Impact of wavy surfaces on particle-laden Poiseuille flow

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Abstract. Flow behavior in a rectangular channel changes significantly when one wall has a wavy shape. The presence of small particles enhances velocity fluctuations, which alters the distribution of flow disturbances. This study investigates the influence of wavy walls on both flow and particle movement by simulating four different geometries with surface wavelengths varying from nearly flat to highly wavy. Our results demonstrate that the wavy walls greatly affect particle transport, leading to different deposition patterns over time. By analyzing the Reynolds number associated with the average velocity fluctuations, we gain insights into the influence of wall undulations on flow disturbances and instabilities.

1 Introduction

Particle-laden flows over wavy surfaces are common in both natural and industrial settings. Examples include sediment transport in rivers, aerosol dispersion in the atmosphere, and particulate movement in chemical processing systems. Surface undulations modify the underlying flow field by altering velocity distributions, inducing secondary flows, and influencing particle trajectories. A comprehensive understanding of these effects is essential for advancing predictive models and improving the design of engineering systems. A number of investigations have addressed the impact of wavy surfaces on particle transport. Lee and Lee [1] examined particle behavior in turbulent flows over undulated boundaries, emphasizing the role of coherent structures in mediating transport phenomena. Hayati et al. [2] employed numerical simulations to uncover deposition mechanisms, demonstrating the influence of surface curvature on particle paths. Schlander et al. [3] utilized resolvent analysis to explore interactions between low-inertia particles and turbulent structures. Within the broader context of particle-laden turbulence, Picano et al. [4] studied dense suspensions of neutrally buoyant spheres, revealing modulation of turbulence and enhanced mixing. Similarly, Peng et al. [5] investigated two-way coupling effects in turbulent channel flows and highlighted the impact of suspended particles on turbulence organization. Experimental work by Haward et al. [6] demonstrated that wavy boundaries substantially modify velocity fields and promote the formation of secondary flow structures. The present investigation adopts a configuration consistent with their validated experimental setup to ensure relevance and comparability. Particle motion in such flow environments is governed by the interplay between fluid and particle inertia. Prior studies suggest that secondary flows induced by surface undulations contribute to particle focusing and preferential concentration [7]. These phenomena are particularly pronounced when the Stokes number approaches unity, where drag and inertial forces are comparable. The particles examined here are modeled as pointwise glass spheres with a Stokes number near one, consistent with experimental parameters reported by Shapiro and Goldenberg [8]. The current work investigates laminar Poiseuille flow laden with particles over sinusoidal wavy walls. The goal is to provide a deeper understanding of dispersion mechanisms in modulated geometries, with implications for both environmental transport processes and industrial flow systems.

2 The Model and Numerical Method

Four separate two-dimensional computational domains are considered, each representing a geometrically identical channel but with a distinct wavy surface wavelength [6]. This configuration allows for a systematic investigation of how periodic surface undulations influence flow behavior. The primary control parameter is the wavelength-to-half-depth ratio, λ/d, which ranges from 10 (representing mild undulations) to 0.625 (highly wavy undulations). Figure 1 is a schematic representation of the flow domain, illustrating key geometric and physical parameters. The corresponding wavenumber, $k = 2\pi/\lambda$, and the non-dimensional parameter, $\alpha = kd$, characterize the flow response to surface modulation. The perturbation amplitude is set at $A/d = 1/40$, ensuring a subtle yet measurable influence on the velocity field.

The governing equations for fluid and particle dynamics are formulated within a coupled framework. The incompressible fluid momentum equation is given by

$$\frac{D\boldsymbol{U}}{Dt} = -\frac{1}{\rho_f}\nabla p + \nu_f \nabla^2 \boldsymbol{U} - \frac{1}{\rho_f}\sum_{n=1}^{N_p} \boldsymbol{f}^n\left(\boldsymbol{x}_p^n\right)\delta\left(\boldsymbol{x} - \boldsymbol{x}_p^n\right),$$

where $\boldsymbol{U}$ is the fluid velocity, p is the pressure, ρ_f is the fluid density, ν_f is the kinematic viscosity, and and δ refers to the Dirac delta function. The last term represents the

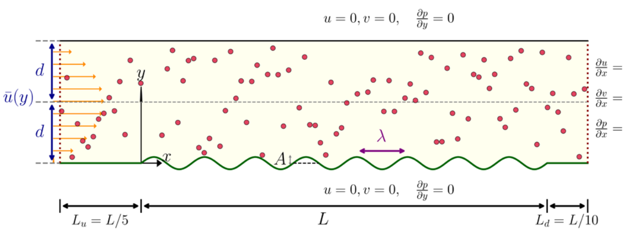


Figure 1: Schematic representation of particle laden Poiseuille flow through a channel with a wavy bottom surface. The channel has a depth $H = 2d$, while the undulating bottom wall is characterized by a sinusoidal perturbation with wavelength λ and amplitude $A \ll d$

momentum exchange between the fluid and the dispersed phase, where $\boldsymbol{f}^n$ is the force exerted by the fluid on the nth particle located at $\boldsymbol{x}_p^n$.

The position of each particle follows $\frac{dx_p^n}{dt} = v_p^n$, where x_p^n and v_p^n denote the position and velocity of the nth particle, respectively. The particle motion is governed by Newton's second law, $m_p \frac{d\boldsymbol{v}_p^n}{dt} = m_p \boldsymbol{g} + \boldsymbol{f}^n \left(\boldsymbol{x}_p^n\right)$, where m_p is the particle mass, $\boldsymbol{g}$ is gravitational acceleration, and $\boldsymbol{f}^n$ represents the fluid forces acting on the particle. Together, these equations describe the coupled interaction between the fluid and dispersed phase. The wavy wall geometry induces streamwise velocity gradients, which, through continuity, generate transverse (v) velocity perturbations. These perturbations influence particle trajectories and lead to wavelength-dependent transport characteristics. The characteristic time scale, is defined as $t_c = L/\bar{U}$, where $\bar{U}$ denotes the mean flow velocity. The non-dimensional time is expressed as $\tau = t/t_c$, where t corresponds to the physical flow time. A structured hexahedral mesh aligned with the wavy boundary is employed to ensure accurate resolution of near-wall flow features. Pressure-velocity coupling is handled using the PISO algorithm, ensuring numerical stability and consistency. Convection terms are discretized using a second-order upwind scheme, while diffusion terms are treated with central differencing to maintain accuracy. A Lagrangian tracking approach is employed to simulate inertial particles introduced at the mean flow velocity.

3 Results

The wavy topography introduces spatially periodic forcing on the flow, giving rise to secondary motion patterns, particularly in the perturbed velocity component. The interaction between the imposed surface shape and the fluid motion leads to the formation of perturbations that propagate downstream. The perturbed velocity field interacts with particle forces, leading to momentum exchange that modifies the flow disturbances induced by the wavy wall. This interaction introduces randomness in the perturbed velocity distribution within the flow domain. Notably, particle-particle collisions are not considered in this study, as the analysis is restricted to a two-way coupling framework [5].

Figure 2 illustrates the particle deposition patterns at two different time intervals for the wavy channels with

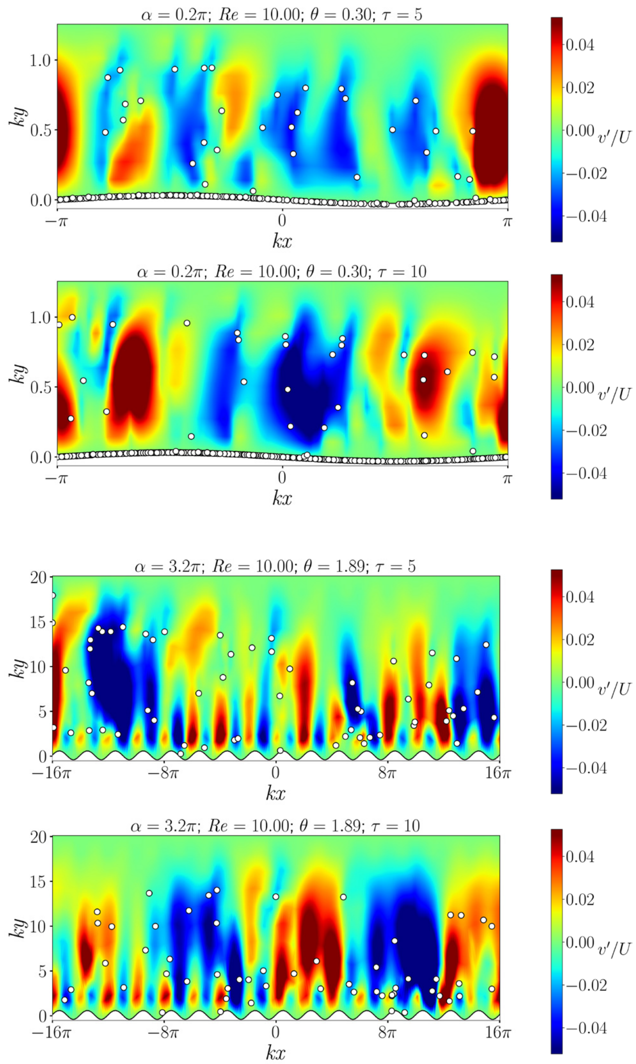
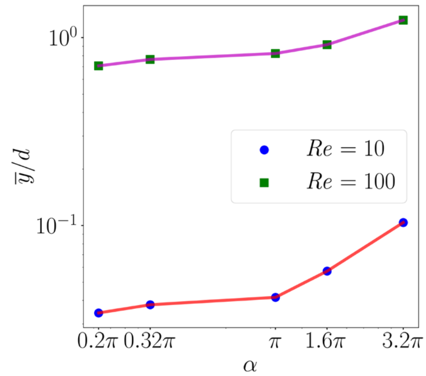


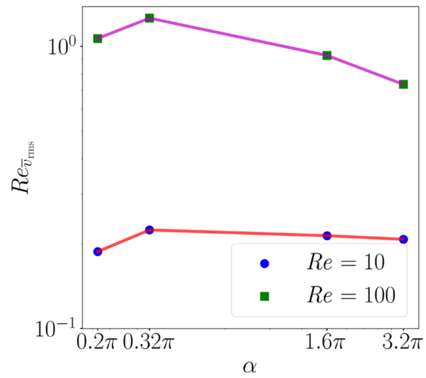
Figure 2: Normalized perturbed velocity field with wavy bottom surface and particle positions (white markers with black outlines) for $\alpha = 0.2\pi$ to $\alpha = 3.2\pi$ after 5 and 10 flow cycles at $Re = 10$. θ represents the viscous length scale.

$\alpha = 0.2\pi$ and $\alpha = 3.2\pi$, showing the evolution of particle distribution over time. The channel with $\alpha = 0.2\pi$, corresponding to the longest wavelength, exhibits significant particle deposition after 5 and 10 flow cycles, whereas in the channel with $\alpha = 3.2\pi$, which has the shortest wavelength, particles remain largely suspended within the flow. The channel characterized by $\alpha = 0.2\pi$ exhibits substantially greater particle deposition at both time instances compared to the channel with $\alpha = 3.2\pi$. Figure 3a illustrates this phenomenon by presenting the average wall-normal position of all dispersed particles at $\tau = 10$, for Reynolds numbers of 10 and 100. The background contour represents the normalized perturbed velocity field, which remains within ±0.04 for the unladen case but slightly exceeds this range in the particle-laden case due to enhanced flow disturbances.The background contour depicts the normalized perturbed velocity field, with the color scale capped at ±0.05. In the unladen case, perturbations generally remain below this limit, while in the particle-laden case, stronger disturbances frequently exceed it.

The Reynolds number based on the area- and time-averaged rms perturbed velocity, $Re_{\tilde{v}_{\mathrm{rms}}}$, provides valuable insights into the influence of wall undulations on flow disturbances, instabilities, and transition mechanisms. This parameter facilitates comparisons between different wavy surfaces and evaluates energy transfer from the mean flow to perturbations. As shown in Figure 3b, for the longest wavelength ($\alpha = 0.2\pi$), $Re_{\tilde{v}_{\mathrm{rms}}}$ initially increases, reaching a peak at $\alpha = 0.32\pi$. This indicates that in the shallow channel regime, $Re_{\tilde{v}_{\mathrm{rms}}}$ increases as the wavelength decreases. However, upon entering the deep channel regime, a further decrease in wavelength causes $Re_{\tilde{v}_{\mathrm{rms}}}$ to decline, reaching its lowest value among all cases studied. Notably, as the channel Reynolds number Re increases, the rise and fall of $Re_{\tilde{v}_{\mathrm{rms}}}$ become more pronounced.



(a) Log-log plot of $\overline{y}/d$ vs. α.



(b) Log-log plot of $Re_{\tilde{v}_{\mathrm{rms}}}$ vs. α.

Figure 3: (a) $\overline{y}/d$ represents the non-dimensional average center of mass of the dispersed phase, indicating particle settling near the wavy wall for different channel wavelengths. (b) $Re_{\tilde{v}_{\mathrm{rms}}}$ peaks in the shallow channel regime and declines in the deep channel regime, with sharper variations at higher Re.

4 Conclusions

The wavy surface generates velocity perturbations that affect particle transport. These perturbations create distinct deposition patterns. Shorter wavelengths increase perturbation energy, which helps keep particles suspended. Longer wavelengths, on the other hand, promote deposition. The perturbed velocity field modifies shear and energy transfer. This influences the balance between inertia and viscosity. The Reynolds number, $Re_{\tilde{v}_{\mathrm{rms}}}$, is based on the area- and time-averaged rms perturbed velocity. It does not follow a simple increasing or decreasing trend. In shallow channels, it increases when the wavelength decreases because of enhanced mixing. In deeper channels, it decreases because the perturbations remain close to the wall. At higher bulk Reynolds numbers, this variation becomes more pronounced. This highlights the growing influence of inertia on perturbation dynamics.

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