Compaction mechanics of soft-rigid mixtures considering realistic particle morphology

Ashik Anwar[1,*], *N.S.S.P.* Kalyan[1,**], and *Ramesh Kannan* Kandasami[1,***]

[1]Department of Civil Engineering, Indian Institute of Technology Madras, India

Abstract. Soft-Rigid granular mixtures (SRM) demonstrate complex compaction behaviour due to the pore-filling mechanism exhibited by the deformable particles. The current study provides a micro-mechanical perspective of the compaction behaviour of these soft-rigid granular mixtures (SRM- 30%, i.e., 30% deformable particles) and pure rigid assembly (SRM-0%, i.e., 0% deformable particles) having rigid realistic angular morphologies. The study first compares the variation of the void ratio during the gravity deposition over a range of inter-particle friction coefficients between zero and one. Furthermore, the study specifically focuses on understanding the evolution of pore size distribution during oedometer compression. A normal pressure at a quasi-static rate is applied on the granular assembly to achieve the 80% of densest volume fraction. The addition of soft deformable particles alters the compaction behaviour by specifically decreasing the void ratio more rapidly under minimal compaction effort, compared to a fully rigid particle assembly. The presence of deformable particles in the mixture enhances the pore filling mechanism which is quantified by computing the pore size distribution at two different stages of compaction. The post-compaction probability distribution of the pore radius infers that the smaller voids are drastically increased due to the addition of deformable particles.

1 Introduction

Heterogeneous soft-rigid granular mixtures, such as the rubber-sand composites, have shown improved performance over pure sand or gravel aggregates in various geotechnical applications, including seismic isolation, reducing ballast degradation in railways, and improving the stability of retaining structures and embankments [1, 2].

While shear strength is often the primary criterion for designing sand-rubber mixture proportions, an equally important property is their compressibility [3], as it directly influences the hydraulic conductivity of these mixtures [4]. In this context, the pore size distribution (PSD) and its evolution under compaction are critical parameters, as they govern the connectivity and constriction of flow pathways within the granular assembly [5]. Therefore, characterizing PSD is essential for predicting and optimizing the hydraulic performance of soft-rigid mixtures when used as filler materials in geotechnical applications [6]. The evolution of the PSD during compaction is governed by the combined influence of soft particle content, particle morphology, and inter-particle friction. The deformation of the soft particles alters the pore structure initially present between the rigid particles, thereby influencing the packing fraction and the internal fabric of the granular assembly. Experimental estimation of PSD from the soft-rigid granular mixtures is extremely difficult and often relied on statistical analysis assuming simple particle shapes [6].

The present study employs 2D discrete numerical simulations that incorporate particle-level deformation to accurately capture the soft nature and contact interactions within granular mixtures. Despite the significant influence of soft particle deformation, previous studies [4, 7] have been constrained by the lack of accurate simulation strategies capable of representing the realistic continuum deformation of soft particles. First, the effect of contact friction on the initial packing of soft-rubber granular mixtures (SRM-30) is examined and compared with a fully rigid granular assembly (SRM-0). Furthermore, an oedometer test is simulated for both purely rigid and soft-rigid granular mixtures. The primary objective of this study is to analyse the influence of soft particle inclusion on the PSD and compressibility characteristics of granular assemblies during an oedometer test.

2 Numerical framework

Traditional soft contact Discrete Element Method (DEM) is limited in its ability to capture the deformation of rubber-like soft deformation, which is crucial for accurately modeling soft-rigid granular mixtures [4]. To address this limitation, the present study employs a coupled DEM and Element-Free Galerkin (EFG) framework. The simulations are conducted using an open-source numerical solver - MELODY 2D [8]. DEM is used to simulate contact interactions within the multi-body system, while the EFG framework simulates the continuum deformation of individual soft particles in the mixture. To simulate the deformation of soft particles, each individual particle is discretized using a unstructured set of nodes. The dis-

*e-mail: ce22b044@smail.iitm.ac.in
**e-mail: ce19s020@smail.iitm.ac.in
***e-mail: rameshkk@iitm.ac.in

placement field within each soft particle is computed by solving the governing equations by applying the external contact forces and the boundary conditions.

To model the rubber-like deformation of soft particles, the Neo-Hookean hyperelastic constitutive model is employed [8]. Each deformable particle is treated as a continuum body, with intrinsic material and contact-related properties such as Young's modulus, Poisson's ratio, and friction. The normal and tangential stiffness between all particles are set to 1 GPa. The Young's modulus (E) and Poisson's ratio (ν) for the soft particles are set to 5 MPa and 0.495 respectively and the coefficient of friction (μ) is 0.5 between all soft-rigid particles. The contact friction (μ_w) between the walls and the particles is set to 0.01. This methodology enables a detailed investigation of the mechanical behaviour of granular mixtures, particularly in systems with diverse particle morphologies and composite additives like rubber.

3 Initial packing

In DEM simulations, the desired packing fraction, ranging from the loosest to the densest state, is typically achieved by controlling the contact friction. This study first examines the packing characteristics of granular mixtures, with a particular focus on the influence of inter-particle friction on packing density. Two types of mixtures are considered: (i) pure rigid granular assemblies (SRM-0) and (ii) composite mixtures containing 30% rubber particles by number (SRM-30). To assess the effect of friction, inter-particle friction coefficients (between rigid-rigid, rigid-soft and soft-soft, $\mu = \mu_{rr} = \mu_{rs} = \mu_{ss}$) are systematically varied from 0 to 1, and their impact on the resulting packing densities are analysed. The minimum and maximum void ratios obtained from this preliminary investigation serve as reference bounds for subsequent oedometer compaction simulations, where the mixtures are compacted to 80% of their maximum relative density.

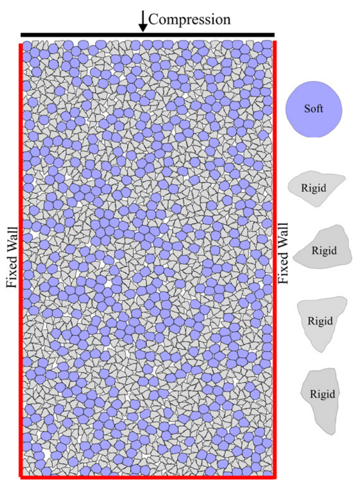

Figure 1: 30% Soft-Rigid Mixture (SRM-30) made of rounded deformable particles and angular non-convex rigid particles.

The granular composite mixtures consist of both rigid and soft particles. The rigid particles exhibit four distinct angular morphologies (as shown in Fig. 1), extracted from high-resolution images using the method described by Kalyan and Kandasami [9]. Based on their measured roundness of 0.3 and sphericity of 0.55, with an equivalent diameter of 4 cm, these particles are classified as highly angular [9]. In contrast, soft rubber particles are modeled as circular and deformable, with a diameter same as the rigid particles (as shown in Fig. 1). Initially a total of 2000 particles are arranged in a uniform spaced grid with randomized orientations and placement of rubber. These particles are then allowed to settle under gravity within a rectangular container (as shown Fig. 1) until a mechanically stable configuration is achieved (velocity $\approx$ 0). To achieve equilibrium condition, contact damping (ζ) for all particle interactions is set to 0.7. The simulations are conducted for six discrete inter-particle friction coefficients: 0, 0.1, 0.25, 0.5, 0.75, and 1.0.

After the packing process, the contour of each particle is extracted, and a tight convex polygon is constructed to encompass all particle boundaries, thereby defining the total occupied area ($A_{particles} + A_{void}$). From this, the total void ratio area is calculated using the precomputed areas of individual particles($A_{particles}$), allowing the packing void ratio to be determined across different friction values. Similar process is employed for the SRM containing 30% of rubber particles by number, to assess the influence of soft particle inclusion on void ratio variations. Three random packing arrangements (SRM-0 and 30) for each friction value are simulated to obtain a range of void ratios. These measurements offer critical insights into the dependence of maximum (e_{max}) and minimum (e_{min}) void ratios on the inter-particle friction for these two-dimensional DEM particles.

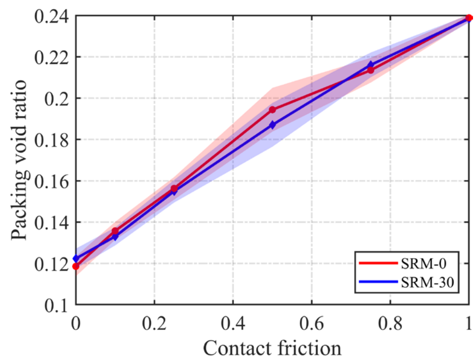

Figure 2: Variation in initial packing void ratio with friction coefficient for completely rigid particles (SRM-0) and 30% soft particle (SRM-30) for three samples each.

In Fig. 2, the trend lines corresponding to SRM-0 and SRM-30 represent the mean void ratios obtained from the gravity deposition simulations, while the shaded bands indicate the standard deviation computed from three independent realizations of random particle arrangements. It illustrates the variation in the initial packing void ratio (e) as a function of friction. Both the rigid granular assembly and the soft-rigid mixture exhibit an upward trend with increase in friction values, with the e_{min} occurring at zero friction. At very low friction values, particles are free to rearrange, leading to the densest possible packing. The maximum void ratio (e_{max}) is observed at the highest friction value of 1.0, where increased inter-particle resistance restricts rearrangement, resulting in the loosest packing under gravity.

Interestingly, the packing void ratio (e) of the rigid and soft-rigid assemblies remains largely similar across all inter-particle friction values. However, in the intermediate friction range of 0.4 to 0.6, SRM-0 shows a slight increase in e, suggesting the development of a relatively loose packing structure. This is attributed to the deformability of the rubber particles, which enables them to compress and occupy interstitial voids that would otherwise remain unfilled in a system comprising only rigid particles.

4 Oedometer test

Using a friction value of 0.5 and the corresponding initial packing density from Fig. 2, the two assemblies discussed in the previous section are subjected to compression in the vertical direction with the side walls of the container fixed with zero lateral strain. The compaction is applied through a piston lid descending at a constant velocity of 5 cm/s (inertial number I $< 10^{-3}$). During the loading process, the normal force exerted on the piston by the particles and the positions of the particles relative to the wall movement are recorded. These force measurements are then correlated with the evolving void ratio over time to construct the pressure–void ratio response of each system (see Fig. 3). The minimum void ratio obtained with zero friction serves as a reference bound for compaction and therefore the simulation is halted when we achieve 80% of the maximum relative density. Excessive compaction beyond this threshold leads to non-physical and significantly large overlaps at the particle contacts specifically post minimum void ratio threshold.

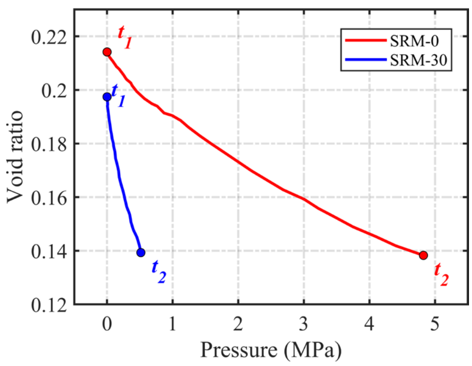

Figure 3: Void ratio *vs*. pressure for 0% and 30% rubber with contact friction $\mu = 0.5$

Fig. 3 illustrates the compaction behavior of the two granular assemblies i.e., the rigid assembly SRM-0 and the composite mixture SRM-30. The void ratio decreases with increasing pressure, reflecting the typical densification response under oedometer compression. A distinct difference is observed between the two mixtures. The rigid particle assembly (SRM-0) exhibits a gradual reduction in void ratio with increasing pressure, indicating that compaction primarily occurs through particle rearrangement and interlocking. The presence of highly angular rigid particles limits extensive densification, requiring a high compaction effort to each densest packing. In contrast, the composite mixture (SRM-30), which includes deformable rubber particles, undergoes a more rapid decrease in void ratio at lower pressures. This behavior suggests that the inclusion of soft particles enhances the compressibility of the granular mixture. The deformability of rubber particles allows them to accommodate local stresses more effectively, leading to a steeper compaction curve. By calculating the slope from the Fig. 3, the compaction indices are 0.0201 for SRM-0 and 0.0352 for SRM-30. The steeper slope observed in the SRM-30 mixture indicates a higher compaction index compared to SRM-0, demonstrating that the inclusion of soft particles significantly improves the packing efficiency under compression. Unlike the rigid assembly that relies on rearrangements and interlocking, the composite mixture exhibits enhanced compressibility as the deformable particles facilitate large coordination number ($Z = 8.3$ at t_2) and stable contacts, resulting in a large reduction in void ratio at lower pressures (see Fig. 4). The coordination number of SRM-0 at t_2 is observed to 4.5. As the rubber fills the voids during compaction, the contact points with rigid increases resulting in high coordination number.

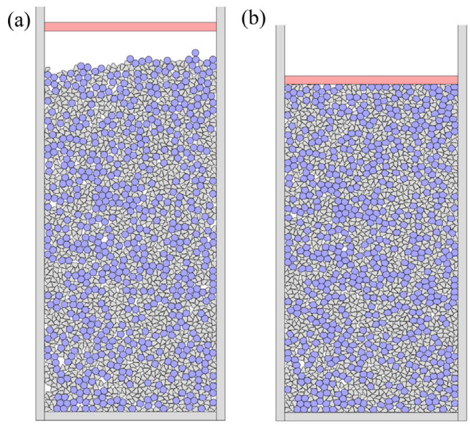

Figure 4: (a) SRM-30 particle arrangement just before compaction (t_1) and (b) Final particle arrangement post compaction stage t_2.

5 Pore size analysis

The current study proposes a novel 2D pore pixelization method to compute pore size distribution from polygonal granular system. The particle surface nodal coordinates are extracted at two stages of compaction (initial t_1 and final t_2). These polygons are then used to generate binary masks, where regions corresponding to particles are assigned a value of zero, while the void spaces remain as ones. The pore space formed by the particles in Fig. 5a represents the segmented void, with each void colour-coded based on its area after extracting the void space boundaries using MATLAB. Connected component analysis (*bwconncomp*) is utilized to segment the void regions and then a unique index is assigned to each distinct

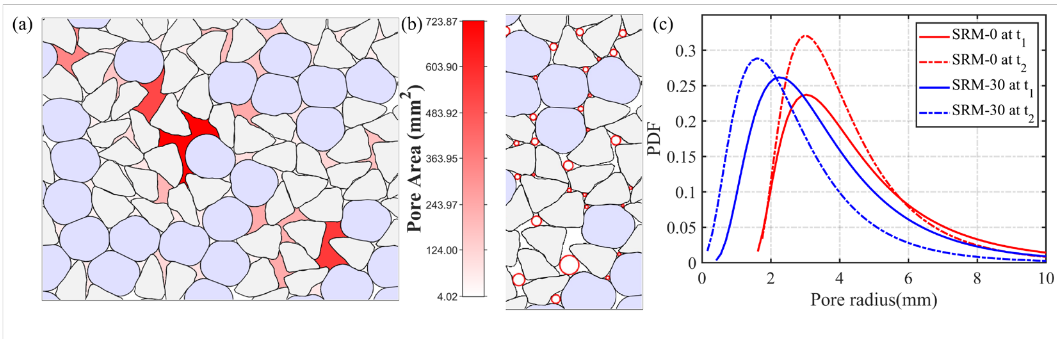

Figure 5: (a) Identification of individual pore spaces formed between particles in SRM-30, (b) Inscribing the largest circle in the identified pore spaces, and (c) Evolution of pore size distribution compared between SRM-0 and SRM-30.

pore region as shown in Fig. 5a. The segmented pores include both pore chambers and connecting throats. Since pore throats only serve as connections, we focus on measuring pore chamber sizes without considering the throats. Additionally for each void space, the Euclidean distance map is quantified to obtain the maximum inscribed circle as show in Fig. 5b. The region inside the inscribed circle represents the pore size, while the area outside it can be considered as pore throats. The void regions are then categorized based on these measured maximum inscribed radius to generate a pore size distribution. Fig. 5c shows the evolution of pore size distribution from the initial state (t_1) to the post-compaction state (t_2) for both SRM-0 and SRM-30. In the rigid assembly, the pore size distribution at t_1 exhibits a slightly skewed peak relative to t_2, indicating only a minor change in pore structure. This limited shift is attributed to the angular shape of the rigid particles, which interlock and maintain the existing void structure during compression. In contrast, the pore size distribution for the mixture (Fig. 5c) shows a noticeable leftward shift, reflecting a more significant reduction in pore sizes. This shift suggests an increased probability of smaller average pore sizes, resulting from the inclusion and deformation of soft particles within the mixture. Rubber particles increase local contact area by deforming into surrounding voids with less compaction effort, reducing pore size and reducing hydraulic conductivity of the granular assemblies. Although the observed changes in pore size distribution highlight the role of soft particles in forming a stable network by increasing coordination number, they may also pose a risk of clogging when such mixtures are used as geotechnical fills.

6 Conclusion

The present study investigates the compaction behavior and pore size distribution (PSD) evolution in soft-rigid mixtures (SRM) using an oedometer test. Results show that SRM-30 exhibits a steeper compaction index of 0.035, approximately 75% higher than the rigid mixture (0.020), indicating greater compressibility. The inclusion of soft particles leads to a significant reduction in void sizes compared to the fully rigid assembly. Moreover, SRM-30 shows a major shift toward smaller pore sizes with a higher probability of voids below 2 mm, achieved with minimal compaction effort. In contrast, the PSD of the rigid mixture remains largely unchanged even under higher compaction effort. The observed behavior is critical for understanding the hydraulic conductivity of these mixtures, as excessive rubber content increases the risk of pore clogging. This highlights the importance of carefully balancing soft particle deformability and content when designing geotechnical fills.

References

[1] B. Madhusudhan, A. Boominathan, S. Banerjee, Cyclic simple shear response of sand–rubber tire chip mixtures, International Journal of Geomechanics **20**, 04020136 (2020).

[2] R. Fu, X. Hu, B. Yang, H. Wang, B. Zhou, An insight into the meso-scale topological structure nature of granular materials subjected to quasi-static shearing, Computers and Geotechnics **137**, 104257 (2021).

[3] M. Cárdenas-Barrantes, D. Cantor, J. Barés, M. Renouf, E. Azéma, Compaction of mixtures of rigid and highly deformable particles: A micromechanical model, Physical Review E **102**, 032904 (2020).

[4] R. Fu, B. Yang, X. Hu, B. Zhou, M.R. Coop, A micromechanical investigation of sand–rubber mixtures using the discrete element method, Engineering Geology **318**, 107106 (2023).

[5] C. Chang, T. Ma, Void size distribution and hydraulic conductivity of a binary granular soil mixture, Acta Geotechnica **19**, 2175 (2024).

[6] B. Indraratna, A.K. Raut, H. Khabbaz, Constriction-based retention criterion for granular filter design, Journal of Geotechnical and Geoenvironmental Engineering **133**, 266 (2007).

[7] M. Asadi, A. Mahboubi, K. Thoeni, Discrete modeling of sand–tire mixture considering grain-scale deformability, Granular Matter **20**, 1 (2018).

[8] G. Mollon, A unified numerical framework for rigid and compliant granular materials, Computational Particle Mechanics **5**, 517 (2018).

[9] N. Kalyan, R.K. Kandasami, Flow kinematics of granular materials considering realistic morphology, Powder Technology **424**, 118516 (2023).