

# Experimental investigation of the effect of grain morphology on packing density of granular assemblies

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**Abstract.** The physical and mechanical properties of granular media at the macro scale are greatly influenced by the morphology of its grains at the micro-scale. The morphology of grains is a multi-level property, with the morphological features at each level being independent. Thus, while understanding the effect of the granular morphology on the macro scale behaviour of granular media like sands, it becomes imperative to understand the influence of individual morphological features. The current study investigates the effects of macro and meso-scale morphological properties of grains on the packing density of granular media. Virtual granular assemblies with controlled shape parameters are generated using the spherical harmonics algorithm. The shapes of the grains in the virtual granular assemblies are varied to understand the effects of individual morphological properties at different levels. Polyjet 3D printing is employed to generate synthetic grains from virtual grain assemblies. Further, experimental investigations are performed using these synthetic grains with controlled shapes to estimate their minimum and maximum packing density. The effects of particle form and roundness on the packing density are analysed. The results highlight that the particle shape significantly affects the packing density of granular media at loose and dense states with packing density values decreasing with a decrease in roundness and regularity.

## 1 Introduction

Packing density is a critical aspect of a granular assembly governing its mechanical properties, thus being an important parameter in several fields, including metallurgy, pharmaceuticals, and civil engineering. In the field of geotechnical engineering, it has been found that the packing properties of soil influence its physical, mechanical, and hydraulic behaviour [1-3]. Hence, understanding the particle packing properties of granular assemblies is of great importance for a comprehensive understanding of fundamental soil behaviour.

The particle packing properties of grains are, in turn, affected by the size of the grains, grain morphology, and the grain size distribution. The effects of these intrinsic properties of the granular assembly on the packing density have been studied through numerical and experimental investigations [3-5]. To understand the effect of particle morphology, which describes the shape of the grains at macro, meso, and micro scales, on the packing density of soil, experimental investigations involving various soil samples with distinct morphological characteristics have been conducted [3]. In addition, attempts have been made to correlate the packing density values previously reported in the literature to the morphology of the soils [5].

However, experimental investigations are restricted by the limited control over particle morphology, as it is extremely difficult to decouple the effect of one morphological parameter from the other in the case of

natural soil. In this context, 3D printing of granular media with control over individual morphological properties has become an extremely useful tool for investigating the effect of particle morphology in the field of geomechanics. 3D-printed granular media have been used to investigate the effects of grain shape on permeability, stress-dilation characteristics, and dynamic behaviour of sands [1, 6-7].

The current study uses 3D-printed granular assemblies with distinct morphological features to study the effect of particle morphology on the minimum and maximum packing densities of granular assemblies. Spherical harmonics (SH) based algorithm is used to generate granular assemblies with controlled morphological parameters. Further, the synthetic granular assemblies are manufactured using polyjet 3D printing technology, which are then used to conduct experimental investigations. The inferences from the current study are critical in understanding the micro-scale behaviour of granular assemblies.

## 2 Generation of virtual granular assemblies

### 2.1 Quantification of granular morphology

The grain morphology was quantified in terms of their aspect ratio ( $AR$ ), roundness ( $R$ ), sphericity ( $S$ ), and regularity ( $\rho$ ). Aspect ratio is defined as the average of the elongation index ( $EI$ ) and flatness index ( $FI$ ) of the

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grains [8] to indicate the form of the grains, where  $EI$  and  $FI$  are calculated as

$$EI=d_2/d_1 \quad (1)$$

$$FI=d_3/d_2 \quad (2)$$

Roundness was calculated using the formula proposed by [9], in which the 3D roundness is defined as the average value of curvatures of all potential corners. In the current study, the potential corners are identified based on the mean curvature value of a given vertex on the SH-reconstructed surface using Equation 3.

$$g(k)=1, \text{ if } r_m \leq r_{ins}, g(k)=0, \text{ if } r_m \geq r_{ins} \quad (3)$$

Then, the 3D roundness ( $R$ ) is calculated as

$$R=(1/N) (\sum g(k) |k_m|^{-1}) \quad (4)$$

where  $N$  is the total number of corners identified, and  $k_m$  is the mean curvature value of a corner calculated as the average of minimum curvature  $k_{min}$  and maximum curvature  $k_{max}$  [10].

Sphericity ( $S$ ) is calculated as the ratio of the diameter of the maximum inscribed sphere to that of the minimum circumscribing sphere [3], as given by Equation 5.

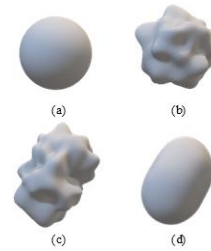
$$S=d_{ins-max}/d_{circ-min} \quad (5)$$

Aspect ratio and sphericity represent macro-scale shape parameters indicating the overall form of the grains, while roundness represents the mesoscale features. A combined shape parameter, regularity ( $\rho$ ), is calculated as the average of sphericity ( $S$ ) and roundness ( $R$ ). Thus, regularity represents the combined effects of macro and mesoscale features [11].

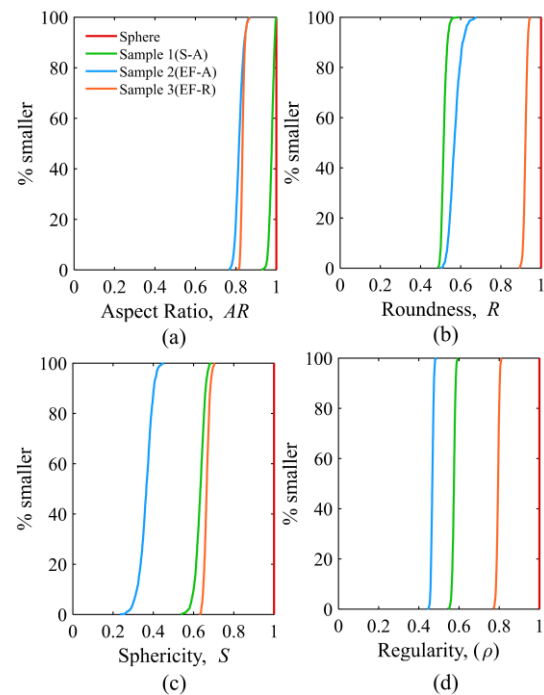
## 2.2 Spherical harmonic reconstruction of grain surfaces

SH analysis has been widely adopted to reconstruct sand particle morphology and quantify the 3D shape parameters [9,12]. In the current study, the SH-based algorithm proposed by [12] is used to generate virtual sand assemblies with controlled morphological parameters. Three samples, each constitutive of 1000 grains were generated using the algorithm. These samples, along with a sample constituted of spheres, were used in the present study to investigate the effects of particle morphology on the packing density. To eliminate particle size effects, all the grains were ensured to have the same equivalent spherical diameter, i.e. the diameter of a sphere with volume equal to that of the grains. Figure 1 shows the typical grain geometries constituting each of the four samples. The cumulative distributions of aspect ratio, roundness, sphericity, and regularity are shown in Figure 2. As can be seen from the figures, sample 1 (S-A) consists of grains that are spherical at the macro-scale and angular at the mesoscale. Sample 2 (EF-A) consists of grains that are

elongated/flat and angular, whereas sample 2 (EF-R) consists of elongated/flat and rounded grains. It should be noted that a terminology indicative of the shapes of grains in each sample is used, with the acronym of the macro and meso scale shape features added in parentheses to facilitate a better understanding of the analysis of results. It can also be seen that the distributions of shape parameters are close to uniform, indicating that the samples consisted of grains with distinct particle shapes, but having similar morphological features.



**Fig. 1.** Virtual shapes generated: (a) Sphere, (b) Sample 1 (S-A), (c) Sample 2 (EF-A), (d) Sample 2 (EF-R)

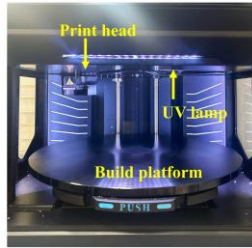


**Fig. 2.** Cumulative distributions of shape parameters: (a) Aspect ratio, (b) Roundness, (c) Sphericity, and (d) Regularity

## 3 3D printing of grain assemblies and packing density estimation

In the current study, polyjet technology, which is a resin-based 3D printing technology, was used to carry out the 3D printing of the virtual granular assemblies. Recently, polyjet technology has been widely adopted to generate granular assemblies consisting of sand particle analogues [1,7]. Further, it has been shown that polyjet printing technology successfully reproduced the shape of sand particle analogues at different levels of detail with sufficient accuracy [13]. In polyjet 3D printing

technology, the nozzles on the print heads deposit drops of photopolymer resin onto the build platform, which is then cured with ultraviolet (UV) lamps. While printing granular particles, two different kinds of resins are deposited, one forming the structure of the grain and the other acting as the support material. Once the 3D printing is completed, the support material is removed from the printed grains by dissolving in a caustic solution. In the current study, 3D printing was carried out using a Stratasys J55 printer, using its Draft White resin and a layer thickness of 19  $\mu\text{m}$ . Figure 3 shows the image of the 3D printer used in the study.



**Fig. 3.** Polyjet 3D printer used in the current study.

Each 3D-printed sample consisted of 10,000 grains and was further used to carry out experimental investigations. The minimum and maximum packing density of the synthetic granular assemblies were estimated by the method delineated by [14]. In this method, the minimum packing density is calculated by gradually rotating a 1000 mL graduated cylinder with the grains by 180 degrees and then measuring the volume of grains occupied by grains in the cylinder. The maximum packing density is calculated by measuring the volume of grains occupied by grains in a 1000 mL graduated cylinder after applying a specified number of blows to the cylinder from a constant distance with a screw driver whose weight is known. This method has been proven to give results similar to those provided by the ASTM D4254 and ASTM D4253 methods [14]. Packing density was calculated as the ratio of the volume of grains to the total volume.

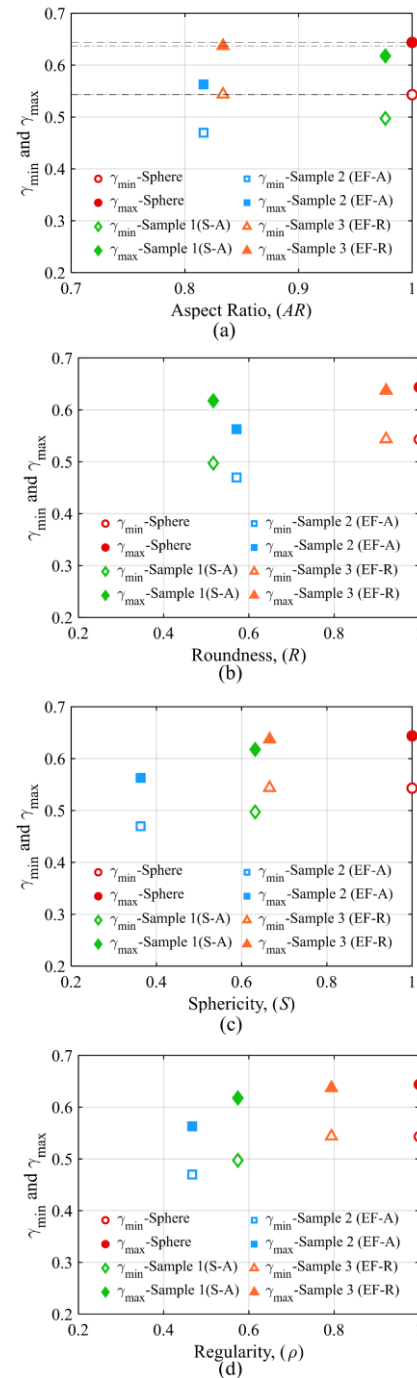
## 4 Results and discussions

Figure 4 illustrates the variation trends in the minimum packing density ( $\gamma_{\min}$ ) and the maximum packing density ( $\gamma_{\max}$ ) as functions of the shape parameters considered in the current study. The mean values of the shape parameters are compared, considering the almost uniform distribution of shape parameter values, as evident from Figure 2. It should be noted any variation in the values of  $\gamma_{\max}$  and  $\gamma_{\min}$  of the samples would have resulted from the combined effect of different shape parameters of the samples.

However, from Figure 2, sphere and sample 2 (EF-R) have roundness values very close to each other, with their aspect ratio values being different. This is also true for sample 1 (S-A) and sample 2 (EF-A). Hence, the effect of aspect ratio can be compared for samples sphere and sample 2 (EF-R) and sample 1 (S-A) and sample 2 (EF-A). The variation of  $\gamma_{\max}$  and  $\gamma_{\min}$  with aspect ratio is depicted in Figure 4a. It can be observed

that while the  $\gamma_{\max}$  and  $\gamma_{\min}$  values of the sphere and sample 2 (EF-R) do not differ much, sample 2 (EF-A) has lower packing density values than sample 1 (S-A).

Similarly, from Figure 2, samples sphere and sample 1 (S-A) have almost the same aspect ratio but a larger variation in roundness, which is also the case with sample 2 (EF-A) and sample 2 (EF-R). Thus, the effect of roundness can be compared for sphere and sample 1 (S-A) and sample 2 (EF-R) and sample 2 (EF-A). From Figure 4b, compared to spheres, sample 1 (S-A) has lower values of  $\gamma_{\max}$  and  $\gamma_{\min}$ , and compared to sample 2 (EF-R), sample 2 (EF-A) has lower values of  $\gamma_{\max}$  and  $\gamma_{\min}$ .



**Fig. 4.** Variation of  $\gamma_{\max}$  and  $\gamma_{\min}$  with shape parameters: (a) Aspect ratio, (b) Roundness, (c) Sphericity, and (d) Regularity

Also, from Figure 2, sample 1 (S-A) and sample 2 (EF-R) have almost the same values of sphericity, but from Figure 4c, their  $\gamma_{\max}$  and  $\gamma_{\min}$  values are different. Thus, it can be inferred that both macro-scale and meso-scale morphological features contribute to the variations in the packing density of the samples. This is also evident from Figure 4d, where the variation of  $\gamma_{\max}$  and  $\gamma_{\min}$  with regularity is plotted. Figure 4d shows a trend of decreasing packing density values with decreasing regularity of the particles. Similar trends between packing density and regularity of the grains have been observed by previous researchers [3,12] for sands and similar granular assemblies. However, the trend is non-linear, indicating possible non-linear trends in the variation of packing density with the morphological parameters at the macro and meso-scales. Also, the contributions of macro and meso-scale morphological features towards the variations in  $\gamma_{\max}$  and  $\gamma_{\min}$  values are different, thus resulting in the trends observed in Figure 4. For example, while the differences between the aspect ratio values of spheres and sample 2 (EF-R) and sample 1 (S-A) and sample 2 (EF-A) are the same, the differences in their respective  $\gamma_{\max}$  and  $\gamma_{\min}$  values are dissimilar. Thus, to understand the effect of particle morphology on the packing density of granular assemblies, the individual effects of shape parameters at different levels of morphological detail have to be studied, which warrants future investigations.

## 5 Summary and conclusions

The current study investigated the effect of the morphology of grains on the minimum and maximum packing density of granular assemblies using 3D-printed grains with controlled morphological features. The results indicate that the particle morphology does affect the minimum and maximum packing densities of granular materials, with distinct contributions towards the packing density variation from the morphological features at macro and meso-scales. Further, it was found that the samples with lower roundness values had lower values of maximum and minimum packing densities, whereas no definite conclusion could be drawn on the effect of aspect ratio on packing density values. Also, the maximum and minimum packing density values were found to decrease with a decreasing value of the regularity of the grains.

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