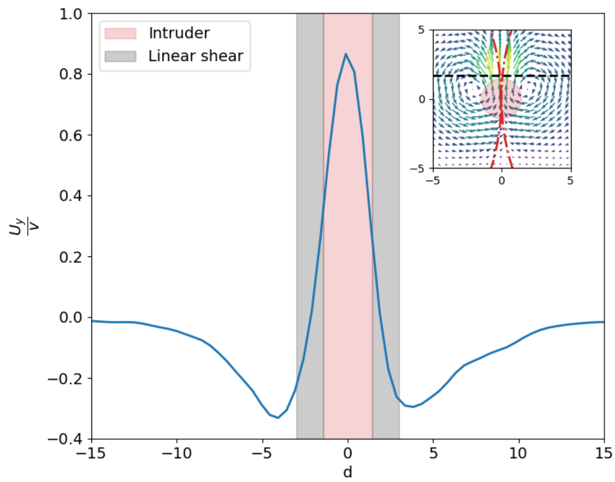


**Figure 4.** Mean velocity profile over 50 cycles measured perpendicular to the rod's direction (indicated by the dashed black line in the inset) at the center of the experiment. A linear shear zone is observed within a dragged region of approximate length scale $\sim 3d \sim d_r$.

## 4 Conclusions

Our study provides new insights into chaotic granular mixing under smooth flow conditions, highlighting strong parallels with classical chaotic advection in fluids. The results suggest that principles from fluid dynamics can be applied to describe granular mixing under certain conditions. In addition, the influence of the intruder's size or density was explored through numerical simulations. Future work could focus on analyzing the continuous velocity field to approximate the discrete granular medium as a continuum, enabling the application of fluid formalism. Specifically, the framework developed by Meunier and Villermaux [13] could be used to describe mixing in granular media, incorporating a characteristic diffusion coefficient as proposed by Rognon and Macaulay [9]. Additionally, future studies could explore increasing the roughness of the experimental slip boundary.

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