

Waves in granular flows down a vertical chute

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Abstract. The flow of granular materials in confined, gravity-driven chute systems is important to various industrial processes. Despite its long research history and practical relevance, solutions to chute flows and their stability continue to be investigated. This study examines the dynamics of granular particles in a vertical chute of a square cross section with nominal side of 50 mm under gravity, focusing on how variations in solid fraction and wall friction influence wave formation in the form of cross-stream oscillations. Using discrete element method simulations, we model the behavior of 2 mm spherical particles that interact through the Hertz-Mindlin contact model with periodic boundaries in the lateral (x) and flow (y) directions. The conditions in the walls in the remaining spanwise direction (z) are varied systematically, including various combinations of walls that are flat-rough and/or bumpy-smooth. Our study indicates that at increased solid fractions, the granular flow transitions to a wave-dominated regime, as the particles descend under gravity. The occurrence and characteristics of these waves are strongly influenced by wall conditions.

1 Introduction

The gravity flow of granular materials through standpipes, hoppers and bunkers is encountered in numerous unit operations, such as reactors, mixers, separators and heat exchangers. Examining the pressure exerted by the flowing material on the walls has practical importance for developing a reliable design. Thus, studying the influence of different wall conditions on granular flow through chutes is crucial.

Granular flows have been extensively studied through experiments, theory and computations, and [1] provides a comprehensive discussion. Scaling laws for granular flows through different geometries have been discussed in [2, 3]. When grains flow through a vertical chute, variations in grain density and size lead to segregation, as reported by [4, 5]. An extended kinetic theory for granular flows in vertical chutes was developed by [6] explaining the formation of distinct flow regimes as particles descend under gravity. The effect of wall roughness on vertical chute flows was studied by [7] who reported the occurrence of cross-stream oscillations when the walls are flat-rough. However, the effects of other wall conditions remain largely less understood. These oscillations appear to arise from thermal agitation at the walls due to particle collisions with it. Because different wall conditions influence the intensity and distribution of these agitations, we expect that they significantly impact the flow behavior.

Here, we investigate the influence of wall conditions on cross-stream oscillations in granular flow down a vertical chute. We find that the agitation produced by a bumpy-smooth wall is more than that of a flat-rough wall for the same solid fraction $\bar{\phi}$ defined as the ratio of the volume of the grains to the volume of the channel. Furthermore, we analyze the scaling of oscillation frequency for flat-rough and bumpy-smooth walls and observe that, with increasing $\bar{\phi}$, the scaled frequency decreases. However, the rate of decrease depends on the wall conditions. Additionally, for a system with one bumpy-smooth wall and the other flat-rough, we examine the thermal agitation, quantified by granular temperature \bar{T} , as a function of time. Our results reveal that, the oscillations gradually dissipate over time.

2 Flow configuration and simulation method

For our simulations we use the discrete element method [8] in three dimensions. Particle-particle and particle-wall contact forces are represented using a nonlinear force model based on the Hertz-Mindlin contact theory along with damping based on the derivation [9]. Coulomb friction (μ) is included in the tangential direction. For the results described here, we use spherical particles 2 mm in diameter with polydispersity of 10 % to impede crystallization. We vary solid fraction $\bar{\phi}$ from 0.20-0.62.; corresponding number N of particles varies from 5981 to 18541. The boundary conditions we use are those of a vertical chute of square cross section of nominal length

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(L) 50 mm in the x, y, and z directions; see Fig. 1. The boundaries are periodic in the x (lateral) and y (vertical) directions. The conditions at the vertical side walls in the z direction conditions are varied systematically, as shown in Fig. 1. The particles in the bulk are modeled as rough, characterized by the parameters: elastic constant for normal contact (κ_n), elastic constant for tangential contact (κ_t), viscoelastic damping constant for normal contact (γ_n), viscoelastic damping constant for tangential contact (γ_t) and μ ; subscripts n and t refer to the directions normal and tangential to the plane of contact, respectively. The values of these parameters may be derived from material properties as described in [9, 10]. Their values are listed in table 1. The contact parameters used in this study correspond to the stiff particle regime, as indicated by the high values of the normal and tangential stiffness constants (κ_n and κ_t). In such regimes, particle deformations are minimal, and the interactions are dominated by collisions rather than sustained contact.

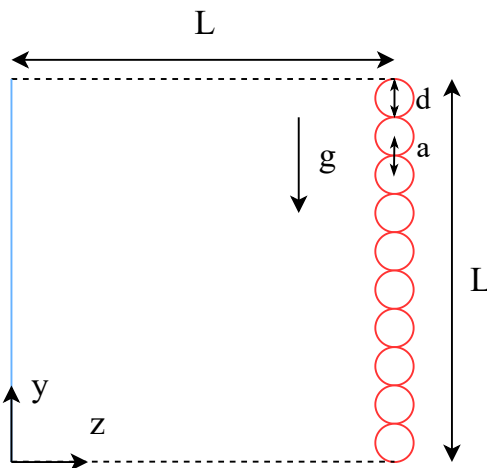


Figure 1. Flow in a vertical channel with periodic boundaries in x (out of plane) and y directions and one bumpy-smooth wall \circ and another flat-rough $|$ in the z direction. In the text we also consider other combinations of sidewalls.

Table 1. Values of contact parameters used in the force model for the DEM simulations

Parameters	values
κ_n	$1.57 \times 10^6 \text{ N/m}^{3/2}$
κ_t	$2.06 \times 10^6 \text{ N/m}^{3/2}$
γ_n	$0.285 \text{ N s/m}^{5/4}$
γ_t	$0.326 \text{ N s/m}^{5/4}$
μ	0.5

In simulating rough surfaces, we account for both tangential and normal interactions between grains and walls, whereas for smooth surfaces, only normal interactions are considered. In each case we use the interaction parameters same as that of bulk particles. Table 2 summarizes the material properties used to characterize the particles-wall interaction. Bumpy surfaces are created by the close packing of 2 mm particles. Bumpiness is characterized by

a , which represents the center-to-center distance between adjacent spheres. In this study, a is taken to be equal to the particle diameter. The boundary particles in the bumpy-smooth wall configuration are the same size as the bulk particles to isolate the effects of wall geometry and roughness. However, varying the boundary particle size can alter the agitation imparted to the flow, as it affects local stress transmission, momentum exchange, and effective roughness. Additionally, the spacing between boundary particle centers, denoted by a , influences the degree of interlocking with bulk particles, thereby impacting flow behavior. Investigating the combined effects of boundary particle size and arrangement on granular flow dynamics constitutes an independent area of research.

We denote the velocity and its components as $\mathbf{u} = \mathbf{u}_x + \mathbf{u}_y + \mathbf{u}_z$ according to directions noted in Fig. 1. For each simulation, the particles are arranged randomly in the chute and then released with small random velocities. After their release, particles collide with one another and with the vertical walls as they accelerate downward. Dissipation of energy via interparticle and wall-particle interactions facilitate a steady state flow.

Table 2. Parameters defining the contact interaction between particles and wall

Bulk particles	$\kappa_n, \kappa_t, \gamma_n, \gamma_t$ and μ
Flat-rough wall & bulk particles	$\kappa_n, \kappa_t, \gamma_n, \gamma_t$ and μ
Bumpy-smooth wall & bulk particles	κ_n and γ_n

3 Results

Consider first the case when both walls in the x direction are flat-rough. Particles descending under gravity collide with the walls, leading to the emergence of different flow regimes depending on ϕ . Fig. 2 illustrates the range of ϕ over which these regimes are observed and these are consistent with the findings of [7] for flat-rough walls. The system exhibits three primary flow regimes. At high solid fractions, the material becomes jammed, leading to a no-flow regime. As the solid fraction decreases, a steady, fully developed flow is observed. Further reduction in solid fraction below a critical threshold results in an accelerated free-fall regime, where the material undergoes unrestricted vertical motion. Between the steady and free-fall regimes, a distinct wave-like regime emerges, characterized by cross-stream oscillations of constant amplitude. Fig. 3 shows the flow in this regime for various wall conditions. This regime is the focus of the present study.

To examine the effect of wall conditions, we conducted a similar analysis when both walls in the z direction are bumpy-smooth. Our results in Fig. 2 indicate that the oscillatory flow regime extends over a broader range of volume fractions in this case. In both wall configurations, oscillations persist for up to the end of our simulations $t \approx 400 \text{ s}$, which is significantly larger than the oscillation time of $t \approx 1 \text{ s}$.

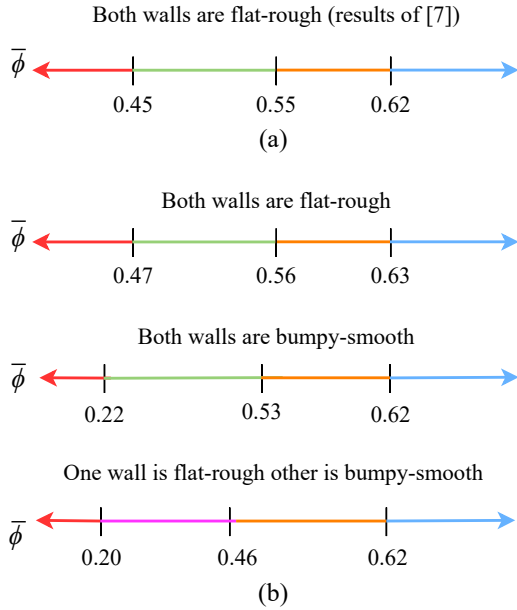


Figure 2. Flow regime map for a vertical chute with different wall conditions. Flow regimes are color-coded as follows: ■ is the accelerated flow regime, ■ indicates sustained oscillatory flow, ■ identifies steady fully developed flow, ■ as no flow, and ■ shows decaying oscillatory flow. (a) Results of [7] for flat-rough walls, and (b) Results for different wall conditions.

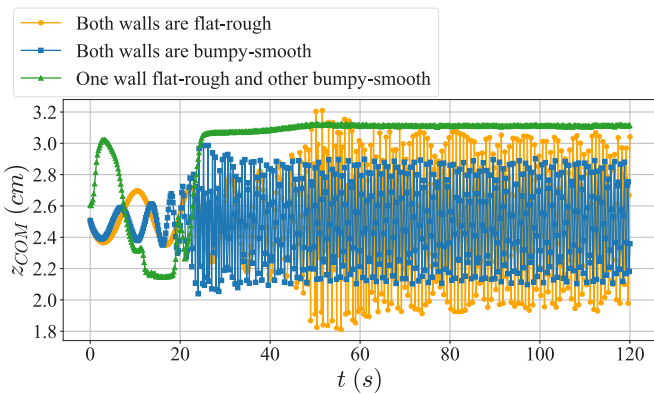


Figure 3. (a) The z coordinate of the centre of mass (COM), z_{COM} of all particles as a function of time t for different wall conditions.

Similar analysis was then performed when one wall is flat-rough and the other is bumpy-smooth as shown in Fig. 4. In this case, within the oscillatory regime, waves were not sustained for the duration of the simulations in contrast to when both walls were either flat-rough or bumpy-smooth. Instead, the oscillations dampened after approximately 50 s. This suggests that asymmetric wall conditions disrupt the sustained energy transfer required for prolonged oscillations.

3.1 Scaling

The sustained oscillations observed in the vertical chute for both flat-rough and bumpy-smooth wall conditions per-

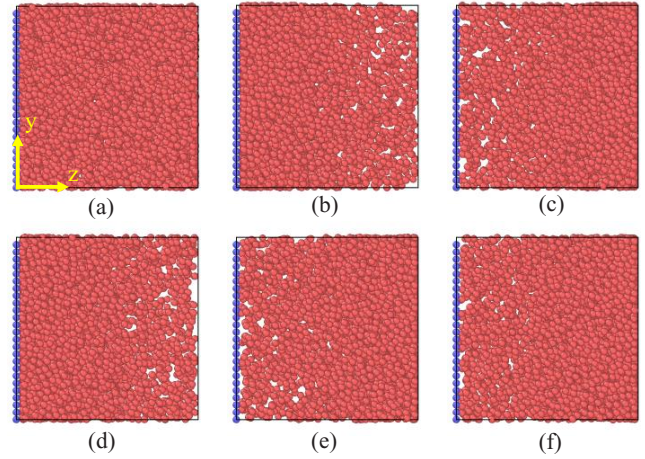


Figure 4. Snapshots of simulation with one wall bumpy-smooth and other flat-rough for $\bar{\phi}=0.35$ at (a) $t = 0$ s (b) $t = 20$ s (c) $t = 35$ s (d) $t = 45$ s (e) $t = 50$ s, and (f) $t = 60$ s

sist over long durations. To characterize these oscillations, their frequency is determined by performing a Fourier transform on the time series data of the z_{COM} as seen in Fig. 3. The frequency corresponding to the peak amplitude in the Fourier spectrum, is identified as the dominant oscillation frequency and the dimensionless scaled frequency $f\sqrt{d/g}$ is plotted against the solid fraction $\bar{\phi}$ in Fig. 5. The results indicate that, as the solid fraction $\bar{\phi}$ increases, the oscillation frequency decreases, with distinct functional dependencies for the two wall conditions. Within the range $0.47 < \bar{\phi} < 0.53$ both symmetric wall conditions exhibit oscillations. However, for the same solid fraction within this range, the flat-rough walls produce oscillations at a higher frequency compared to the bumpy-smooth walls. Furthermore, by adjusting the roughness and bumpiness parameters, we hypothesize that it is possible to identify wall parameters that suggests results comparable to both the flat-rough and bumpy-smooth configurations.

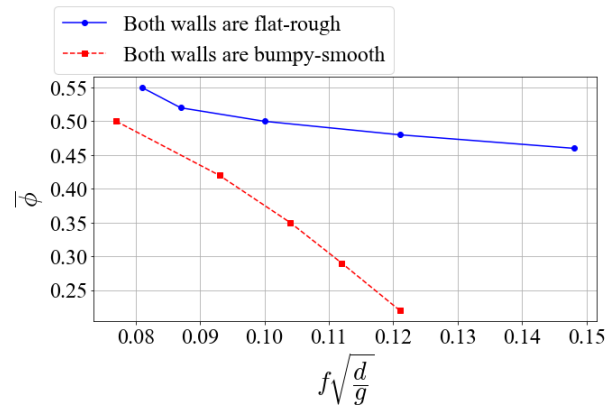


Figure 5. Scaled frequency $f\sqrt{d/g}$ as a function of solid fraction $\bar{\phi}$ for systems with both walls flat-rough and both walls bumpy-smooth.

3.2 Granular temperature

For the asymmetric system with one flat-rough wall and one bumpy-smooth wall, the granular temperature at each wall, defined as $\bar{T} = (\overline{u'u'} + \overline{v'v'} + \overline{w'w'})/3$, where u' , v' and w' denote the deviations of particle velocities from the mean velocities in x , y , and z directions, respectively (these fluctuations are calculated over a thin region adjacent to the wall to capture the near-wall dynamics), is analyzed as a function of time in Fig. 6. The results confirm that oscillatory behavior persists for time $t \approx 50$ s which is much less compared to when both walls are flat-rough or bumpy-smooth, where oscillations persist up to 400 s at least. Beyond this point, the oscillations in the asymmetric system dissipate, and the flow transitions to a steady state. In this state, the velocity fluctuations near the flat-rough wall nearly vanish, whereas those near the bumpy-smooth wall stabilize at $\bar{T} \approx 200$ m²/s².

These findings indicate that the bumpy-smooth wall transfers greater thermal fluctuations compared to the flat-rough wall. By carefully tuning the roughness and bumpiness parameters, it is possible to design a new wall condition that results in equivalent fluctuations at both walls. This remains an important avenue for future research.

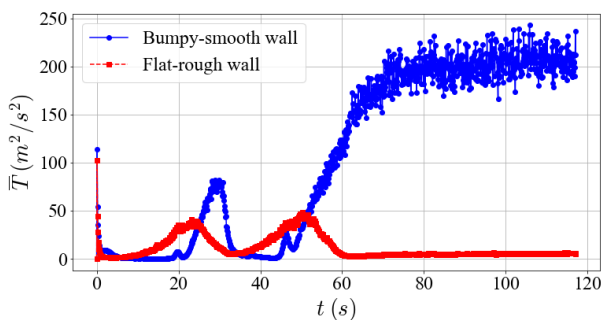


Figure 6. Granular temperature \bar{T} as a function of time t for a system with one wall flat-rough and the other bumpy-smooth at $\phi = 0.35$.

4 Conclusion

The flow of granular material through a vertical channel is investigated, with wall conditions varied as follows: both walls flat-rough, both walls bumpy-smooth, and one wall flat-rough while the other is bumpy-smooth. The analysis is conducted under periodic boundary conditions in both the flow and one spanwise directions. Different flow regimes emerge as we vary the average solid fraction, defined as the ratio of the solid volume to the total channel volume.

The system exhibits three primary flow regimes: accelerated, oscillatory and steady fully developed. Dissipation mechanisms, particularly damping and friction, play a crucial role in determining whether the system reaches a steady state or sustains oscillatory motion. The oscillation frequency decreases with increasing solid fraction, although the specific nature of this dependence varies with wall conditions. Additionally, the granular temperature is analyzed for the case where one wall is flat-rough and

the other bumpy-smooth. The granular temperature at the walls oscillates at a very low frequency before stabilizing which corresponds to the cessation of oscillations. In general, oscillations are strongly influenced by particle-wall friction, highlighting its essential role in sustaining the oscillatory regime. Thus, this study emphasizes the significance of wall conditions in controlling granular flow behavior.

The oscillatory behavior and wave formation observed in vertical granular chute flows have practical relevance across diverse fields. In space exploration, they offer insights into regolith behavior under confinement and gravity, which is critical for infrastructure development on extraterrestrial bodies. Similar flow dynamics are essential in civil engineering for the efficient design of silos and hoppers, where they help prevent clogging and structural failure. In industries such as pharmaceuticals, food processing, and mining, understanding these granular flow regimes enables better transport system design, reducing energy consumption, and minimizing material degradation or loss.

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