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Horizontal size-based segregation in a vertically oscillating box with a sloped bottom

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Abstract. We study the size-based segregation in a vertically oscillating box with a sloped base connecting two flat regions at different elevations, as shown in Figure 2. Some boulders (large size) are placed at random positions inside the box, which is filled with grains (small size) to a certain depth. On vertical oscillation of the box for a long time, boulders segregate towards the flat portion at higher elevation. We attempt to understand this experimental observation by studying the velocity profile of grains and boulder trajectories through Discrete Element Method (DEM) simulations.

1 Introduction

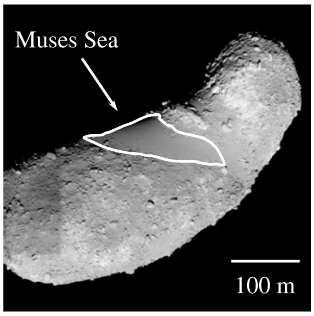

Figure 1: Itokawa as seen by the Hayabusa spacecraft (credits: ISAS-JAXA/NASA Goddard). The smooth patch called Muses sea is at a lower gravitational potential compared to its surroundings.

It is well known that vertically oscillating a box containing two different-sized grains moves the larger ones towards the surface through the eponymous and well-studied Brazil-nut effect. Various mechanisms, including granular convection [2] and void filling, were found to be active, with different mechanisms becoming dominant in different parameter regimes. However, in a typical geophysical scenario involving, say, grains on a shaking topography, we see that the gravity is not aligned with the topographic normal. For example, on the asteroid Itokawa in Figure 1, the regolith, containing large boulders, lies in an undulating topography. During its lifetime, the surface grains are affected by impact-induced seismic vibrations [1], and it is of interest to understand the outcome. This motivates us to study the effect of vertically shaking a mixture of a few boulders (large size) and many grains (small size) in a box with a sloped bottom, i.e. a bottom whose normal is not always aligned with the vertical gravity; see Figure 2.

2 A typical experiment

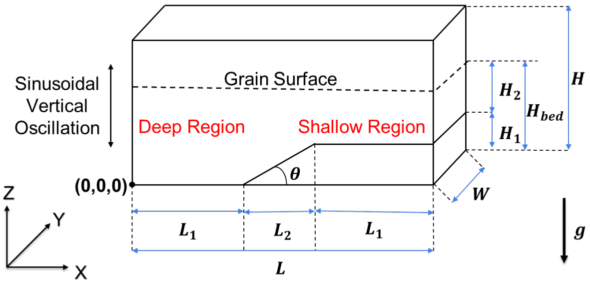

Figure 2: Vertically oscillating box with a sloped bottom, filled with grains, up to a bed depth H_{bed}

Grains (Mustard seeds) of 2 mm average diameter are mixed with some "boulders" (white Peas) of 6 mm average diameter. This mixture is poured into a plexiglass box with a sloped bottom, similar to the one shown in Figure 2. The entire box is oscillated vertically with an amplitude of 2 mm, equal to grain diameter, and a frequency of 12 Hz. Most boulders move toward and settle in the shallow region, i.e., a region at a higher elevation, as shown in Figure 2. After approximately 30 minutes, the system reaches a steady state as shown in Figure 3. Thus, boulders in a vertically oscillated box with a sloped bottom segregate horizontally towards the shallow region.

3 Simulation Methodology

All simulations are performed using LAMMPS on a HPC cluster. The box geometry, as shown in Figure 2, is used. The box is open on top. The walls parallel to XZ plane are periodic and all other walls are fixed. Grains with an average diameter $d = 2\ m$ are filled up to a height $H_{bed} = H_1 + H_2 = 32.5\ d$. Boulder/s of diameter $D = 6\ d$

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Figure 3: Initially, the pale yellow coloured boulders are distributed randomly in the black coloured grain bed. The above figure shows a top view of the final state after oscillating the box for a long time. Most boulders accumulate in the shallow region. The direction of oscillation is normal to the image.

is/are placed at random position/s inside the grain bed.
The box oscillates sinusoidally along the z direction. The key parameters governing sinusoidal oscillation properties are non-dimensional amplitude of oscillation $A/d = 1$ and non-dimensional acceleration $\Gamma = Aw^2/g = 2$, where w is the angular frequency of the oscillation and $g = 9.81\ m/s^2$ is the acceleration due to gravity. So the time period of oscillation T can be computed as, $T = 2\pi\sqrt{A/(\Gamma g)} = 2.006s$. The height H of the box is taken large enough to prevent grains from escaping the box during oscillation.
The interactions are classified into four types: grain-grain, grain-boulder, grain-wall and boulder-wall interactions. For all these interactions, Hertzian spring of stiffness $k = 8.2\times10^7\ N/m^2$ is used to simulate normal interactions and normal coefficient of restitution $e = 0.5$. The tangential interactions are modelled by a tangential frictional force with coefficient of friction $\mu = 0.7$.
The stiffness of the Hertzian spring is taken such that the maximum penetration in any collision is less than 1% of the grain radius [3]. The timestep dt is taken close to $1.2\times10^{-3}\ s$ such that the grain-grain collision event can be resolved in 30 steps [3]. All simulations are performed with the same parameter values as mentioned in this section.

4 Results

4.1 Effect of box dimensions

As shown in the Figure 2, the parameters L_1, L_2, W, H_{bed} and θ uniquely determine the dimensions of the box and the static grain bed. Some of the box parameters are fixed as follows: $L_2 = 40d$, $H_{bed} = 32.5d$ and $\theta = 15°$. These are chosen to imitate the conditions near Muses sea in Figure 1. Ideally, to replicate typical geophysical conditions, L_1 and W should be as large as possible. However, simulating large boxes containing many grains is not computationally feasible. Thus, we performed various simulations to find the critical values of L_1 and W from which the boulder segregates horizontally.

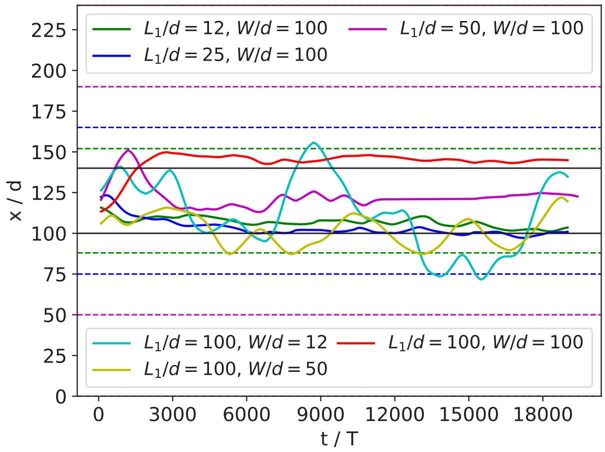

Figure 4: **Solid lines:** Non-dimensional x coordinate of center of mass of boulders (x/d) vs number of oscillations (t/T). **Black-coloured solid horizontal lines:** The extremes of the slope in x direction at $x/d = 100$ and $x/d = 140$. **Dashed horizontal lines:** The extremes of a box in the x direction at $x/d = 100 - L_1/d$ and $x/d = 140 + L_1/d$.
The solid lines are coloured to distinguish simulations with different box dimensions L_1/d and W/d as shown in legend. The dashed horizontal lines are coloured the same as their corresponding solid lines.

Figure 4 shows the non-dimensional x-coordinate of center of mass of boulders (x/d) vs number of oscillations elapsed (t/T) for various W/d and L_1/d values. For all box dimensions with $L_1/d < 100$ or $W/d < 100$, the non-dimensional x coordinate of the center of mass of the boulders exhibits one or a combination of the three behaviours as a function of time: oscillates, decreases, or remains close to its initial value. But for $L_1/d = 100$ and $W/d = 100$ the non-dimensional x coordinate of the center of mass of boulders increases and then remains constant, which clearly indicate a horizontal segregation towards shallow region. Thus, we may conclude that $L_1/d = 100$ and $W/d = 100$ are the critical values from which the horizontal segregation can be observed. Hence, we will proceed with these box dimensions in all simulations.

4.2 Motion of grains

A DEM simulation was performed with only grains in the box with dimensions $L_1/d = 100$ and $W/d = 100$. The box is divided into 3-dimensional grid containing cubical cells of side length 3d. Velocity is defined at each cell by assigning velocity of center of mass of grains contained in it. The time-averaged velocity field is computed and found to be two-dimensional, with insignificant velocity component or variation along the y-direction. The velocity field is non-dimensionalised with reference velocity $v^* = d/T$. A thin slice of this non-dimensional velocity field, parallel to the XZ plane, is visualised as shown in Figure 5.
A pair of counter-rotating circulation loops of different sizes can be observed in Figure 5. The bigger circulation loop spans across the deep region and approximately half of the shallow region. The smaller one approximately

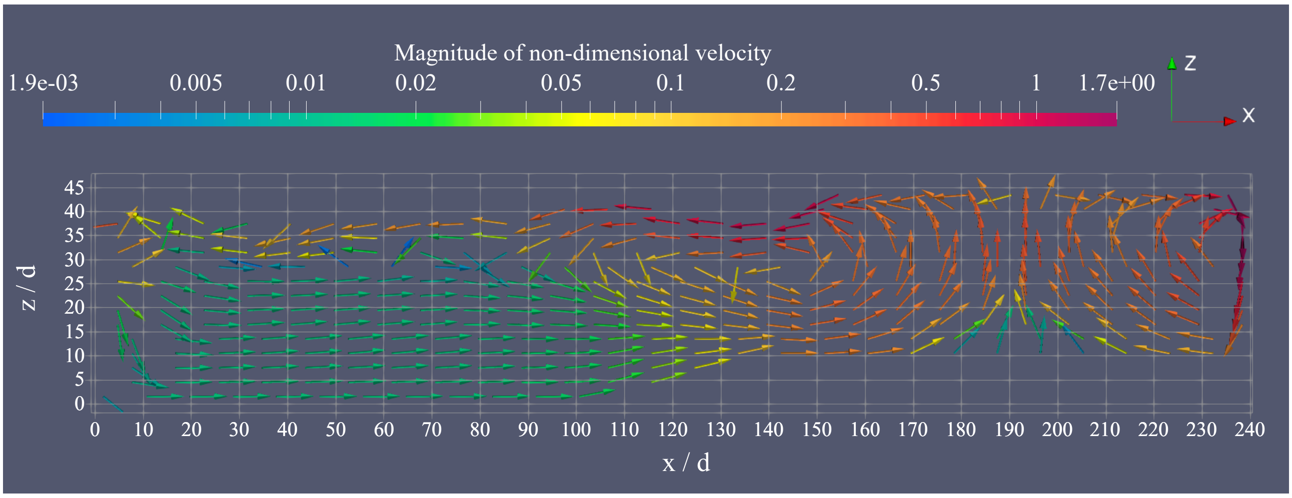

Figure 5: Cross-section parallel to XZ plane showing flow field. 1 Unit length = 1 grain diameter, i.e., the length scales(x,z) are non-dimensionalised by dividing with grain diameter d. The arrow and its colour represent the direction and magnitude, respectively, of the non-dimensional time-averaged velocity. The colour bar is taken on a log scale to clearly distinguish the order of magnitude difference in the values in different regions.

spans the other half of the shallow region. The flow on the surface and near the right wall ($x/d = 240$) has a higher speed. The flow in the bulk has a relatively lower speed. The bulk flow speed in the deep region is around 10 times lower than the bulk flow speed in the shallow region. Similar circulation loops are observed in experiments with a quasi-2D box.

4.3 Motion of a single boulder

Now, simulations were performed by inserting a boulder inside the grain bed. The boulder's position data is recorded, smoothed, and non-dimensionalised using the grain diameter d, and the results are shown in Figure 6a and Figure 6b. The velocity of the boulders is computed from the smoothed position data, non-dimensionalised using reference velcoity $v^* = d/T$ and compared with the time-averaged non-dimensional velocity of nearby grains as shown in Figure 6c. The time-averaged non-dimensional velocity of nearby grains is computed as mentioned in the previous section.

From Figure 6a we can see that a boulder starting from any initial position reaches close to a particular region where both the circulation loops meet, as shown in Figure 5. We can also observe that in the deep region, the boulders travel in the XZ plane without any significant motion along the y direction. The boulders starting from the deep region, irrespective of their initial position, always get pulled towards the slope and start travelling up the slope.

Figure 6b shows that all the boulders, irrespective of their initial position, reach close to a particular line in the y direction ($(x/d, z/d) \approx 170, 23$) at the same time ($t/T \approx 2800$). This attractor line doesn't move much until 4500 oscillations. After that, it evolves to $(x/d, z/d) \approx (200, 22)$ by 5000 oscillations.

Figure 6c clearly shows that the boulder's velocity is significantly higher than nearby time-averaged grain velocity. Instead of being carried by grains, the boulders appear to move faster than grains around them.

5 Conclusions

Vertically shaking a mixture of a few large grains (boulders) and many small grains in a sloped-bottom box caused horizontal segregation of boulders towards the shallow region at a higher elevation. A small circulation loop of grains spanning half of the shallow region and a large circulation loop of grains spanning the remaining region were observed.

We found that to achieve movement of boulders towards the shallow region, the width (W) and flat portion's length (L_1) should be at least close to 100-grain diameters ($100d$).

In the single boulder simulations, all the boulders in independent simulations reach close to an attractor line along the y-axis at the same time. We also observed that, instead of being carried by the flow of grains, the boulders move faster than the grains around them.

It is possible that on Itokawa, seismic shaking may have driven boulders upslope from the low-lying muses sea [1]. We are presently examining this hypothesis more thoroughly.

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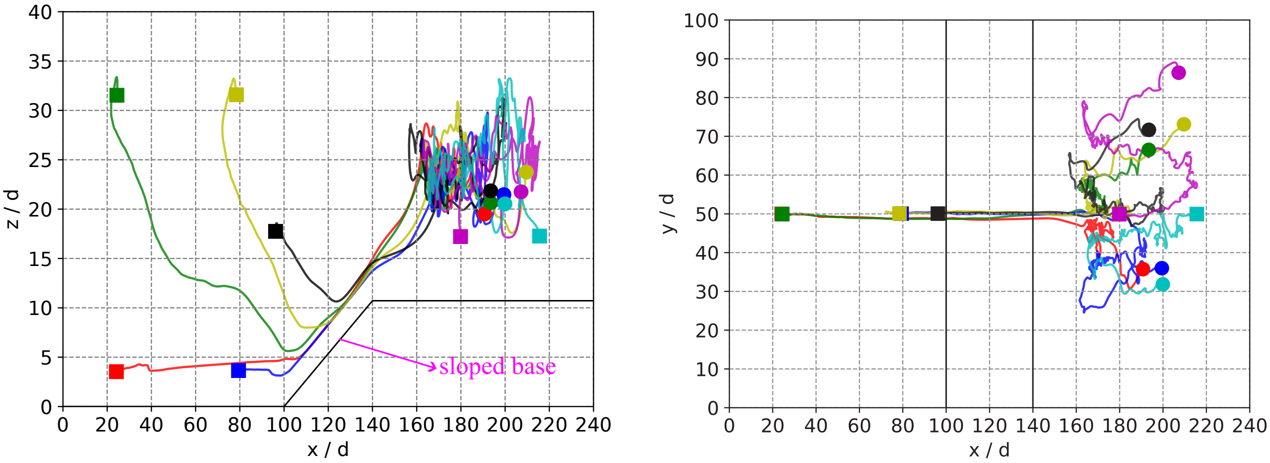
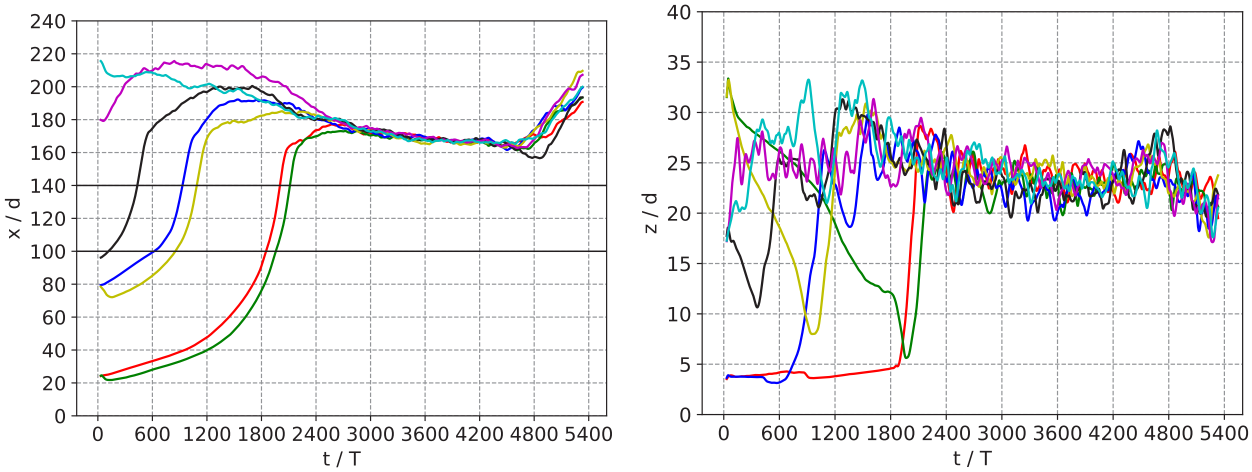
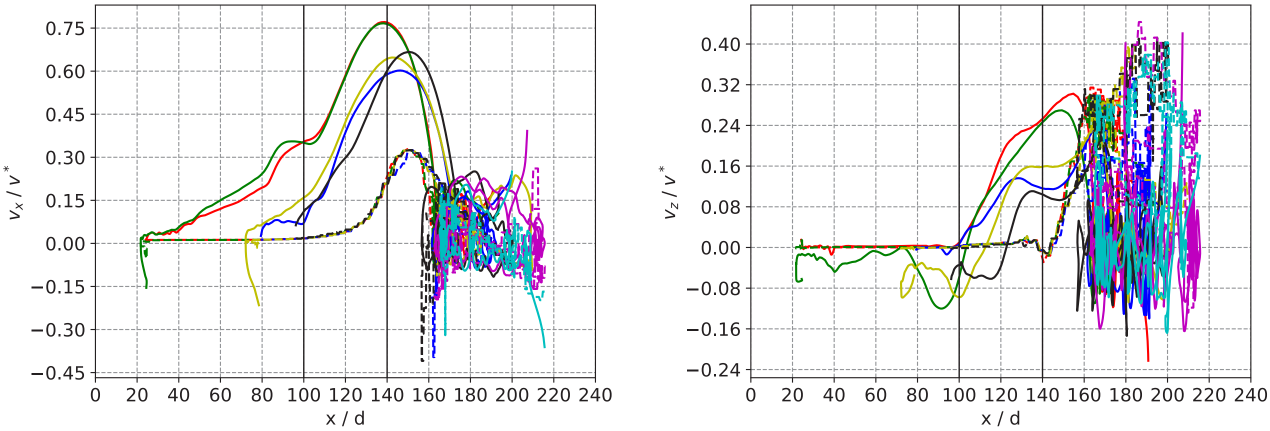


(a) Side view(X-Z plane) and top view(X-Y plane) of paths of a single boulder placed inside a grain bed in an oscillating box with a sloped bottom. The square and circular markers indicate the boulder's position at the start and end of a simulation, respectively.



(b) Non-dimensional coordinate of a boulder vs Number of oscillations elapsed.



(c) Non-dimensional velocity components of boulder and nearby grains vs non-dimensional x coordinate of a boulder. Solid and dashed lines correspond to the velocity of a boulder and time-averaged velocity of its nearby grains, respectively. Dashed lines have the same colour coding as their corresponding simulation's solid line.

Figure 6: Lines and markers are coloured to distinguish simulations with different initial positions of the boulder. The same colour coding is used in all the sub-figures.