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Discharge of spheres and dumbbells mixed with fine grains from a 3D silo

Sandip H. Gharat[1,3,*], *Shyam* P. Tekade[1,**], and *Luis* A. Pugnaloni[2,***]

[1]Department of Chemical Engineering, Gharda Institute of Technology Lavel, Khed, Ratnagiri 415708 India
[2]Departamento de Física, Facultad de Ciencias Exactas y Naturales, Universidad Nacional de La Pampa, CONICET, Uruguay 151, 6300 Santa Rosa (LaPampa), Argentina
[3]Department of Chemical Engineering, Shroff S.R. Rotary Institute of Chemical Technology, UPL University of Sustainable Technology, Ankleshwar, Gujarat 393001 India

Abstract. The flow of mixtures of non-spherical particles from silos is extensively observed in various industries. In this paper, we study the flow of binary mixtures of large grains mixed with fine spherical particles during the discharge of a silo with a circular orifice in its base. In particular, we focus on the effect of the shape of the large grains by using either spheres or dumbbell-shaped grains (mung beans). The flow rate is measured for various diameters of the orifice (D) and mix ratio of small and large particles (χ). The pure systems (no added fine grains) show equivalent flow rates for spheres and dumbbells. When fine grains are added to a sample of large particles, the effective flow rate of the big grains (diameter d_b) can increase. We observed that this is the case for both spheres and dumbbells. We propose a scaling of the flow rate, for binary mixtures, that allows us to assess the effect of the particle shape by removing the particle size contribution. The results suggest that for $D/d_\mathrm{b} \lesssim 6.0$ the shape of the large particles does not affect its flow rate for these low aspect ratio dumbbell shapes. However, for larger orifices and intermediate mix ratio, the spheres flow faster than the dumbbells.

1 Introduction

The study of granular flow in confined systems such as hoppers and silos is crucial for various industrial applications, including agriculture, pharmaceuticals, and mining. While the flow behavior of spherical particles has been extensively investigated, real-world granular materials are often non-spherical and consist in a mix of different particle shapes, leading to more complex phenomena. Understanding the behavior of more realistic granular samples is essential for optimizing material handling, preventing blockages, and improving efficiency in bulk solid processing.

In the past, researchers have focused extensively on the flow rate of spherical particles discharging from a silo though a circular orifice [1, 2]. Ashour et al. [3] have considered the impact of particle shape. They have found that for moderately anisometric particles (long axis/short axis < 4), the discharge rate agrees to some extent with that of a sphere of equal volume. For large aspect ratios, however, the flow rate is much smaller than that of the same-volume spherical particles. Liu et al. have also reported a reduced flow rate for non-spherical grains [4]. Surprisingly, Li et al. have found an increased flow rate when spheres are replaced by disk-shaped grains [5]. These and similar more recent studies [6–8] have focused on monodisperse systems. Recently, we have shown that the presence of a second fine species can significantly alter the flow and clogging of the main coarse species [9, 10]. The focus of the present work is to analyze the effect that the shape of the large grains may have on the flow behavior in the presence of the finer grains.

We conduct an experimental analysis of dumbbell-shaped particles (mung beans) discharging from a 3D silo. These particles are mixed with a finer species. We investigated the effects of shape on flow rate by comparing results with those of similar systems where the coarse dumbbells are replaced by spheres (glass beads). We present a scaling of the flow rate that allows direct comparison for binary mixtures in which the coarse species do not have the same particle size.

2 Experimental

All the experiments were carried out using a cylindrical acrylic silo with an inner diameter of 9.4 cm and height 29.5 cm (see Fig. 1). The bottom of the silo features a 3D-printed, 3.0 mm-thick plastic disk with a circular opening in the center. The disk is interchangeable, allowing the granular material to flow through holes of different sizes (9.00 mm < D < 28.00 mm in diameter). These openings are designed with a conical shape, where the lower end is wider than the upper end, to prevent grains from getting stuck due to the thickness of the plastic piece. The silo is placed on a supporting table, and a container is used to collect the material as it flows out.

For the coarse (big) particles, we use either glass beads (spherical, of mean diameter d_b = 4.4 mm) or mung beans

*e-mail: sandipgharat78@gmail.com
**e-mail: shyamtekade@gmail.com
***e-mail: luis.pugnaloni@gmail.com

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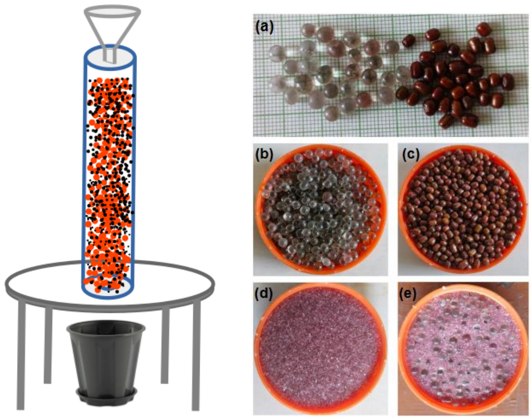

Figure 1. (Left) Schematic diagram of the experimental setup. The red ovals correspond to the coarse particles (spherical glass beads or mung beans) and the black dots to the fine particles (glass beads). Images of materials used in the experiments: (a) close-up of glass beads and mung beans, (b) large glass beads (4.4 mm), (c) mung bean (3.8 mm), (d) fine glass beads (0.75 mm), and (e) sample binary mixture of glass-glass beads (0.75 mm & 4.4 mm).

(dumbbell shape, of mean diameter d_b = 3.8 mm, long axis of ≈ 4.5 mm and short axis of ≈ 3.3 mm, aspect ratio ≈ 1.4). For the fine species we use glass beads (spherical) of d_f = 0.75 mm in mean diameter. The mean diameters where measured by total volume of fresh water displaced by a known number of grains in a graduated cylinder. The coarse and fine glass beads have same material density (ρ = 2.5 g/cm^3), while the density of mung beans is 1.27 g/cm^3. Images of sample materials used in the experiments are shown in Fig. 1. It is worth mentioning that mung beans have a reddish color that in contact with the glass beads transfer the dye and color the glass as observed in the images.

We have used two types of binary mixtures to perform this study: mixtures of coarse glass beads–fine glass beads (GB-GB) and mixtures of mung bean–fine glass beads (MB-GB). A mixture is characterized by the fraction of volume occupied by the big grains $\chi = (M_b/\rho_b)/(M_b/\rho_b + M_f/\rho_f)$, where $M_{b/f}$ is the total mass of the corresponding species (big or fine) and $\rho_{b/f}$ is the corresponding material density. We used the fraction of volume instead of the mass fraction because of the density differences between the glass beads and the mung beans. Mixing is done by stirring the samples in a shallow container.

Samples are poured into the silo through a funnel from the top in small batches to minimize segregation and ensure a uniform distribution. The filling process continues until the desired height of the material inside the silo is reached (≈ 2 kg of material is used in each experiment). Then, the discharge hole at the bottom is opened, allowing the material to flow out due to gravity. A stopwatch is used to measure the time taken for the silo to completely discharge. The mass flow rate of the coarse particles (Q_b) is calculated by dividing the total mass of these particles in the mixture by the total discharge time. Each experiment is repeated two to three times, and the average value is recorded. The minimum and maximum values from these trials are presented as error bars to indicate variability in the results.

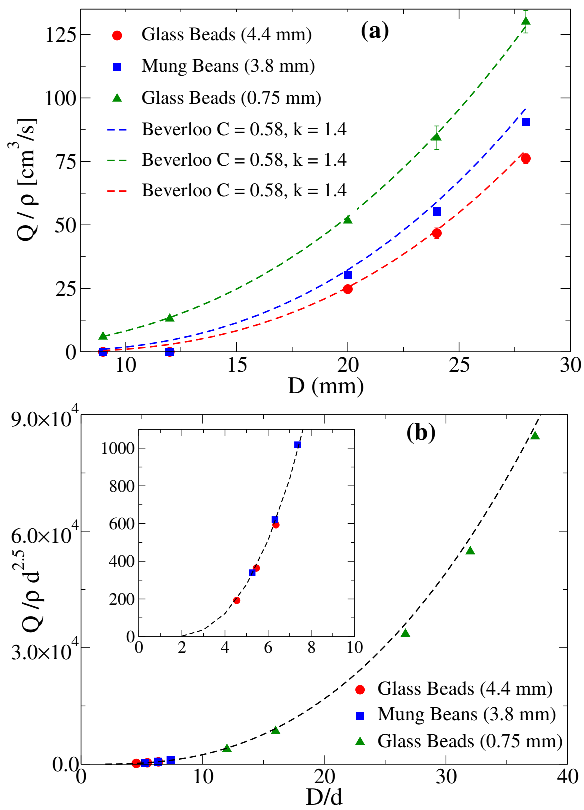

Figure 2. (a) Mass flow rate (Q) of the pure systems (large glass beads, mung beans and fine glass beads) scaled by the material density (ρ) of each material as a function of orifice diameter. The dashed lines corresponds to the Beverloo model Eq. (1) with ϕ = 0.62, C = 0.58, and k = 1.4. (b) Same data as in part (a) with Q/ρ normalized by $d^{2.5}$ and D normalized by d. The dashed lines correspond to the rescaled Eq. (2). Inset: close-up for the data on the coarse particles.

3 Results

3.1 Flow rate of pure species

To characterize the flow behavior of each separate species, we show in Fig. 2(a) the mass flow rate (Q) scaled by the material density (ρ) as a function of the orifice diameter (D) for pure samples of each material. We can observe that, after scaling by material density, smaller grains flow faster. This is consistent with the Beverloo equation [1]

$$\frac{Q}{\rho} = C\phi\sqrt{g}(D - kd)^{2.5}. \quad (1)$$

In Eq. (1) C ≈ 0.58 and k ≈ 1.4 are empirical constants, and d is the mean particle diameter (assuming a narrow size dispersion). We used the material density (ρ) and the packing fraction (ϕ) in replacement of the more traditional notation in terms of the bulk density ($\rho_{bulk} = \rho\phi$).

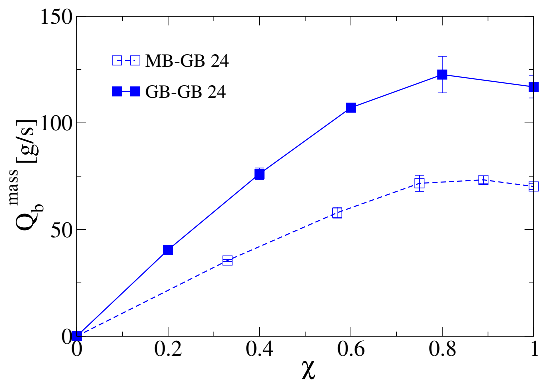

Figure 3. Mass flow rate Q_b^{mass} of coarse grains as a function of mixture composition χ for glass-glass and mung bean-glass beads systems for an orifice diameter $D = 24$ mm. Error bars correspond to the maximum variation among realizations.

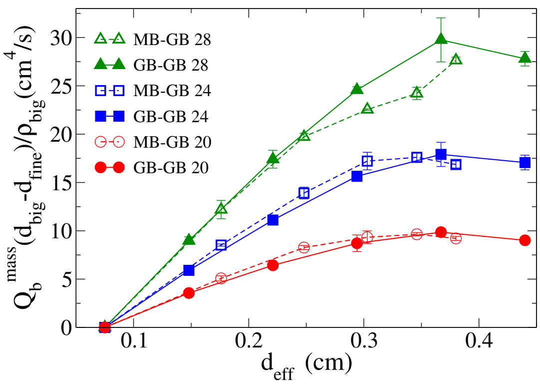

Figure 4. Scaled flow rate $[Q_b^{mass}(d_b - d_f)/\rho_b]$ as a function of the effective diameter (d_{eff}) for mixtures of mung beans-glass beads (MB-GB) & glass-glass (GB-GB) for three different orifices (20, 24 and 28 mm). Error bars correspond to the maximum variation among realizations.

In order to compare results from different mean particle sizes one can rescale Eq. (1) by $d^{2.5}$ to yield

$$\frac{Q}{d^{2.5}\rho} = C\phi\sqrt{g}(D/d - k)^{2.5}. \quad (2)$$

In Fig. 2(b) we plot $Q/(d^{2.5}\rho)$ as a function of D/d. As suggested by Eq. (2), the rescaled flow rate corresponds to a single functional form that depends only on D/d. Interestingly, mung beans and coarse glass beads, which are close in size and cover a similar D/d range, show the same rescaled flow rate despite their different particle shapes (see the inset in Fig. 2(b)). We note that Eq. (2) was developed for spherical (or close to spherical) grains. In view of the inset in Fig. 2(b), it seems that the small deviation of our dumbbell-shaped grains from a sphere does not make a significant impact on the effective flow. It is worth mentioning that the coarse species used do not differ only on shape, but also on stiffness, friction, etc. Nonetheless, all these differences do not affect the rescaled flow of the pure samples. In the next section, we will see that the shape of the particles can have a discernible effect when considering mixtures.

3.2 Mixed systems

Figure 3 shows the mass flow rate of the big grains (Q_b^{mass}) when the mixtures with fine grains are discharged from the silo as a function of χ (the fraction of the volume occupied by the coarse grains). Q_b^{mass} is naturally higher for the large glass beads than for the mung beans due to different material density. For $\chi \longrightarrow 0$ the system contains no large grains and therefore $Q_b^{mass} \longrightarrow 0$. For $\chi \longrightarrow 1$ the system contains only large grains and therefore Q_b^{mass} corresponds to the pure coarse particle system. As observed in previous studies for spherical grains [9, 10], Q_b^{mass} shows a maximum at intermediate $\chi \approx 0.8$, implying that coarse particles can be discharged faster by adding some fine grains to the sample. This is true also for the non-spherical mung beans. To compare results from different material mixtures, we need to consider a different scaling to the one used in the previous section for pure system. If we vary χ in the experiments as in Fig. 3, the mean particle size varies. Let us consider a simple model for Q_b^{mass}. The mass flow rate of the coarse grains Q_b^{mass} can be calculated from the volume flow rate Q_b^{vol} as

$$Q_b^{mass} = Q_b^{vol}\rho_b, \quad (3)$$

with ρ_b the density of the material of the big grains. If the total volume flow rate of the mixture Q_{tot}^{vol} is known, we can calculate Q_b^{vol} as $Q_b^{vol} = \chi Q_{tot}^{vol}$. In turn, the total volume flow rate of the mixture can be calculated by the modified Beverloo equation [9, 11]

$$Q_{tot}^{vol} = C\phi\sqrt{g}(D - kd_{eff})^{2.5}, \quad (4)$$

with d_{eff} the effective size of the grains in the mixture. There are different possible definitions for d_{eff} in a mixture, but they all depend on χ [12]. Let us take the simple definition $d_{eff} = \chi d_b + (1-\chi)d_f$. If we put Eqs. (3) and (4) in terms only of χ we get

$$Q_b^{mass} = \rho_b\chi C\phi\sqrt{g}[D - k(\chi d_b + (1-\chi)d_f)]^{2.5} \quad (5)$$

Since we run experiments with different large particles (different d_b and ρ_b), we cannot rescale Eq. (5) to make the RHS independent of these two parameters and use a plot with χ as control parameter as in Fig. 3. Then, we try writing Eq. (3) in terms of d_{eff} instead of χ. It is easy to show, from the above definition of d_{eff}, that $\chi = (d_{eff} - d_f)/(d_b - d_f)$, hence

$$Q_b^{mass} = \rho_b\frac{d_{eff} - d_f}{d_b - d_f}C\phi\sqrt{g}[D - kd_{eff}]^{2.5} \quad (6)$$

In this case, we can easily eliminate the dependency on ρ_b and d_b from the RHS by rearranging as

$$\frac{Q_b^{mass}(d_b - d_f)}{\rho_b} = (d_{eff} - d_f)C\phi\sqrt{g}[D - kd_{eff}]^{2.5} \quad (7)$$

In Eq. (7), the RHS depends only on d_f and D, which are fixed parameters for a given orifice and fine grain material, and we can use d_{eff} as the control parameter of the mixture. Certainly, Eq. (7) can be put in terms of χ by using the definition $d_{\mathrm{eff}} = \chi d_{\mathrm{b}} + (1-\chi)d_{\mathrm{f}}$. However, this would make the RHS to depend on d_{b}, which is different in experiments with large glass spheres and mung beans. Therefore, if results with different large particles disagree as a function of χ, the difference could be attributed to particle size rather than particle shape.

Figure 4 displays $Q_{\mathrm{b}}^{\mathrm{mass}}(d_{\mathrm{b}}-d_{\mathrm{f}})/\rho_{\mathrm{b}}$ as a function of d_{eff} for the different mixtures and three different orifice diameters. As we can see, the rescaled flow rate indicates that the large glass beads and mung beans flow at the same rate when mixed with the fine material, in most cases. This indicates that the different shape of the large particles has no effect in the flow rate in the mixtures; in a similar way as we observed for the pure systems in the previous section. However, for the largest orifice explored (D = 28 mm) and at 0.30 mm $< d_{\mathrm{eff}} <$ 0.40 mm, we can appreciate a clear deviation between the results of glass beads and mung beans. Glass beads flow faster. Since the rescaling used removes the effect of the size and the density of the large particle, the deviation could be attributed to the particle shape. It is important to recall that the two coarse species utilized differ not only on shape, density and size; they also have different mechanical properties like Young modulus, friction, etc. Some factors other than shape can be involved in the difference observed at D = 28 mm. Also, segregation can be an issue in some cases [9]. We have not quantified the degree of segregation in our experiments, but this was not appreciable, at least by simple visual inspection.

4 Conclusions

We have studied the flow rate of binary mixtures of grains (coarse and fine) discharging from a silo where the coarse species was changed between spherical (glass beads) and dumbbell-shaped (mung beans) grains. We explored a range of mix ratios and orifice diameters.

We have shown that a special scaling is needed to compare the flow rate of binary mixtures when the particle size and the density of the large species varies. The replacement of glass beads by mung beans seems to have no effect on the effective flow of the large grains under most conditions. However, for large orifices we observed a clear deviation, with the glass beads flowing faster than the mung beans in a narrow range of effective particle sizes d_{eff} (corresponding to volume mix ratio $0.7 \lesssim \chi \lesssim 0.9$).

Future works need to explore if the observed deviation persists at larger orifice diameters. In addition, the presence or absence of segregation during discharge needs to be quantified. The study can be extended to other more anisometric particles, for the coarse species, that do show flow rate differences even for pure samples.

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