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Dilute gas-particle flow through a vertical axisymmetric pipe

Mudasir Ul Islam[1,*], *James* T. Jenkins[2,**], and *Sovan* Lal Das[1,***]

[1]Physical and Chemical Biology Laboratory and Department of Mechanical Engineering, Indian Institute of Technology, Palakkad, Kerala, India
[2]School of Civil and Environmental Engineering, Cornell University, Ithaca, NY, USA

Abstract. We investigate the flow of a dilute gas-particle flow through a vertical axisymmetric pipe under the influence of gravity. A simplified gas turbulence model is considered for the flow of relatively heavy particles within the turbulent gas, whereas the particle phase is described using constitutive relations from kinetic theory. Featureless frictional and bumpy frictionless boundary conditions are applied to analyze any resulting changes in flow behaviour. We present and compare the radial profiles of mean particle and gas velocity, particle and gas velocity fluctuations, and particle concentration for the two diferent wall boundary conditions.

1 Introduction

In this paper, we apply the model of Pasini and Jenkins [1] to study gas-particle system through a vertical circular pipe, utilizing the constitutive equations of kinetic theory for the dilute particle phase. In our previous work, kinetic theory was applied to granular flows in a vertical chute and a circular pipe without a gas phase in the entire range of solid volume fractions [2, 3].

Previous investigations relevant to the present study have focused on the influences of the particles on the fluid turbulence in dilute flows [4–11]. Here, we formulate a boundary value problem for particles and gas flowing through a vertical circular pipe under the influence of gravity. The pipe walls are assumed to be featureless, frictional [12] and bumpy, frictionless.

We consider a steady, fully developed flow of gas and particles in a vertical circular pipe. The z-axis is oriented against gravity, meaning that an upward flow corresponds to a positive velocity. The radial coordinate is denoted by r. The variables ν, u, s, p, and w represent the particle volume fraction, mean velocity, shear stress, collisional pressure, and fluctuation velocity (where $w = \sqrt{T}$, with T being the granular temperature, which equals 1/3 of the mean square of particle velocity fluctuations). Similarly, U, S, and P denote the mean velocity, shear stress, and pressure of the gas phase. The material densities of the particles and fluid are given by ρ_s and ρ_f, respectively, and lengths are nondimensionalized using the particle diameter d. The gravitational acceleration is g, and for a large density ratio $\bar{\rho} = \rho_s/\rho_f$, which is typical for solid-gas systems, buoyancy corrections are negligible. Velocities, stresses, and volume flow rates are nondimensionalized by $(gd)^{1/2}$, $\rho_s gd$ and, $g^{1/2}d^{5/2}$ respectively.

We employ an effective coefficient of restitution, $e = \max\{\epsilon - 6.9(1+\epsilon)/St,\ 0\}$, which accounts for the coefficient of normal restitution e_n and the effect of sliding friction μ on the dissipation rate of translational fluctuations [13, 14], where $\epsilon = e_n - (3/2)\mu\exp(-3\mu)$ [15]. The influence of viscous resistance due to the lubrication layer between two colliding spheres is characterized by the Stokes number, $St = \bar{\rho}T^{1/2}R_e/9$ [16, 17]. Here, $R_e = \rho_f d(gd)^{1/2}/\eta_{mol}$ is the Reynolds number based on the terminal velocity of a single particle, where η_{mol} represents the molecular viscosity of the gas.

2 Governing equations

Upon rearranging the balance equations and the constitutive relations [1, 18, 19] by considering the vertical axisymmetric pipe, we obtain the system of first-order differential equations governing the flow, phrased using the fluctuation velocity, $w = \sqrt{T}$, the differential equation for the volume fraction obtained is given by

$$\nu' = f_1 q\left[w^3\left(\frac{\partial f_1}{\partial \nu}f_4 - f_1 f_5 - \frac{f_4}{w^2}\frac{D\eta_{turb}}{1-\nu}\right)\right]^{-1}, \qquad (1)$$

where prime denotes the derivative with respect to r, $p = f_1 w^2$, $f_1 = \nu[1+2G(1+e)]$, $f_4 = 4M\nu G/\sqrt{\pi}$ and $f_5 = 25\sqrt{\pi}N/(128\nu)$ with

$$M = \frac{1+e}{2} + \frac{9\pi}{144(1+e)G^2} \times \left\{\frac{\left[5+3G(2e-1)(1+e)^2\right]\left[5+6G(1+e)\right]}{16-7(1-e)}\right\}$$

*e-mail: 132004002@smail.iitpkd.ac.in
**e-mail: jtj2@cornell.edu
***e-mail: sovan@iitpkd.ac.in

and

$$N = \frac{96\nu(1+e)}{25G(1+e)}\frac{5+6G(1+e)}{16+3(1-e)} \times \left\{\frac{20\left[5+3G(2e-1)(1+e)^2\right]}{48-21(1-e)}\frac{\nu}{G}\frac{\partial G}{\partial \nu} - (e+e^2)\left(G+\nu\frac{\partial G}{\partial \nu}\right)\right\}.$$

The radial distribution function at contact, g_0, enters these expressions as, $G = \nu g_0$. The function g_0 has the information about the probability of having two particles in contact [20]

$$g_0 = \frac{2-\nu}{2(1-\nu)^3}. \tag{2}$$

$D = [0.3|U-u| + 18/R_e]/(1-\nu)^{3.1}$ is the drag coefficient [1]. The turbulent viscosity is given by

$$\eta_{turb} = (1-\nu)\frac{1}{\bar{\rho}}\left(\frac{0.09}{0.165}\right)k^{1/2}l, \tag{3}$$

where l is the mixing length, and $k^{1/2}$ represents gas velocity fluctuations (turbulent kinetic energy), related to the fluid shear stress S as

$$k^{1/2} = \frac{(0.09|S|\bar{\rho})^{1/2}}{0.165(1-\nu)^{1/2}}. \tag{4}$$

This expression follows from the fluid fluctuation energy balance, neglecting energy diffusion [1]. The absolute value of S prevents complex values of $k^{1/2}$ due to sign changes in shear. Following [7], the mixing length l varies as

$$l = \begin{cases} \kappa(1-r), & \text{for} \quad r \geq 0.7; \\ 0.3\kappa, & \text{for} \quad r \leq 0.7, \end{cases} \tag{5}$$

where von Kármán's constant κ is set to 0.41.

The differential equation for the particle shear stress is

$$s' = -\frac{s}{r} - \nu\frac{\bar{\rho}}{\bar{\rho}-1} - \frac{\nu D}{\bar{\rho}}(U-u). \tag{6}$$

The particle mean velocity across the pipe follows

$$u' = \frac{1}{f_2}\frac{s}{w}, \tag{7}$$

where $s = f_2 T^{1/2} u'$ and the coefficient $f_2 = 8J\nu G/(5\sqrt{\pi})$ with

$$J = \frac{1+e}{2} + \frac{\pi}{32}\frac{[5+2(1+e)(3e-1)G]\,[5+4(1+e)G]}{[24-6(1-e)^2-5(1-e^2)]\,G^2}.$$

The differential equation for the particle fluctuation energy flux is obtained from the fluctuation kinetic energy balance [1]

$$q' = -\frac{q}{r} + \frac{1}{f_2}\frac{s^2}{w} - f_3 w^3, \tag{8}$$

where $q = -2f_4 w^2 w' - f_5 w^3 \nu'$ is the diffusive flux of particle fluctuation energy. The coefficient $f_3 = 12(1-e^2)\nu G/\sqrt{\pi}$.

The governing equation for the particle fluctuation velocity is obtained as

$$w' = -\frac{q}{2f_4 w^2} - \frac{q f_1 f_5}{2f_4} w\left[w^3\left(\frac{\partial f_1}{\partial \nu}f_4 - f_1 f_5 - \frac{f_4}{w^2}\frac{D\eta_{turb}}{1-\nu}\right)\right]^{-1} \tag{9}$$

The differential equation for the shear stress in the gas phase is given by

$$S' = \frac{dP}{dz} - \frac{S}{r} - \frac{1-\nu}{\bar{\rho}-1} + \frac{\nu D}{\bar{\rho}}(U-u). \tag{10}$$

For the variation of mean fluid velocity across the pipe [18, 21], we have

$$U' = \frac{S}{\eta_{turb} + \eta_{gran}}, \tag{11}$$

where the tubulent gas viscosity is given in eq. 3 and $\eta_{gran} = (1+2\nu)f_2 w/[(1-\nu)\bar{\rho}]$ is the grain-induced viscosity associated with the random fluctuations induced in the gas by fluctuations in the particle velocity [21].

The differential equation associated with the particle volume flow rate across the pipe, $i(r) = \int_0^r \nu u(2\pi r)dr$, permits the use of the particle volume flow rate as one of the boundary conditions:

$$i' = 2\pi R^2(\nu u)r. \tag{12}$$

3 Boundary conditions

3.1 Featureless Frictional Boundary

Jenkins [12] derived boundary conditions for the particle phase by considering collisions between inelastic, frictional spheres and a frictional wall. Assuming small friction coefficients and continuous slipping at contact points, the tangential momentum and energy balances are given by:

$$s = \mu p \tag{13}$$

$$q = \frac{3}{8}p(3T)^{1/2}\left[\frac{7}{2}(1+e_w)\mu^2 - (1-e_w)\right], \tag{14}$$

where e_w is the restitution coefficient for particle-wall collisions.

Additional boundary conditions include the vanishing of particle shear stress, fluid shear stress, and energy flux at the centerline, along with the no-slip condition for gas velocity at the boundary.

3.2 Bumpy Boundary

Instead of a smooth frictional wall, we now consider a bumpy frictionless boundary. The slip velocity of the particles at the bumpy boundary is given by [22]

$$\frac{u_b}{w_b} = \left(\frac{\pi}{2}\right)^{1/2} f\frac{s_b}{p_b}, \tag{15}$$

where

$$f = \left[\frac{3}{2^{5/2}J_b}\frac{2^{3/2}J_b - 5F_b(1+B)\sin^2\phi}{2(1-\cos\phi)/\sin^2\phi - \cos\phi} + \frac{5F_b}{2^{1/2}J_b}\right],$$

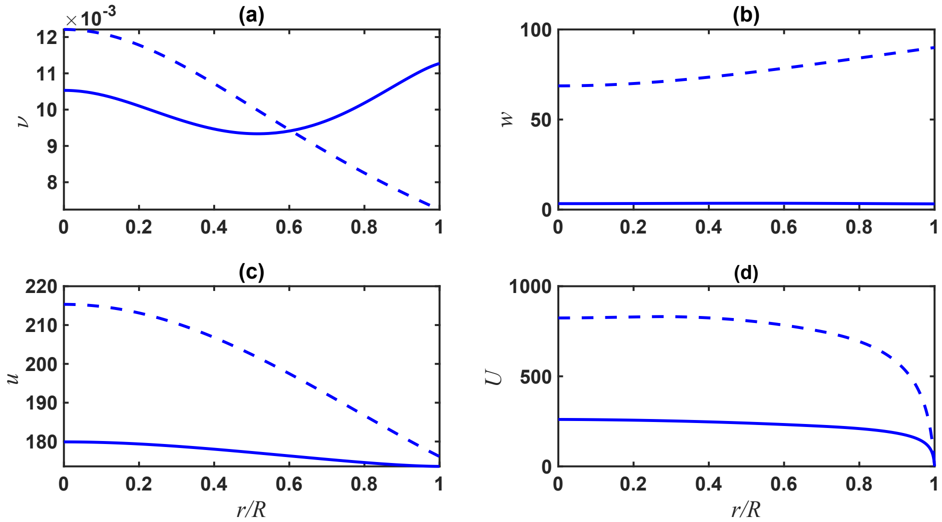

Figure 1. Comparison of profiles between featureless, frictional (solid lines) and bumpy boundaries (dashed lines). The flow is maintained at an average solid volume fraction of 0.01 and total particle volume flow rate of 5000. The pressure gradients $dP/dz = 1.12$ (frictional wall) and $dP/dz = 51.34$ (bumpy wall).

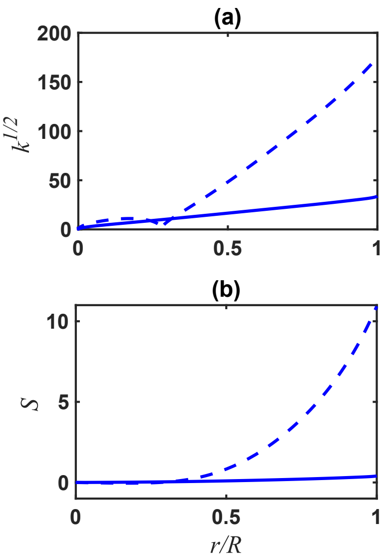

Figure 2. Comparison of strength of gas velocity fluctuations and gas shear stress between featureless, frictional (solid lines) and bumpy boundaries (dashed lines).

with $B = \pi\left[1 + 5/(8G_b)\right]/(12\sqrt{2})$, and $F_b = (1 + e_n)/2 + 1/(4G_b)$. The subscript b denotes the boundary, and ϕ measures boundary roughness. For flow spheres of the same diameter as boundary spheres, $\sin\phi = (d + l_s)/(2d)$, where l_s is the gap between boundary spheres.

The energy flux balance at the boundary is [22]

$$q_b = s_b u_b - \gamma_b, \tag{16}$$

where the collisional dissipation rate is:

$$\gamma_b = \left(\frac{2}{\pi}\right)^{1/2} p_b w_b (1 - \epsilon) \frac{2(1 - \cos\phi)}{\sin^2\phi}.$$

All other boundary conditions are the same as in the featureless frictional wall.

4 Results and discussion

The governing differential equations, along with the specified boundary conditions, form a two-point boundary value problem, which we solve numerically using MATLAB's bvp4c solver. We present results for a particle flow rate of 5000, assuming restitution coefficients of $e_n = e_w = 0.92$ and parameter values $\phi = \pi/6$, $\mu = 0.1$, and $\bar{\rho} = 840$ (e.g., air-particle flow with $\rho_s \approx 1020$ kg/m^3 and $\rho_f \approx 1.21$ kg/m^3). The pipe radius is set to $R = 30$ particle diameters.

In Figure 1, the profiles of solid volume fraction ν, particle fluctuation velocity w, particle velocity u, and gas velocity U are presented for two different boundary conditions. With a bumpy boundary, particle fluctuations are more pronounced, leading to a less uniform distribution of ν and a more dilute flow near the walls. Additionally, the granular temperature (particle fluctuation velocity) is significantly higher for bumpy boundary. This is due to the mean slip of particles relative to the wall, which causes frequent collisions with the bumps. The pressure gradient required to sustain the flow is considerably higher for bumpy boundary, as enhanced particle fluctuations lead to greater momentum exchange between the gas and particles, intensifying turbulence and increasing pressure drop.

Figure 2 illustrates the impact of boundary conditions on the strength of gas velocity fluctuation and gas shear stress. When the boundary is bumpy, the increased particle agitation induces a higher turbulence in the gas as shear is high (see Eq. 11). Note that for bumpy boundary, a kink appears in the profile of $k^{1/2}$ at the location where the shear stress S changes sign. This is due to the use of the absolute value of S in the expression of $k^{1/2}$ to avoid complex numbers (see Eq. 4).

5 Conclusions

We implemented a simplified gas turbulence model for the dilute flow of relatively massive particles in a turbulent gas through a vertical axisymmetric pipe. Both featureless frictional and bumpy wall boundary conditions were applied to analyze their impact on flow behavior. The results indicate that boundary conditions play a crucial role, with frictional walls promoting a more uniform particle distribution, while bumpy walls enhance particle fluctuations near the boundary.

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