

## Memory effects in a vibrated thin granular layer

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**Abstract.** We present in this work the first experimental evidence of the thermal memory effect in a granular fluid. In particular, we observe here the Kovacs memory effect (an anomalous evolution of at least one macroscopic variable) in the granular temperature of the fluidized granular monolayer. The experimental set-up consists here in a vertically shaken granular monolayer. The evolution of the granular temperature curves clearly displays the characteristic Kovacs humps. Furthermore, it appears that, at experimental level, the shaken monolayer displays the so-called *anomalous* Kovacs effect; i.e., and upwards hump for cool down protocol (or a downwards hump for a heating up process). The experimental results are also supported by molecular dynamics simulation data which use a realistic computational model for both the dynamics and tribology properties of the oscillatory top and bottom walls that are present in our laboratory.

### 1 Introduction

From the physical point of view, *memory* is the ability of a system to store information on the properties and features of the system in the past, which can have an effect on its later evolution [1]. The study of the different processes of memory formation in physical systems has a number of applications and, as it is known, it has spawned a large number of publications in the recent years. Most notably, the *Mpemba memory effect* has received much attention [2], with a high number of works analyzing this effect in a variety of different systems. The Mpemba effect consists in an anomalous cooling or heating process, in the sense that an initially hotter system may cool down faster to a lower temperature state than another initially cooler state (and conversely for heating processes). One of these systems is the granular fluid, where initial theoretical works on both the Mpemba [2] and *Kovacs* memory effects [3, 4] contributed to generate interest in this type of phenomenology. The Kovacs memory effect consists in a tendency of the system to go back, during a relaxation process, to a past value/state of a given physical property, as if the system remembered this past state. It was originally observed in the volume of a sample of polymer material [4]. Interestingly, experimental evidence of both memory effects in the granular fluid had not been yet reported, in spite of the fact that subsequent experimental work has shown the existence of the Mpemba effect in other systems [5]. We provide now experimental evidence of the Kovacs effect in fluidized granular matter. In particular, our results support the evidence of a thermal memory effect in a granular fluid, this referring to the ability of the system to encode or

remember features of the granular temperature past. Previous works on memory effects in granular fluids focus on other properties, such as particle density [6] or stress [7].

But, why these memory effects emerge in matter?

In this sense, let us comment that, in the context of granular fluids, recent work on kinetic theory shows that the transfer of information from past to future states occurs always during *fast* transients, for which there is a sufficiently long *kinetic stage*; i.e., a succession of states that cannot be considered as *normal states* in the sense of kinetic theory. Certainly, as it is known, kinetic theory establishes [8, 9] that the distribution function for a system composed by many particles can fall into a family of states where all the time and space dependence of the distribution function can be described through the average or *hydrodynamic* fields (such as temperature, density, flow velocity...) [10]. Normal states can only emerge under not too extreme non-equilibrium conditions, though. If, on the contrary, the system is very far away from equilibrium, the distribution function  $f$  resorts to a kinetic stage before eventually entering a normal state. Thus, in this context, the kinetic stage denotes a set of states for which there is no valid hydrodynamic description [8]. During these states, the distribution function can only be fully described if both the hydrodynamic fields and the complete set of moments of  $f$  combined are specified. This situation is known in kinetic theory and fluid mechanics as *lack of scale separation*, meaning that the microscopic scale (particle dynamics) and the macroscopic scale (hydrodynamics) are coupled, which means that a fluid movement cannot be studied from the hydrodynamic equations.

But what is of interest for the analysis of memory effects is the following: while in the kinetic stage, and as

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the distribution function  $f$  relaxes towards equilibrium, the time evolution of the hydrodynamic fields and the cumulants (reduced moments of the distribution function) remains coupled. This means that even if one of the hydrodynamic fields has reached the equilibrium value, it will continue to evolve and thus to depart from its equilibrium value. This departure can last until all the other hydrodynamic variables and cumulants have completely relaxed to their equilibrium values. And only then, the distribution function will stop to evolve. Therefore, the memory effect in a macroscopic variable is expressed as an anomalous departure from its equilibrium value due to the information stored in (the coupling with) the other hydrodynamic fields and/or cumulants of the distribution function. In this sense, physical memory can be considered as information about a variable or set of variables that is stored in the set of microscopic variables. The fact that physical memory can only occur during kinetic stages means that physical memory has a non-persistent nature since it can only last until the normal states that live close to equilibrium are attained. Thus, the memory effect can only be permanent if the system is permanently under far from equilibrium conditions. This picture is schematically represented in Figure 1. In fact, this kind of explanation was already hinted by Kovacs in the original work [4] on the nowadays known as Kovacs memory effect, but can account for a generic memory effect.

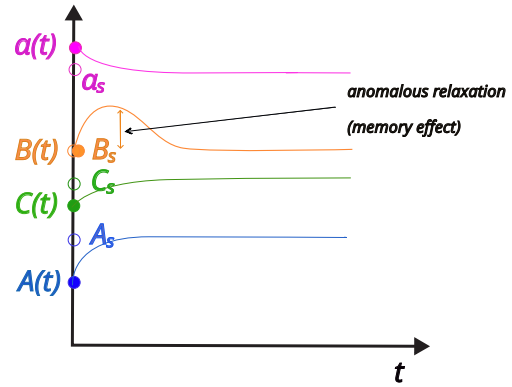
As a consequence, the distribution function of the granular fluid may keep evolving from a point where, for instance, the granular temperature  $T$  has reached a steady state value, by means of further evolution of the cumulants [3]. This is exactly what happens in the Kovacs memory effect for the granular temperature (see diagram). In summary, let us call  $t_m$  the time scale for which the fast (microscopic) variables become stationary, the memory effect can take place as a consequence of scale separation during fast transients due to the fact that, for instance, for the temperature  $T_0 = \mathcal{F}(A, B, \dots, a, b, \dots, t \rightarrow \infty)$  is a mono-valued function but  $T(t) = \mathcal{F}(A(t), B(t), \dots, a, b, \dots, t \rightarrow t_m)$  is not (uppercase letters stand for macroscopic variables and lowercase for the microscopic ones).

## 2 Kovacs protocol. Experimental set-up and simulations

As we said in the introduction, this is a work based on both laboratory and computational experiments. The experimental part is based on the set-up originally designed by J. S. Urbach and collaborators [11] and the computational part is based on molecular dynamic simulations. The molecular dynamics code used for this work has been used in previous works on the thin vibrated granular layer [11]. The specific of both parts of the work are described below.

### 2.1 Experimental set-up

The experimental set-up consists of a vibration system Brüel & Kjaer, that is composed by amplifier (LDS



**Figure 1.** Schematic representation of the mechanism for the emergence of a memory effect. Average (or hydrodynamic) fields are denoted with uppercase and microscopic variables (cumulants). Here, field  $B$  is the one that undergoes the memory effect. At the initial time, this field is already in its steady value ( $B(t) = B_s$ ). However,  $B(t)$  departs from  $B_s$  as a consequence of the other magnitudes not being in their stationary values. As a consequence,  $B(t)$  undergoes a non-monotonic time further evolution, which is considered as the emergence of the memory effect.

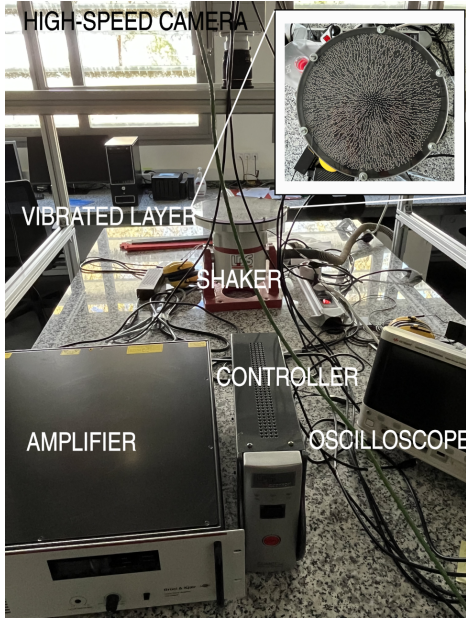
LPA600 model), controller (Comet, COM200 model), electromagnetic shaker (LDS V406 model, with internal amplifier PA500L, maximum force of 196 N, max peak to peak displacement 17.6 mm) and triaxial accelerometer (4535-B model with 10 mV/g, and 700 g range). These devices are configured by means of specific Brüel & Kjaer software (SCO-02V) in a feed-back loop so that acceleration is kept constant within a great degree of precision (fluctuations of less than 1%). The shaker is cooled by a fan (works at 50Hz) so it can keep its performance during the experiments. For particle tracking, a high-speed camera Phantom VEO is used, at a rate of 1000 fps. The timing of the on/off events of the camera and changes in the acceleration input are both controlled with the use of a high precision oscilloscope, in such a way that the recording is synchronized with the changes in the input acceleration. This allows for identification of the exact movie frame where the acceleration changes occur and so we characterize with precision the Kovacs protocol timing.

The experiments were performed with stainless steel spheres (with coefficient of normal restitution  $\alpha = 0.95$  [12]). All spheres are of diameter  $\sigma = 2.5 \text{ mm} \pm 0.5 \mu\text{m}$  of diameter and mass  $m = 1 \text{ g}$ . Molecular dynamics simulations were performed trying to mimic the experimental conditions [11], by adjusting the friction coefficients to the values that approximately would correspond to steel spheres [13].

An image of our set-up can be found in Figure 2.

### 2.2 Molecular dynamics

The algorithm for molecular dynamics simulations has been used elsewhere [11]. The boundaries are configured so that there is a pair of parallel vibrating walls that are perpendicular to gravity (Z direction)  $g = 9.8 \text{ m/s}^2$ . The



**Figure 2.** Here we can see an image of the experimental set-up in our laboratory (at Instituto de Computación Científica Avanzada, ICCAEx, in Badajoz, Spain). Inset: close-up of the thin layer (top view).

system is periodic in the horizontal directions (XY coordinates). The particles interact through a combination of a conservative restoring force and normal and tangential frictional forces, in such a way that particle-particle forces can be expressed as

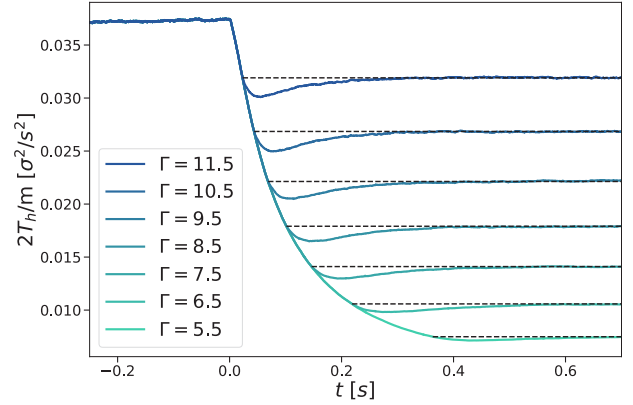
$$\mathbf{F}_{ij}^{\text{rest}} = Ym_i(|\mathbf{r}_{ij}| - \sigma)\hat{\mathbf{r}}_{ij}, \mathbf{F}_{ij}^{\text{diss}} = -\gamma_n m_i \mathbf{v}_{ij}^n, \quad (1)$$

$$\mathbf{F}_{ij}^{\text{shear}} = -\gamma_s m_i \mathbf{v}_{ij}^t. \quad (2)$$

In the above equations (2), subscripts  $i, j$  stand for particles;  $m_i$  is the particle mass;  $\mathbf{r}_{ij}$  are relative positions:  $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ , and  $\hat{\mathbf{r}}_{ij} = \mathbf{r}_{ij}/|\mathbf{r}_{ij}|$ . Analogously,  $\mathbf{v}_{ij}$  stands for relative velocities, and  $\mathbf{v}_{ij}^n, \mathbf{v}_{ij}^t$  stand for the projections of relative velocities in the normal and tangential directions respectively. Analogous equations would rule for the wall-particle interactions, where one of the  $i, j$  subscripts would be replaced by  $w$ , which stands for walls. The coefficient  $Y$  is the Young modulus that characterizes the (conservative) restoring force whereas  $\gamma_n, \gamma_s$  account for the dissipation in the normal and tangential directions respectively. There is also a set of analogous particle-wall interactions, but with coefficients  $Y_w, \gamma_{nw}, \gamma_{sw}$ . In this work we have used values of parameters that mimic the behavior of metallic balls with a coefficient of normal restitution  $\alpha = 0.95$  for steel balls [14]. Also, in order to simplify, we have used the same values of force parameters for the wall-particle interactions. Thus, use here  $Y = Y_w = 10^7 \text{ s}^{-2}$ ,  $\gamma_n = \gamma_{nw} = 200 \text{ s}^{-1}$ ,  $\gamma_s = \gamma_{sw} = 200 \text{ s}^{-1}$ . See [15] for further reference. Other simulation parameters are  $\nu = 640 \text{ Hz}$ , and  $h = 1.75 \sigma$ . Results for a simulation series of the Kovacs protocol can be found in Figure 3.

### 3 Results

Figures 3, 4 show that the Kovacs humps are clearly measured in both MD simulations and experiment, respectively. As it can be seen, there is good qualitative agreement between the humps observed in molecular dynamics and those for the experiments.



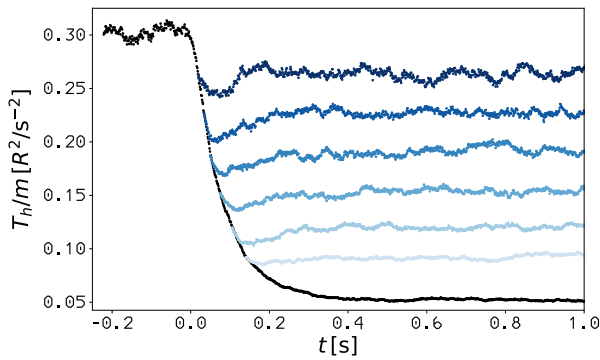
**Figure 3.** Evolution of the horizontal temperature,  $T_h$  for a simulation series of the Kovacs protocol at a frequency  $\nu = 643.22 \text{ Hz}$ . The top and bottom acceleration inputs of the Kovacs protocol are here  $\Gamma_{\text{top}} = 12$  and  $\Gamma_{\text{bottom}} = 5.0$  respectively. Intermediate values of the input acceleration for the protocol are here  $\Gamma = 11.5; 10.5; 9.5; 8.5; 7.5; 6.5; 5.5$ . As it can be seen, the memory effect appears for all intermediate curves, with the Kovacs hump being more deep if the waiting time is shorter (since the system has had less time to adapt its distribution function to that of the final steady state value, corresponding to  $\Gamma_{\text{bottom}}$ ).

It is worth to note also that we can only detect in both cases the so-called *anomalous* Kovacs effect (a downwards hump for a cooling protocol), contrary to what has been observed in an unbounded granular fluid, where the *normal* Kovacs effect (an upwards hump for a cooling protocol) is observed for a sufficiently low inelasticity [16]. We think that the absence of the normal Kovacs hump in the thin layer can probably be due to the fact that there is an inherent coupling between horizontal and vertical temperature in the thin vibrated granular layer, and a minimum degree of inelasticity due to experimental conditions. In this sense, we think that more insight can eventually be provided from a theoretical point of view, in the context of kinetic theory for a confined thin layer [17–19].

### 4 Conclusion and Outlook

In the introduction, we have explained the physical origin of memory effects. To this matter, we have focused on providing a rationale for the memory effect that emerges after the Kovacs thermal protocol (which we have also described). We have provided strong experimental evidence of this for the granular temperature of a thin vibrated granular layer.

For this, we have built a new laboratory set-up where we perform experiments with a vertically vibrated granular monolayer. Our experimental results are additionally supported by molecular dynamics simulations of a



**Figure 4.**  $T_h(t)$ , from experiments at shaking frequency  $\nu = 180$  Hz. The top and bottom acceleration inputs of the Kovacs protocol are here  $\Gamma_{\text{top}} = 7.5$  and  $\Gamma_{\text{bottom}} = 3.15$  respectively. Intermediate values of the input acceleration for the protocol are here  $\Gamma = 6.5; 6.0; 5.5; 5.0; 4.5; 4.0$ . Particles are steel spheres with diameter  $\sigma = 2.5$  mm.  $R$  is the particle radius ( $R = \sigma/2$ ). A similar behavior to that described in the simulations is found experimentally.

thin layer with vibrating walls that have been realistically simulated. At experimental level, since there is an intrinsic minimum degree of friction and inelasticity due to the effects of particle-particle and particle-wall contacts, we have not detected the normal Kovacs effect (i.e., an upwards hump for the cooling protocol) that tends to appear at low inelasticity and equilibrium systems.

In a forthcoming work, we will discuss on the comparison of experiment, molecular dynamics and event driven simulations with recent results from kinetic theory [17] that will be applied to the Kovacs memory effect in the thin vibrated granular layer.

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