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Compressive wave speeds in dry, confined granular mixtures

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Abstract. The behavior of waves in granular aggregates holds significance in engineering and geotechnical applications alike. Earlier studies have investigated wave propagation in granular media, often estimating bulk elastic properties from measured wave speeds and proposing scaling laws relating wave speed to confining pressure. Here, we look at wave transmission through monodisperse granular aggregates of various particle sizes, and mixtures composed of grains of two different sizes. In particular, we will report findings from experiments conducted on a) monodisperse granular assemblies composed of particles with diameters ranging from 0.1 mm to 4 mm; b) bidisperse granular mixtures composed of particles with diameters d_1 and d_2, mixed in equal proportions by mass, with mean diameters ranging from 0.175 mm to 3.5 mm and diameter ratios $d_{\mathrm{small}}/d_{\mathrm{large}}$ between 0.125 and 0.75.

1 Introduction

Wave propagation in continuum solids has been extensively studied and is well understood within the framework of classical elasticity and continuum mechanics [1]. However, the propagation of waves through granular media remains comparatively less explored. Because of their discrete and heterogeneous nature, granular materials exhibit complex behaviors that differ fundamentally from those of continuous media, particularly in how they transmit and dissipate mechanical energy.

Understanding wave dynamics in granular media is of significant practical importance in a range of scientific and engineering applications. For instance, wave speed and energy dissipation characteristics in such media play a critical role in the study of small solar system bodies that have significant amounts of regolith [2], acoustic damping materials, and the interpretation of seismic wave propagation through Earth's heterogeneous crust.

Earlier studies have investigated wave propagation in granular media, often estimating bulk elastic properties from measured wave speeds. While theoretical models predict a $v_p \sim p^{1/6}$ scaling [3–5], experimental observations typically report exponents ranging from 1/6 to 1/3, with $v_p \sim p^{1/4}$ being most common [5–8]. In this work, we present a series of controlled laboratory experiments designed to measure the speed of compressional (p-) waves in dry granular materials under varying levels of uniaxial compression by systematically varying the particle size. For these measurements, a custom experimental setup was developed that allows the application of compressive loads on the granular assembly, while avoiding direct loading of the input and output transducers. This design consideration is essential, as piezoelectric transducers are sensitive to high axial loads and can be easily damaged under direct compression.

Using this setup we have conducted experiments on both monodisperse and bi-disperse granular aggregates. The monodisperse samples consisted of spherical grains whose diameters were varied from 0.1 mm to 4 mm. For bi-disperse mixtures, particles with diameters in the range of 0.1–4 mm were combined to produce aggregates with mean diameters ranging from 0.175 mm to 3.5 mm.

We report the measured p-wave speeds in these aggregates as a function of particle size and compressive load. These experimental results offer insights into the behavior of granular materials and provide a basis for validating numerical models that seek to simulate wave propagation and energy dissipation in granular aggregates under compression.

2 Experimental Setup

Figure 1 shows the schematic of the experimental setup. We have used a stainless steel cylindrical cell of diameter, D=50 mm, and height, H=35 mm and filled it with grains (glass beads). To generate and detect the $p-$ waves we utilised two identical Olympus V102-RB contact transducers. These transducers have 1 inch (25.4 mm) detector face diameter. The source and receiver transducers are connected to a digital oscilloscope (Keysight DSOX1202G 200 MHz). The oscilloscope also has a built-in wave-

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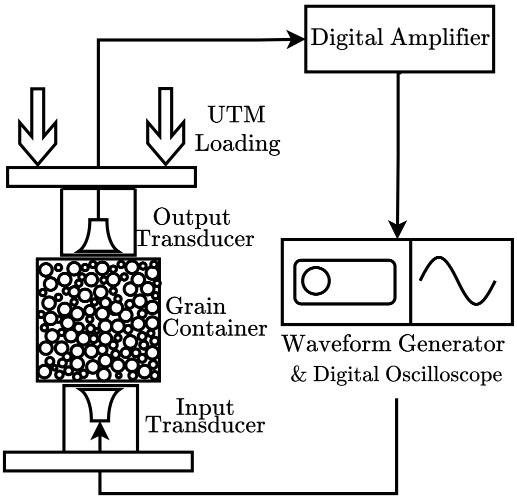

Figure 1. Schematic of the experimental setup.

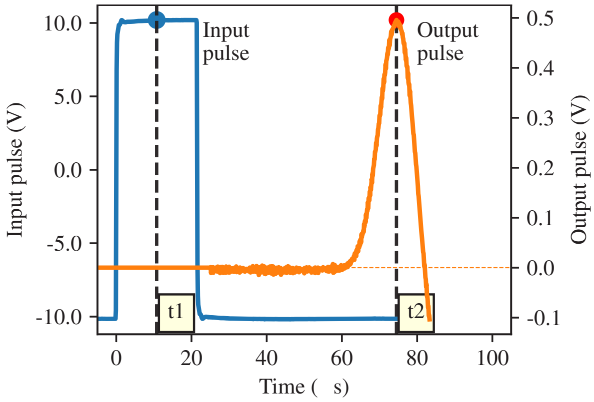

Figure 2. Representative input and output waveforms for a granular aggregate composed of monodisperse grains (diameter = 0.25 mm) under a confining pressure of 150 kPa. The figure illustrates the identification of t_1 and t_2, which are used for calculating the wave speed. For this particular sample, the height is 38.08 mm, the measured travel time is 67.7 µs, corresponding to a p-wave speed of v_p = 597.33 m s^{-1}.

form generator. A 10 V square pulse with a pulse width of 21.5 µs was generated using the waveform generator to serve as the input excitation. The input signal is passed through the sample at a frequency of 100 Hz. The signal frequency was chosen to be high enough so that each pulse propagates through the entire sample before the next pulse arrives, and any reflected waves are sufficiently damped. This was visually confirmed from the oscilloscope display. The excitation pulse was fed into the input transducer. The face of the input transducer was kept in contact with the base of the cylinder containing the grains under compression, thus setting up a longitudinal (p-) wave that propagated through the granular sample. Upon reaching the opposite end of the sample, the transmitted wave was detected by an output transducer. A thin layer of glycerine was applied between the transducer face and the contacting surfaces to ensure better contact. Due to the typically low amplitude of the detected signal, it was routed through a digital amplifier (NF Electronics 5307) to enhance its magnitude. The amplified signal, along with the original input pulse, was then recorded with a digital oscilloscope. Both waveforms were stored digitally for subsequent analysis and post processing. To apply compressive loading on the granular samples, we employ a Universal Testing Machine (Lloyd Instruments EZ-50), which allows fine control on the applied confining force.

3 Experimtental Methods and Protocols

The experimental procedure begins by placing the sample onto the Universal Testing Machine (UTM). Before taking the actual wave speed measurements, the sample is subjected to three initial load–unload cycles: the load is gradually increased up to 1 kN ($\approx$ 500 kPa) at a rate of 5 mm min^{-1}, held at 1 kN for one minute, and then unloaded to 0 N; the sequence is repeated three times. This preconditioning step helps ensure a compact sample and minimizes variations in packing fraction across different experimental repetitions. Following such preconditioning, the sample is loaded to specific load steps, and at each step, an input pulse is transmitted through the sample while the corresponding output pulse is recorded. After each measurement, the sample is unloaded to 0 N before being reloaded to the next load step. The chosen load steps (in N) are: 50, 100, 150, 200, 300, 500, 750, and 1000. Once all measurements were collected for the specified load steps, the positions of the transducers were interchanged without disturbing the sample, and the measurements were repeated at the same load levels. This transducer inversion was carried out to assess the homogeneity of the sample; if the measured wave speeds remained consistent when the direction of wave propagation was reversed (i.e., from top to bottom versus bottom to top), the sample could be considered homogeneous. The wave speeds measured before and after transducer inversion were found to be comparable, confirming that no significant size segregation occurred within the bi-disperse mixtures during sample preparation or loading.

The above process completes one full repetition of the experiment. For each subsequent repetition, the sample is removed, stirred thoroughly in a steel bowl using a tablespoon to randomize its configuration, and carefully poured back into the cylindrical container. The experiment is then repeated following the same protocol, including the initial three load–unload preconditioning cycles, measurements at each load step, and the transducer inversion. This process is repeated a total of three times. After completing three repetitions for a given sample, a new sample — comprising a fresh combination of grain sizes d_1 and d_2 is prepared for the next set of experiments.

4 Results

We have conducted a series of experiments to measure compressional (p-wave) velocities in both monodisperse

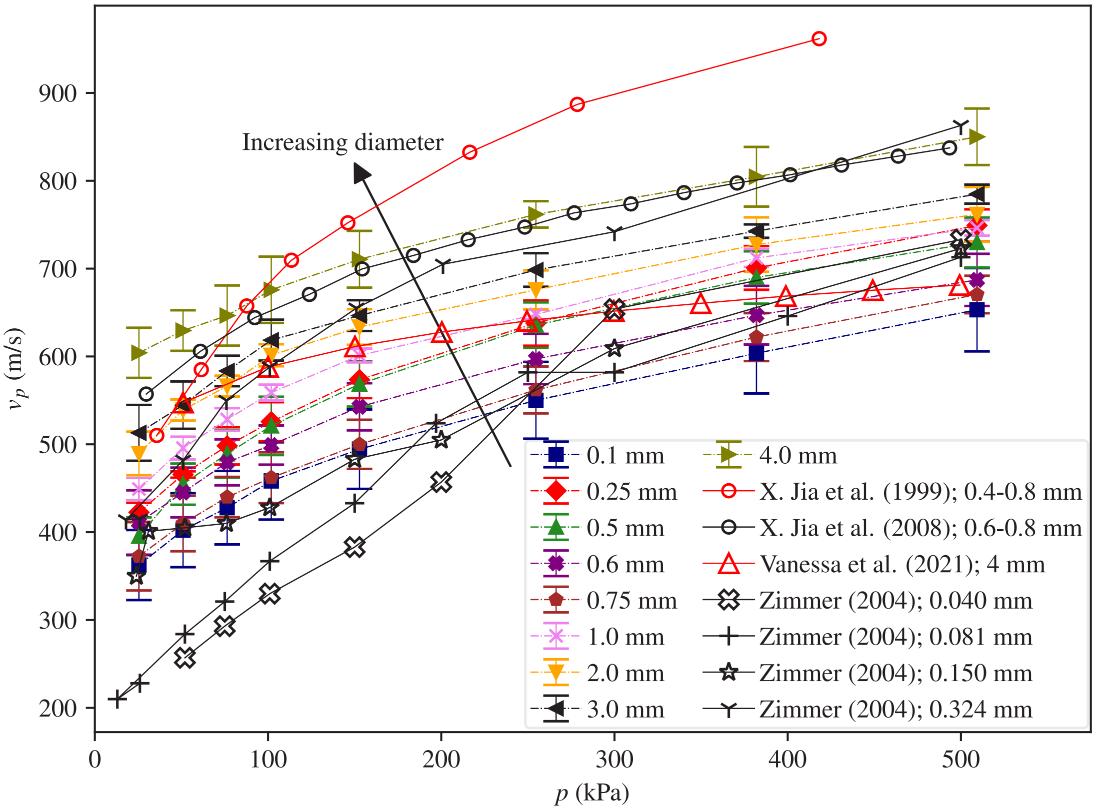

Figure 3. Variation of the p-wave speeds (v_p) in monodisperse granular aggregates with confining pressure (p) for several grain sizes.

and bi-disperse granular aggregates subjected to varying levels of uniaxial compression. The cylindrical cell was filled with granular material to a level slightly below the brim, leaving sufficient space for lid placement. The height of the sample, denoted as H, was then measured using vernier calipers.

The wave arrival time t_2 is defined as the time of the first peak in the recorded output signal. The time t_1 is taken as half the width of the input square pulse. Identification of t_1 and t_2 in the input and output waveforms is illustrated in figure 2. The p-wave speed is then calculated from the relation:

$$v_p = \frac{H}{t_2 - t_1}. \tag{1}$$

4.1 Monodisperse Granular Aggregates

We considered a series of experiments with aggregates of same-sized spherical particles with diameters of 0.1, 0.25, 0.5, 0.6, 0.75, 1, 2, 3 and 4 mm. Each experiment was conducted under compressive stresses ranging from 50 kPa to 500 kPa, and the corresponding p-wave velocities were recorded.

The results from our experiments are shown in figure 3. We also show p-wave velocity measurements from existing literature [6, 8–10] in figure 3. For larger diameter samples (3–4 mm), our velocity measurements show good agreement with those reported by [10] for 4 mm glass beads, particularly in the lower pressure regime (~100 kPa). Experimental v_p data from [8, 9] is slightly higher than our measurements for equivalent grain sizes, but exhibit a similar $p - v_p$ trend. While these comparisons show fair agreement, we note that experimental conditions between our setup and those in the literature are not completely identical. Differences in input pulse signals, grain container geometries, and arrival time definitions can influence measured velocities. For instance, experiments by [9] were conducted on slightly polydisperse granular materials, and [6] has reported the D50 (median) diameter of the grains. The input pulse and wave arrival time identification also vary across studies. Furthermore, as noted in [6], precise determination of the wave arrival times is difficult, which may affect the velocity calculations. These factors should be considered in these comparisons.

Our experimental results reveal a clear trend: p-wave speed rises with increasing grain diameter. This trend indicates a strong dependence of bulk p-wave velocity on grain size. Although some deviations from this trend are present — for example, samples with grain diameter d=0.25 mm exhibit v_p values exceeding those measured for certain samples with larger grain sizes, the overall relationship between grain diameter and wave speed remains consistent. We attribute the rise in wave speed with increasing grain diameter to the reduction in the number of grain–grain contacts per unit length of the sample. Fewer contacts

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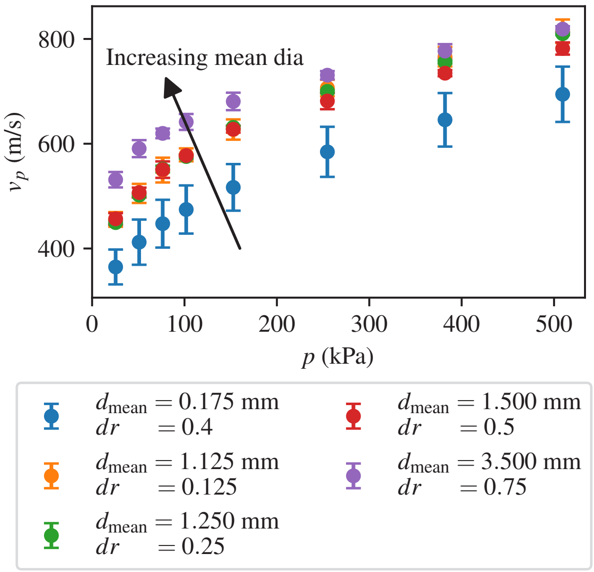

Figure 4. p-wave speeds in bi-disperse granular mixtures. The general trend shows higher v_p in mixtures with higher d_{mean}.

in larger-grain aggregates result in reduced scattering, allowing the wave to propagate more efficiently through the medium. It is well established that wave speeds in continuum solids are significantly higher than in granular assemblies [5]. In the limiting case where the grain size approaches the height of the sample, the system effectively contains a single grain, and the wave propagates through its interior as it would in a continuum body. Therefore, the observed trend is physically meaningful.

This behavior is observed across the entire range of compressive stresses investigated in this study. We observe a similar trend of particle size dependence on wave speeds in the data reported by [6] as well—an aspect not explicitly discussed in their work but evident from their results in this pressure regime.

4.2 Bi-disperse Granular Mixtures

In bi-disperse mixtures each experimental sample consisted of two different-sized grains mixed in equal mass proportion. With d_1 and d_2 denoting the diameters of the larger and smaller grains, respectively, five different combinations of grain pairs were investigated: $(d1, d2) = (0.25, 0.1)$, $(2, 0.25)$, $(2, 0.5)$, $(2, 1)$ and $(4, 3)$ mm. For each mixture, we define, diameter ratio $dr = d_2/d_1$ and mean diameter $d_{mean} = (d_1 + d_2)/2$.

Analysis of the experimental results shown in figure 4 indicate that p-wave velocities do not appear to exhibit a clear dependence on the diameter ratio. However, a consistent trend is observed with respect to mean particle size: mixtures with larger mean diameters generally exhibit higher bulk p-wave velocities. We observe that the mixtures with the largest d_{mean} have the highest v_p, the lowest v_p is measured in the mixture with the smallest d_{mean}, with the remaining three mixtures exhibiting almost similar wave speeds. This trend aligns with the previous observations in monodisperse aggregates and reinforces the role of particle size in governing wave propagation characteristics in granular media.

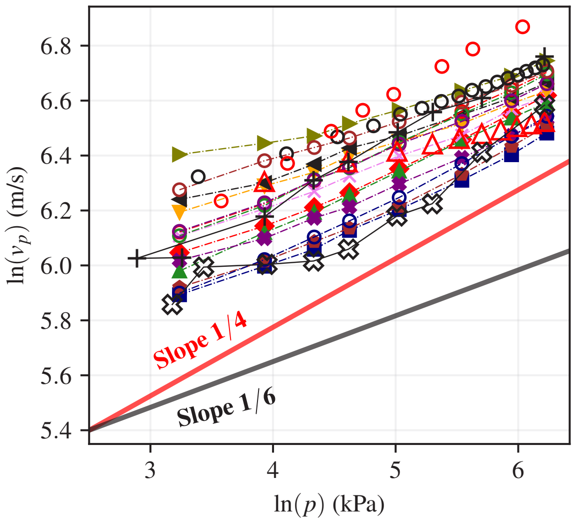

Figure 5. Variation of compressive wave speeds (v_p) with confining pressures presented in figures 3 and 4 are shown on a logarithmic plot. The markers have same meaning as in the legends of figures 3 and 4. Reference lines with slopes 1/4 and 1/6 are drawn for comparison of trend.

Further, the $p - v_p$ results shown in figures 3 and 4 are plotted on a log-log scale in figure 5, along with reference lines of slopes 1/4 and 1/6. The data appears to align more closely with the slope 1/4 line, indicating a $v_p \sim p^{1/4}$ scaling, consistent with the experimental trends discussed in section 1.

Note on wavelength: As can be seen from figures 3 and 4, the typical compressive wave speed is approximately $\bar{v}_p \approx 500\ \mathrm{m\,s^{-1}}$, and the wave duration (pulse width) is about $\bar{t} \approx 20$ µs (see figure 2). Using the relation $\lambda_w = \bar{v}_p\bar{t}$ (as defined in [11]), the corresponding wavelength is calculated to be $\lambda_w \approx 10$ mm, giving a ratio $\lambda_w/d \approx 10$ for the experiments carried out in this study.

5 Conclusions

In this work we have experimentally investigated p-wave speeds in dry granular media under controlled compressive loading, using a custom-built apparatus. Our study included both monodisperse and bi-disperse granular aggregates with grain diameters ranging from 0.1 mm to 4 mm.

For monodisperse aggregates, we observed a clear trend of rising p-wave velocity with increasing grain size, which we attribute to a reduction in the number of grain–grain contacts. This behavior is consistent with the expectation that, in the limit of very large grain diameters, the medium must approach continuum-like behavior, where wave speeds are inherently higher.

In bi-disperse mixtures, we found that p-wave velocity showed no clear dependence on diameter ratio. However, a general trend of increasing wave speed with increasing mean grain diameter was observed. These findings emphasize the importance of grain-scale structure and particle size distribution in influencing wave transmission in granular materials.

The experimental data presented here provides a foundation for validating discrete-element simulations aimed at further exploring how heterogeneity and confining pressure affects wave propagation in granular aggregates.

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