

# Experimental observations of traction on grooved rods pulled from within cylindrical silos containing grains

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**Abstract.** In this work we present experimental results on the subject of frictional traction forces acting on cylindrical grooved rods, lying axi-symmetrically centered in a cylindrical silo. The experiments carried out for this study allowed us to measure the traction acting on the grooved rod for several filling heights,  $H$ , of a granular material. The experimental results showed how the traction forces were affected by the grooves on the rod. Comparisons could be made taking into account the traction measured experimentally for different grooved rods and to measure the effect of the lifting of the weights of the mass of grains and the outer cylinder for a smooth rod and grooved rod.

## 1 Introduction

Recently in [1], frictional forces arising in the annular space between two concentric vertical cylinders were examined. The study focuses on the outer surface of a cylindrical rod and the inner wall of a larger cylinder, both filled with dry, non cohesive granular material. The research took into account the Janssen's model in order to describe the pressure saturation effect, this implies the stabilization of horizontal and vertical pressure over a characteristic length. The experimental work allowed measuring of the traction force acting on the rod for different heights of filling, providing good agreement between theoretical and experimental values when assuming an active state Rankine's coefficient.

In addition, an interesting feature was observed, when attempts were made to extract the rod immersed in the granular material, the traction force was sufficient to lift the combined weights of the rod, the granular material, and the outer cylinder, showing rigid upward movement of the entire system. This study explores the frictional traction forces exerted on cylindrical grooved rods placed symmetrically within cylindrical silos. In a series of experiments, we measured the traction forces for different filling heights ( $H$ ), revealing the influence of the rod's grooves on these forces. Furthermore, we compared the traction values obtained for various grooved rod designs and examined how both the granular mass and the outer cylinder were simultaneously lifted when using a smooth rod versus a grooved one.

## 2 Traction forces acting on rods in cylindrical silos

The Janssen model is a fundamental theory for understanding how pressure distributes in silos that contain different granular materials such as grains or seeds. For liquids, it is well established that the pressure effect mainly depends on the height of the fluid column, meanwhile in granular materials friction against the walls of the silo is generated altering the load distribution. Janssen in his work proposed a mathematical model based on force balance, taking into considering the weight of the granular material stored in the silo, the resistance of the wall, and the friction coefficient between the material and the silo walls. As a conclusion, this model shows that the vertical pressure at the bottom of the silo does not increase indefinitely according to the height of the material but instead stabilizes due to the transfer of loads to the walls. Numerous works extensively discuss the Janssen model. An authoritative work that thoroughly addresses the physical and mathematical development of this model was presented by Boutreux et al. [2].

The study of pressure distribution in silos has important implications, mainly in the structural design of silos and the structures that support them. A better understanding of the pressure distribution allows a more accurate calculation of the loads that the walls and the silo base must withstand. The risk of structure failure is lowered by the wall's ability to resist part of the granular material load through friction. In addition, the Janssen model has important applications in agricultural and industrial engineering helping to optimize the silo design and construction based on the type of the stored material, its density, and the silo's features. Its applications enhance safety and efficiency in bulk material storage. A compressive review of experimental and numerical methods for silo design can be found in [3].

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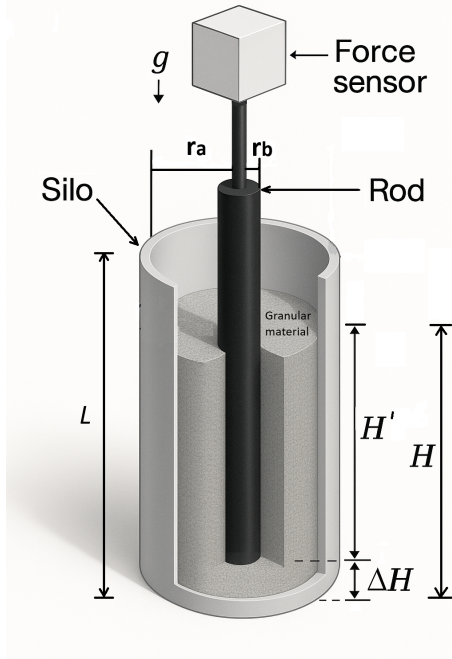
The silo is represented as a cylindrical tube of radius  $r_a$  with a flat bottom and a wall thickness  $w$ , and height  $L$ . The silo contains a granular material of density  $\rho$ . The vertical pressure  $p_z(z)$  is the force exerted on a horizontal section of the silo at a depth  $z$  according to Janssen's model. The silo wall also exerts an horizontal pressure  $p_r(z)$  different in value to  $p_z(z)$ . Both pressures are related by  $p_z(z) = kp_r(z)$ , where  $k$  is the Janssen parameter [3–5]. The vertical friction stress  $\tau$  between the grains and internal silo wall is generated and can be related to  $p_r(z)$  and the wall friction coefficient  $\mu_w$  as  $\tau = p_r(z)\mu_w$ . Carrying out a force balance analysis, a first order linear differential equation is obtained

$$\pi r_a^2 \frac{dp_z}{dz} = \pi r_a^2 \rho g - 2\pi r_a \tau. \quad (1)$$

The solution of (1) subject to the condition  $p_z(0) = 0$  is given by

$$p_z(z) = \rho g \lambda \left[ 1 - \exp\left(-\frac{z}{\lambda}\right) \right] \quad (2)$$

where  $\lambda = r_a / (2\mu_w k)$ . Expression (2) describes the increment of the vertical pressure with  $z$ .



**Figure 1.** Schematic depicting a rod of radius  $r_b$  concentrically aligned within a larger cylinder of radius  $r_a$ . The rod is positioned at a small height  $\Delta H$  above the bottom of the silo.

As a consequence, the vertical shear stress  $\tau(z)$  yields on an overall traction force  $T(H)$ , acting on the sidewall of the cylinder, filled up to a height  $H$ , that is

$$T(H) = \pi r_a^2 \rho g \left\{ H - \lambda \left[ 1 - \exp\left(-\frac{H}{\lambda}\right) \right] \right\} \quad (3)$$

Now, considering the presence of a rod of radius  $r_b$  located at the center of the silo of radius  $r_a$  as in Fig. 1. For this

case, the vertical force balance gives the following equation

$$\pi(r_a^2 - r_b^2) \frac{dp_z}{dz} = \pi(r_a^2 - r_b^2) \rho g - 2\pi(r_a \tau_a - r_b \tau_b), \quad (4)$$

where the vertical shear stress on the rod is  $\tau_b = \mu_{wb} p_r$  and on the inner wall of the cylinder is  $\tau_a = \mu_{wa} p_r$ . Solving (4) and following the previous methodology, the traction force on the wall of the rod can be expressed as

$$T_{rod}(H') = 2\pi\mu_{wb} K_b \rho g \lambda r_b \left\{ H' - \lambda \left[ 1 - \exp\left(-\frac{H'}{\lambda}\right) \right] \right\} \quad (5)$$

with  $\lambda = (r_a^2 - r_b^2) / (2k(r_a \mu_{wa} - r_b \mu_{wb} k))$ , where  $K_b$  is the Janssen parameter close to the rod and  $H'$  is the effective height where the traction force is exerted on the rod under the effect of the granular column.

### 3 Experimental set up

Some previous works have considered the study of the effect that traction force has on rods under granular flow or in the presence of obstacles, including the study of acoustic effects as can be found in [6–8]. The expression (5) is valid for smooth rods inside a sand-filled silo. The experimental results presented in this work will take into account grooved rods at different distances as showed in Fig. 2.

For these experiments, we used a short cylindrical glass tube as a silo. The silo is open at the top and has a bare edge with an external diameter  $D_E = 4.5 \pm 0.1$  cm and internal diameter  $D_I = 3.6 \pm 0.1$  cm. The bottom is a flat closed end. The length of the silo is  $L = 46.5$  cm and the wall thickness  $0.9 \pm 0.01$  cm. Wooden rods were used to perform the experiments with a length  $L_r = 50$  cm and a diameter  $d_r = 1$  cm. The silo was filled using Ottawa sand with a bulk density of  $\rho = 1.60 \times 10^3$  Kg/m<sup>3</sup>, mean value of friction angle of  $\theta = 31^\circ$ , a friction coefficient of 0.51 and a mean diameter of  $d_s = 0.05103$  m.



**Figure 2.** Rods used in the experiments. Left to right: a) smooth rod. Rods with grooves every: b) 0.5 cm, c) 1 cm, d) 1.5 cm.

The rod can be held to quantify the vertical pressure acting on the grains and the partial weight supported by

the rod and the sidewall of the outer cylinder, considering a rod of radius  $r_1$ , concentrically aligned with a larger cylinder of radius  $r_0$ , and raised to a small height  $\Delta H=1$  cm above its bottom. The rod is held by a force sensor (see Fig. 3). This condition is relevant in experimental setups since, when sand is poured into the annular space, the vertical shear stress exerted by the grains tends to drag the rod downward toward the small empty space beneath it, allowing for direct traction measurements. It is important to note that the height of the sand relative to the bottom is  $H$ , when measured relative to  $\Delta H$  we get  $H'$ , so that  $H = \Delta H + H'$ .

To perform the experiments, a force sensor, model Pasco CI-6537, with range  $\pm 50$  N and a resolution of 0.03 N was employed. The sensor was configured according to the manufacturer's specifications. The recorded signals were time-domain signals. The sampling frequency chosen for the experiments was 100 Hz. Each rod has a hook on one end that allows connection to the sensor. The section of the laboratory in which the experiments were carried out was climate controlled ( $25 \pm 1$  C° and 28 % R.H.).



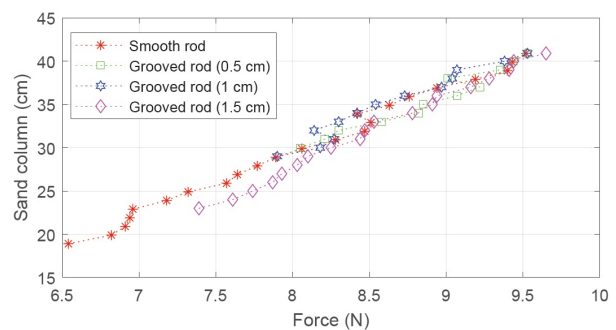
**Figure 3.** Example of how the experiment was set up to perform the measurements.

#### 4 Some observations

The first series of experiments were conducted with a wooden rod without grooves. In this case, the maximum required force to lift the entire system was recorded to have a value of  $9.52 \pm 0.03$  N. The force was opposite to the direction of gravity. The sand column within the annular section formed by the inner walls of the silo and the outer wall of the rod had a height of 40.9 cm. A characteristic of the system was that the rod did not touch the bottom and was positioned 1 cm above the bottom of the silo. Series

of experimental tests were carried out for different heights of sand column, where it was observed that the minimum height of the sand column to pull up the system was 18.9 cm applying a force of  $6.54 \pm 0.03$  N. The rod managed to exit the column by applying a force of  $7.5 \pm 0.03$  N with a sand column height of 17.9 cm. In Fig. 4, the time averaged pull up forces to lift the system with the smooth rod are represented by red stars marks.

A second series of experiments were conducted in the same manner as it was previously in the above paragraph. In this case, the rod is equally grooved at a distance of 0.5 cm along the rod surface. The original sand column had a height of 40.9 cm. The force required to lift the entire system was  $9.53 \pm 0.03$  N. However, after conducting this series of experiments, it was observed that by applying a force of  $8.23 \pm 0.03$  N in a 29 cm sand column, the rod could be extracted from the sand-silo system. The minimum height of the sand column to pull up the system was 30 cm applying a force of  $8.05 \pm 0.03$  N. The behaviour of these series of experiments are indicated by green square marks in Fig.4.



**Figure 4.** Time averaged pull up forces for different sand column height and different types of rods.

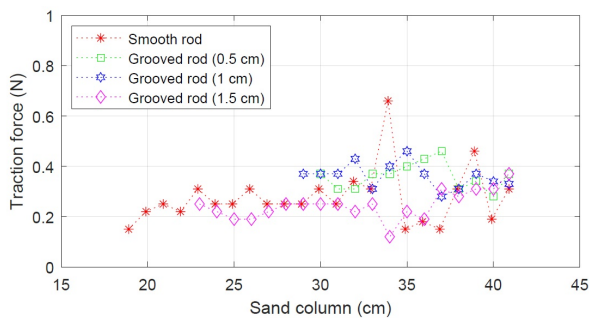
A third series of experiments are represented by blue star marks in Fig.4. For this case, the rod was grooved each 1 cm over its surface. The original sand column had a height of 40.9 cm. The force required to lift the system was  $9.53 \pm 0.03$  N according to sensor readings. The minimum height of the sand column to pull up the system was 29 cm applying a force of  $7.90 \pm 0.03$  N. After conducting this series of experiments, it was found that a force of  $8.90 \pm 0.03$  N was needed to extract the rod from the system with a sand column of 28 cm height.

The last series of experiments dealt with a rod with grooves spaced 1.5 cm apart. The procedure implemented was the same as in previous series of experiments. The experiment began with a sand column of 40.9 cm. The force required to lift the system with this configuration was  $9.65 \pm 0.03$  N. Subsequently, after several trials reducing the sand column in which the rod was buried, it was observed that the force required to extract the rod was 7.5 N in a sand column of 22 cm. Also, the minimum height of the sand column to pull up the system was 23 cm applying a force of  $7.39 \pm 0.03$  N. Table 1 shows the necessary force to extract the rod from the sand column for each case.

**Table 1.** Lift force required to extract the rod

Kind of Rod	Sand column (cm)	Force (N)
Smooth	17.9	7.5
Grooved (0.5 cm)	29	8.23
Grooved (1.0 cm)	28	8.90
Grooved (1.5 cm)	22	7.5

In the annular region between the inner sidewall of the silo and the surface wall of the rod, the traction force can be directly measured from the data recorded by the force sensor in time domain. The value of this traction force ( $T_{rod}$ ) changes according to the mass flow rate at which the annular region is being filled. Due to friction, as the grains flow and interact with the rod's surface, the traction force evolves from a minimum to a maximum value. For this series of experiments, a fluctuation of  $T_{rod}$  values was observed. However, for all cases, values remained reasonably close to the mean, as can be seen in Fig. 5 where maximum values of  $T_{rod}$  are plotted. The mean  $T_{rod}$  values for every case where: a) 0.2752 N, b) 0.36 N, c) 0.3623 N, d) 0.2447 N, in agreement with Fig. 2.



**Figure 5.** Traction forces for every kind of rod used in experiments.

## 5 Conclusions

These experiments show that the force required to extract a rod within a rod-sand-silo system. The experiments were conducted using four different types of rods (one smooth rod and a couple of grooved rods). According to Table 1, it was observed that the smooth rod and the rod with grooves every 1.5 cm required a force of 7.5 N to extract the corresponding rod from the system.

The rods with grooves every 0.5 cm and 1 cm displayed similar behaviour, exhibiting a greater resistance for the rod to be extracted from the sand column. The explanation for this phenomenon appears to be linked to in the effective radius over which the traction forces act. The

smooth rod also has a larger surface area on which the system's internal forces act. On the other hand, grooved rods have a variable radius over which these internal forces are distributed. This paper presents all relevant results derived from recent experimental observations, however, no definitive conclusions can yet be drawn. Further analysis, intended at adjusting the proposed mathematical model for it to account for the influence of the groove pattern, will be carried out in subsequent studies.

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## References

- [1] A. López Villa, J. Hernández-Juárez, A. Medina, R. Diez-Barroso, A. Serrano, Traction on Rods within Cylinders Containing Grains: An Analogy with the Upward Movement of Trees in Tornadoes. *Fluids*, 9(10) 234 (2024). <https://doi.org/10.3390/fluids9100234>
- [2] T. Bouteux, E. Raphael, P. G. de Gennes, Propagation of a pressure step in a granular material: The role of wall friction, *Physical Review E* 55(5) 5759 (1997). <https://doi.org/10.1103/PhysRevE.55.5759>
- [3] J. Tejchman, *Confined Granular Flow in Silos*, Springer (2013). <https://doi.org/10.1007/978-3-319-00318-4>
- [4] B. Andreotti, Y. Forterre, O. Pouliquen, *Granular Media: Between Fluid and Solid*, Cambridge University Press (2013).
- [5] B. A. Ismain, N. Blaise, A. Amine, G. Fouad, Stress distribution during a silo filling or a discharging process, *Revue Nature et Technologie*, 2(2) (2010). Janssen
- [6] T.D. Atkinson, J.C. Butcher, M.J. Izzard, R.M. Nedderman, The forces on obstacles suspended in flowing granular materials. *Chemical Engineering Science* 38(1) 91-105 (1983). [https://doi.org/10.1016/0009-2509\(83\)80138-9](https://doi.org/10.1016/0009-2509(83)80138-9) Varilla
- [7] K. Wieghardt, Experiments in Granular Flow, *Annual Review of Fluid Mechanics*, Vol.7:89-114 (1975). <https://doi.org/10.1146/annurev.fl.07.010175.000513> Drag rod
- [8] K. Nishiyama, S. Mori, Frequency of Sound from Singing Sand, *Jpn. J. Appl. Phys.* 21 591 (1982). 10.1143/JJAP.21.591