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Macroscale signatures of crushing from resonant column tests

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Abstract. The resonant column apparatus has been widely used to measure the dynamic properties of particulate geomaterials, and to describe their variation as a function of confining stress and strain. This study explores the use of resonant column testing to detect signatures of crushing from measurements of the shear modulus (G) and damping ratio (D) through testing of a highly crushable model granular material. As values of G and D measured at small strains are highly sensitive to the nature and distribution of the contacts and the interparticle forces, their measurement provides the means to assess the effects of particle breakage on the fabric. The paper discusses various testing protocols that can be employed to capture the occurrence of crushing resulting from isotropic compression and/or torsional vibrations, leveraging recent developments in resonant column testing that allow analysis of off-resonance data.

1 Introduction

Brittle porous granular materials are an important class of granular materials encountered in the food processing, infrastructure and the pharmaceutical industries. In addition to the unique characteristic properties of granular materials, intriguing mechanical behaviors arise due to the additional porosity and the brittle nature of the individual particles. Researchers [1] have examined localized behaviors through one dimensional compression tests and demonstrated the formation and periodic propagation of compaction bands in brittle porous media. The interaction of fluids with these materials has also been shown [2] to generate localized crushing and collapse that leads to incremental global compaction. These experiments have contributed in no small measure to developing an insightful conceptual model for predicting the cascading propagation of compaction bands.

We seek to investigate signatures of particle crushing during isotropic compression and torsional shear through wave based non-destructive testing in a resonant column apparatus. Resonant column testing has been widely used for several decades to investigate the dynamic properties (shear modulus, G, and damping ratio, D) of geomaterials providing unique insights into the role played by mineralogy, particle characteristics, stress level, density and time. G depends on both the shear wave velocity and the density, while D expresses the amount of energy dissipated during a full loading cycle relative to the stored elastic strain energy.

Unique to the resonant column apparatus is the ability to probe material response over a broad range of strains (10^{-5}% to 0.5%).

2 Model material

The tests presented in this paper were performed on specimens of a breakfast cereal made from puffed grains of rice, which has been often used as a model material for porous brittle granular materials [e.g. 1-2]. Based on sieve analysis (see below) D_{50} and C_u are equal to 4 mm, and 1.9, respectively. Individual particles are platy-shaped with thickness of 2.67 ± 0.52 mm, and aspect ratio of 1.56 ± 0.34, the latter determined by fitting an ellipse to the particle boundary using ImageJ.

Cylindrical specimens (diam.=70 mm, height=140 mm) for resonant column tests were prepared through dry pluviation using a funnel with spout diameter of 20 mm, and drop height of less than a cm. This procedure yielded specimen dry densities of 0.124-0.128 g/cm^3.

3 Use of resonant column test for probing particle crushing

3.1 Test setup

Figure 1 shows a schematic of the fixed-free Drnevich type resonant column apparatus used in this work. Torsional vibration is applied to the top of a membrane-enclosed specimen through four magnets placed inside electrical coils. A source is used to drive the coils with the desired input, and signals of the torque and of the accelerometer mounted on the top platen are acquired through a spectrum analyzer. Specimen shear modulus (G) and damping ratio (D) are derived from the real and imaginary components of the complex frequency response function reported by the analyzer. The accelerometer signal is also used to determine the

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rotation of the top plate. A LVDT measures axial deformation of the specimen, from which approximate values of the volumetric strain can be derived. In this work all tests were conducted on dry material, controlling the specimen's isotropic effective confinement by the application of vacuum through the base pedestal. While in conventional resonant column testing, *G* and *D* are derived exclusively at resonance, this study leverages recent developments [3] that do not restrict data analysis to resonance (see more below when discussing testing protocol *TP-3*).

As shown in Figure 1, a wireless microphone is mounted on the resonant column base immediately next to the specimen. The output from the microphone is used as a surrogate indicator of crushing events, which manifest as popping sounds, providing the means to monitor the physical process (particle crushing).

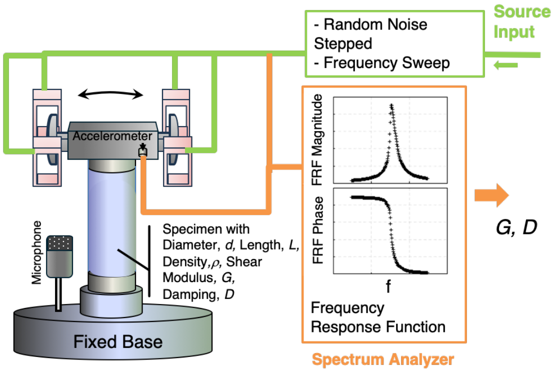

Fig. 1. Schematic of fixed-base resonant column apparatus

3.2 Testing protocols and results

We put forth resonant column testing protocols aimed at generating particle crushing and assessing its effects on the structure of the particulate ensemble.

The first two protocols (*TP-1* and *TP-2*) involve the measurement of *G* and *D* at very small shear strains ($<7\cdot10^{-5}$% in the tests presented here), which represent the initial maximum shear modulus (G_{max}) and minimum damping ratio (D_{min}). These quantities are sensitive to the nature and distribution of the contacts and the interparticle forces, and thus represent a means to examine the fabric of the particulate ensemble.

Measurements of G_{max} and D_{min} are achieved through the application of small amplitude random noise torsional vibrations. In these tests, power is distributed over a frequency bandwidth (~ 50 Hz in the tests presented here) that comprises the resonant frequency, overcoming limitations in the minimum power that can be applied by the source. While it is possible to analyse data obtained across the entire bandwidth (see below), only the data at resonance are used here. Due to the applied strain level, these measurements can be considered effectively non-destructive.

In the first testing protocol (*TP-1*) (Fig. 2), random noise torsional vibrations are applied following the step-wise increase/descrease of the isotropic confining stress (σ'_c) (Fig. 2a). The objective of such a protocol is to identify the confining stress that defines the onset of crushing and to quantify its effects on G_{max} and D_{min}. Based on the microphone data presented in Fig. 2b, for the material examined in this work, the most significant evidence of crushing occurs when the specimen is loaded into the virgin compression region to $\sigma'_c \geq 40$ kPa. Below this stress level, and in all unload-reload stages the microphone signal remains relatively low.

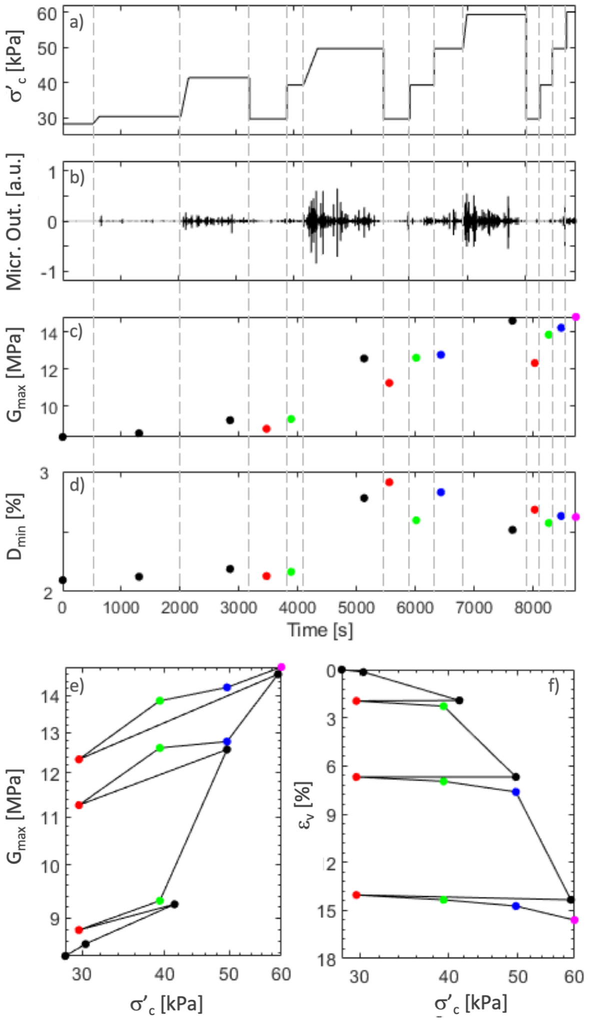

Fig. 2. Test performed following *TP-1*: a) σ'_c, b) microphone output, c) G_{max} and d) D_{min} over the duration of the test; dependence of e) G_{max} and f) ε_v on σ'_c.

Associated with the manifestation of crushing captured by the microphone recording are large volumetric strains (Fig. 2f) and a sharp increase in G_{max} (Fig. 2c). We conjecture that this reflects the generation of a new granular material with increased polydispersity in size and shape. Beyond 40 kPa the stress level exponent, *n*, which describes the sensitivity of G_{max} to changes in confining stress ($G_{max} \propto \sigma_c'^{n}$) increases by a factor of five to ~ 1.2, significantly exceeding the theoretical value for elastic Hertzian contacts, as well as values typically reported for granular materials, including previous data for crushable vulcanic soils [4,5]. This is thought to arise not only from plastic deformations occurring at the particle contacts, but also from the increased number and different nature of the contacts produced by fragmentation of the particles.

Post-mortem analysis of resonant column specimens confirms that particle fragmentation occurs under isotropic confinement in this stress range, as illustrated by the particle size distributions and images of the material both in its intact state and after testing at σ'_c = 50 kPa (Fig. 3). Larger shift in the particle size distribution occurs after testing at higher confinement.

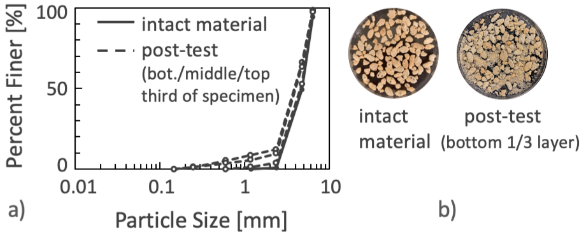

Fig. 3. a) Particle size distributions and b) images of 1.5 g of rice cereal before and after isotropic confinement at 50 kPa.

Values of D_{min} measured across all stress levels, even at low confinement prior to any significant evidence of crushing, are substantially higher than those typically reported for granular geomaterials, including crushable volcanic soils [4,5]. It is generally assumed that attenuation at very small strains cannot be attributed to frictional losses [6], and that in the case of dry granular materials damping in this strain range arises primarily from thermoelastic relaxation. The higher values of D_{min} measured on the puffed rice cereal suggest that additional mechanisms may come into play and that a contribution from frictional losses cannot be excluded.

As in the case of G_{max}, D_{min} also shows a sharp increase when σ'_c > 40 kPa. This is inconsistent with previous studies on a range of granular materials (e.g. [4,5]) that report constant or slightly decreasing values of D_{min} with increasing stress level, and supports the conjecture above regarding the transition at higher confinement to a new granular material.

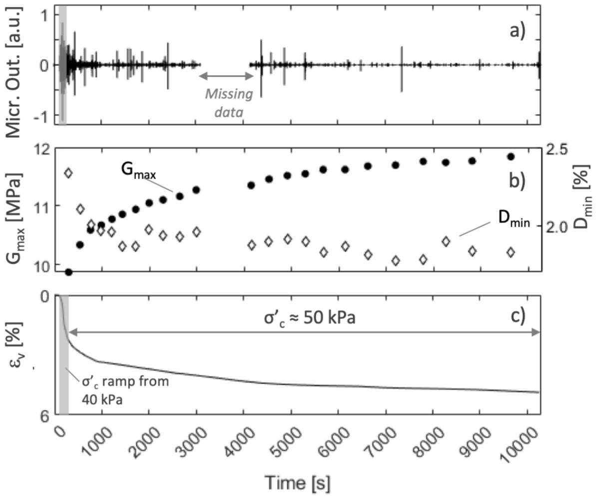

Fig. 4. Data from test performed following *TP-2*: a) output from microphone, b) G_{max} and D_{min} (measurements at 10' intervals) and c) ε_v during stress ramp-up and aging stage.

The second testing protocol (*TP-2*) involves the application of small amplitude torsional vibrations as the effective confinement, σ'_c, is isotropically increased to a given value and then maintained constant for an extended "aging" period. Figure 4 shows the results of such a test with σ'_c increasing from 40 to 50 kPa. While the microphone data (Fig. 4a) reveal the strongest evidence of crushing (accompanied by the most rapid increase in volumetric strain – Fig 4.c) during the ramp-up of the confining stress, additional crushing "events" (and continued porosity loss) are recorded over time at constant σ'_c, as a result of the continuous redistribution of the contact forces. The increase in G_{max} and decrease in D_{min} with time (Fig. 4b) can be attributed to the contact force homogenization process that occurs with aging as forces are redistributed from the strong force network to the weak force network [7]. While similar behavior has been reported for a wide range of geomaterials, particle crushing leads to greater changes in both G_{max} and D_{min} with time, larger volumetric strains during aging and an extended time scale for contact force redistribution/homogenization.

In alternative to a microphone, the accelerometer mounted on the top platen (Fig. 1) can be used during periods of no torsional excitation of the specimen to record vibrations produced by crushing/rearrangement of the grains. The comparison (Fig. 5) of the output of the microphone and of the accelerometer during a test similar to that above shows that the two sensors provide generally consistent data.

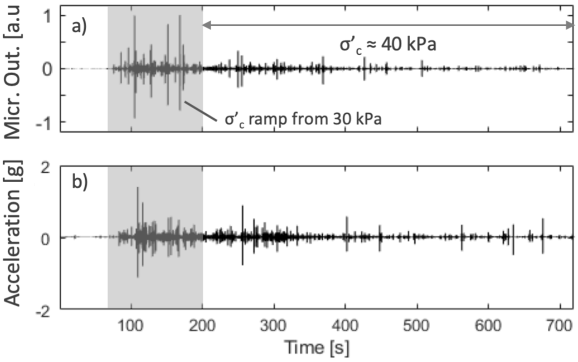

Fig. 5. Comparison of a) microphone and b) accelerometer output during ramp-up and constant stress stage in *TP-2*.

The third testing protocol (*TP-3*) is aimed at establishing the level of distortion required to produce particle breakage, and examine its effects on the fabric of the granular assembly. *TP-3* relies on stepped frequency sweeps, which represent a novel approach [3] to dynamic torsional shear testing. They consist in the application of a fixed sinusoidal torque at discrete frequencies over a bandwidth that encompasses the resonant frequency. The analysis of the off-resonance data is enabled by advances in resonant column theory [8]. As under a given excitation the shear strain amplitudes for off-resonant frequencies are much lower than at resonance, *G* and *D* can be characterized semi-continuously as a function of shear strain.

Figure 6 shows data from a test in which a series of six frequency sweeps were performed over approximately 2.5 hours on a single specimen, each time increasing the applied torque (from ~1 to 110 mN·m over the 6 sweeps). The tests was performed at σ'_c=35 kPa, as at this stress level particle crushing arising from isotropic confinement is not expected to be significant (Fig. 2), allowing shear induced effects to be isolated. In each of the sweeps, the frequency bandwidth was set at 20 Hz, with measurements performed at ~60 frequencies (with greater frequency resolution in proximity to resonance). At each frequency, testing involved 200 settling cycles, and 100 integration cycles. The control of the number of cycles applied to the specimen represents another key advantage of stepped frequency sweeps relative to conventional resonant column tests.

As shown in Figure 6a, the shear strain range probed in the frequency sweeps varies over three orders of magnitude, with a maximum γ~0.2% measured at resonance under the highest torque. After each frequency sweep, a low amplitude random noise test, similar to those described above for *TP-1* and *TP-2*, was performed to measure the small strain values of G_{max} and D_{min} and assess the impact of the shearing process on the specimen's fabric. The microphone output was recorded over the entire duration of the test, including during the periods of no torsional excitation (Fig. 6b).

In general, the microphone signal remains close to the baseline (and volumetric strains are negligible) during the first few frequency sweeps, with small peaks observed only close to resonance (in correspondence to the maximum shear strain amplitude). Higher outputs are recorded in the latter sweeps as the strain amplitude exceeds 0.1%. Measurements of *G* and *D* were derived at all strain amplitudes, but are not discussed here.

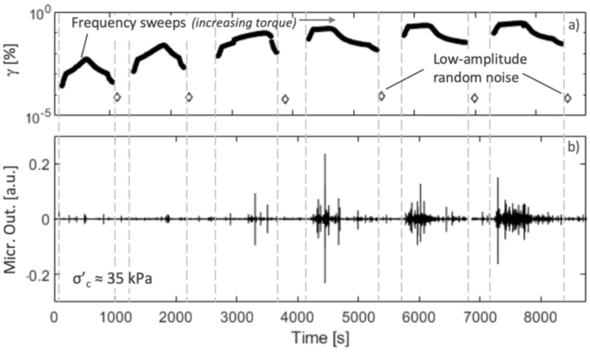

Fig. 6. a) Shear strains imposed during stepped frequency sweeps and b) resulting microphone output (note different y-axis scale in b) compared to Figs. 2-4).

Overall, the G_{max} and D_{min} data extracted from the low amplitude random noise tests performed after each sweep remain relatively constant (G_{max} = 9.27 ± 0.19 MPa, D_{min} = 1.86 ± 0.14 %), an indication that the crushing events experienced by the specimen did not significantly alter the fabric.

4 Summary

This work examines the use of the resonant column apparatus, supported by recordings collected by a wireless microphone, for capturing signatures of crushing produced by isotropic compression and torsional shear in porous brittle solids. The testing protocols put forward rely on non-destructive measurements of *G* and *D* at very small strains to identify the stress and shear strain levels that define the onset of crushing and to investigate force redistribution during aging. As the dynamic properties are strongly influenced by the formation of new contacts, the tests also provide insight into fabric changes produced by crushing. The protocols were not designed to study localized phenomena occurring during crushing.

While the paper largely focuses on small strain data, measurements of *G* and *D* during the frequency sweeps introduced in *TP-3* can provide rich information on the variation of these parameters and on mechanisms responsible for energy dissipation at higher shear strains. Moreover, the application of higher torques can further extend the strain range probed. Advances in resonant column theory [8] and associated testing procedures [3] that enable analysis of off-resonance data and control of the number of applied cycles, can be further leveraged to tailor testing protocols to examine the role played by strain and frequency vibration history. We conclude that resonant column testing holds promise for advancing the understanding of the crushing behavior of brittle porous granular materials.

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