Electromechanical coupled deformation analysis of a hyper-elastic dielectric membrane under combined pressure and electric loading

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Abstract. Dielectric elastomers are soft, lightweight, and highly deformable electroactive polymers that undergo large deformations when subjected to mechanical and electrical loading. This unique property makes them promising candidates for applications in artificial muscles, soft robotics, biomedical actuators, and sensors. In this study, the large deformation behavior of a flat, circular dielectric elastomer membrane, clamped at the boundary, is investigated under the combined influence of lateral pressure and an applied electric potential across its thickness. The resulting model provides a framework for predicting deformation profiles and electromechanical stability limits in dielectric elastomer actuators.

1 Introduction

Dielectric elastomers (DEs) Dielectric elastomers (DEs) are a class of electroactive polymers that can convert electrical energy into mechanical work through large, reversible deformations [1]. Their high energy density, low weight, and compliance make them attractive for diverse applications, including artificial muscles in biomechanics, flexible actuators in soft robotics, energy harvester and highly sensitive deformation sensors [2- 5].

A typical DE actuator consists of a thin elastomeric membrane sandwiched between compliant electrodes. When an electric field is applied, electrostatic (Maxwell) forces cause the membrane to expand in-plane and contract in thickness. This electrically induced actuation can be further coupled with external mechanical loading to achieve complex deformation modes [6].

Understanding the coupled electromechanical deformation of dielectric elastomers is crucial for the design and optimization of such devices. In many practical scenarios, DE membranes are subjected to both mechanical pressure and electrical excitation. For example, in pneumatically driven artificial muscles, a pre-stretched elastomer membrane is inflated by internal pressure before an electric field is applied to enhance or modulate the deformation.

In the recent advances in modelling electromechanical behavior of a membrane the dielectric permittivity of the membrane material which is an important parameter in designing the soft DE actuation model is kept constant [4-5].

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Recent study shows that permittivity is a strong function of deformation. The relative permittivity of the elastomer often varies with stretch [7], introducing additional coupling between the mechanical and electrical responses as DE requires pre-stretch for tuning their actuation. Hence it requires also to include this phenomenon in the model.

In this work, we consider a flat, circular dielectric elastomer membrane clamped at its circumference, first inflated by uniform lateral pressure and then subjected to an electric potential across its thickness. The material is modeled as an incompressible Mooney–Rivlin hyperelastic solid, with the strain energy density expressed in terms of the first and second strain invariants. The total potential energy of the system is formulated by combining the stored elastic energy and the work done by mechanical and electrical loads, with the electrical contribution modeled through Maxwell stress [8] . Using the principle of minimum potential energy, we derive the governing equilibrium equations in terms of radial stretch and membrane slope. This formulation captures the interplay between large deformation mechanics and electrostatics and provides a predictive tool for the analysis and design of dielectric elastomer-based actuators and sensors.

2 Kinematics of Deformation

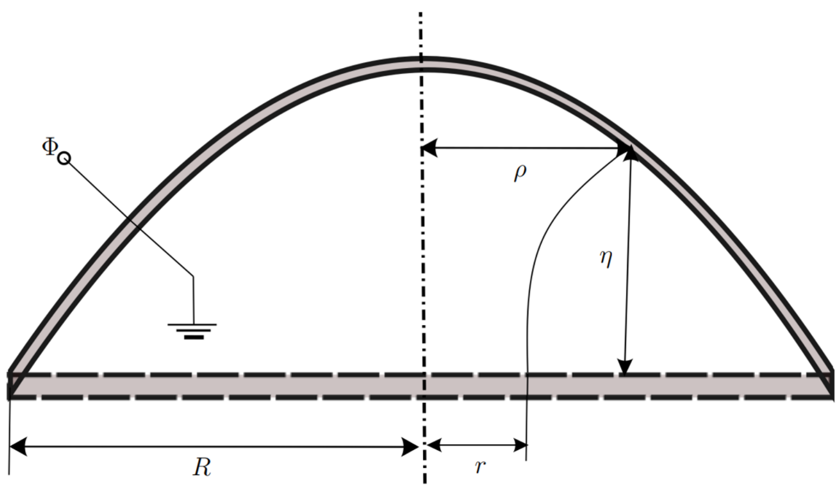

Fig. 1. Schematic illustration of a flat circular dielectric elastomer membrane, where the undeformed configuration is indicated by the broken line. A material point located at r in the reference state is displaced to a radial position ρ and an elevation η in the deformed configuration.

In this study, the non-linear large deformation actuation of initially flat isotropic DE membrane is explored. As shown in figure 1, the membrane is first allowed to inflate due to a constant pressure p and then an electric potential is applied across the membrane thickness. The membrane is incompressible, so that its volume remains unaffected before and after deformation.

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The meridional and circumferential stretch ratios in membrane are, respectively, given by

$$\lambda_1 = \sqrt{\rho'^2 + \eta'^2} \text{ and } \lambda_2 = \frac{\rho}{r} \quad (1)$$

To determine total potential energy, we begin with the strain energy density of the dielectric elastomer which is homogeneous, isotropic and incompressible obeying Mooney-Rivlin hyperelastic membrane material model. The strain energy density for this material model is

$$\widehat{U} = C_1(I_1 - 3) + C_2(I_2 - 3) = C_1[(I_1 - 3) + \alpha(I_2 - 3)] \quad (2)$$

where $\boldsymbol{C_1 and\ C_2}$ are material constants, and $\alpha = C_2/C_1$ which is some time referred as rigidity of membrane, The I_1 and I_2 are the invariant of right Cauchy green tensor and is given by, $I_1 = \lambda_1^2 + \lambda_2^2 + 1/\lambda_1^2\lambda_2^2$ and $I_2 = \lambda_1^2 + \lambda_2^2 + 1/\lambda_1^2\lambda_2^2$.

The above equation is obtained by using the incompressibility for which

$$\lambda_3^2 = \frac{1}{\lambda_1^2\lambda_2^2}$$

The total strain energy of the membrane is then obtained by integrating the strain energy density function $\widehat{U}$ over the (undeformed) volume of the membrane material as

$$\mathrm{U} = 2\pi \int_0^{\mathrm{R}} \widehat{\mathrm{U}}\ \mathrm{r\,dr} \quad (3)$$

The electric potential across the deformed membrane is given by $E = \frac{\Phi}{h}$ with $h = \lambda_3 h_0$ where h and h_0 are the deformed and undeformed thickness, respectively, and λ_3 is the stretch ratio across the thickness. The in-plane components of electric field is zero. The electric energy per unit deformed volume is written as

$$W_e = \frac{1}{2}\epsilon E^2 \quad (4)$$

which is also known as Maxwell stress from electric field. Maxwell stress acts as an effective negative pressure across the thickness, reducing the total resistance of stress. Thus, the effective pressure for the membrane is $p - \frac{1}{2}\epsilon E^2$ (where p is the applied pressure). Therefore, pressure potential is given by

$$W_p = \int \left(p - \frac{1}{2}\epsilon E^2\right) dv = -2\pi \int_0^R \frac{1}{2} p\,\rho^2\,\eta'\,dr \quad (5)$$

Now we find total potential energy as

$$\Pi = U - W_P \quad (6)$$

We derive governing equation of equilibrium using principle of minimum potential energy $\delta\widehat{\Pi} = 0$ where $\widehat{\Pi} = \widehat{U}\,r - \frac{1}{2} p\,\rho^2\,\eta'$. For this purpose, we choose λ_2 and η' as field variable (it can be observed from Eq. (6) η is a cyclic variable). The other variable λ_1 is replaced by $\sqrt{(\lambda_2 + r\,\lambda_2')^2 + \eta'^{\,2}}$ using Eq. (1).

The governing equations are

$$\frac{d}{dr}\left(\frac{\partial\widehat{\Pi}}{\partial\lambda_2'}\right) - \frac{\partial\widehat{\Pi}}{\partial\lambda_2} = 0, \frac{d}{dr}\left(\frac{\partial\widehat{\Pi}}{\partial\eta'}\right) = 0 \quad (7)$$

Next, we substitute $w = -\eta'$ in Eqs. (7) the governing equations read as

$$\lambda_2'' = f_1(\lambda_2, \lambda_2', w, r),\ w' = f_2(\lambda_2, \lambda_2', w, r) \quad (8)$$

where f_1 and f_2 are appropriate functions obtained from Eqs (7).

The boundary conditions for the set of differential equations are

$$\lambda_2'\big|_{r=0} = 0,\ \lambda_2\big|_{r=R} = 1,\ w\big|_{r=0} = 0 \quad (9)$$

here the boundary condition $\lambda_2'\big|_{r=0} = 0$ as the membrane remain flat at the center [9].

3 Numerical Approach

We have solved this problem for the relative permittivity model given by a Gaussian polymer chain [7] as

$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon_0}\left(\boldsymbol{\alpha_1} + \boldsymbol{\alpha_2}\left(\boldsymbol{\lambda_1^2} + \boldsymbol{\lambda_2^2} - \frac{\mathbf{2}}{\boldsymbol{\lambda_3^2}}\right)\right) \tag{10}$$

This should be noted that deformation obtained here for the DE membrane, the permittivity of the DE material is not constant instead it is function of stretch ratios, unlike previous study. [4-5]

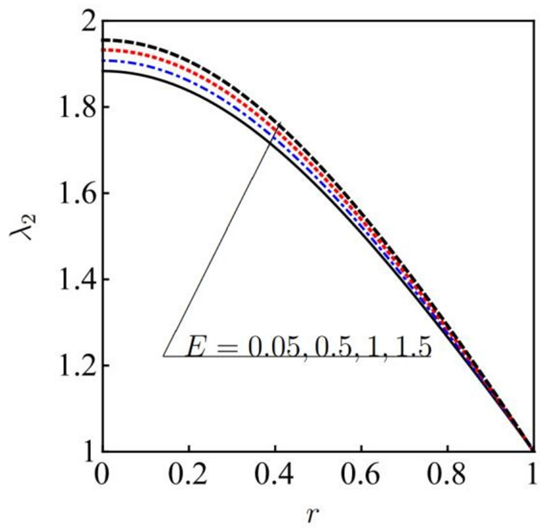

Fig. 2. Variation of circumferential stretch λ_2 v/s radius r.

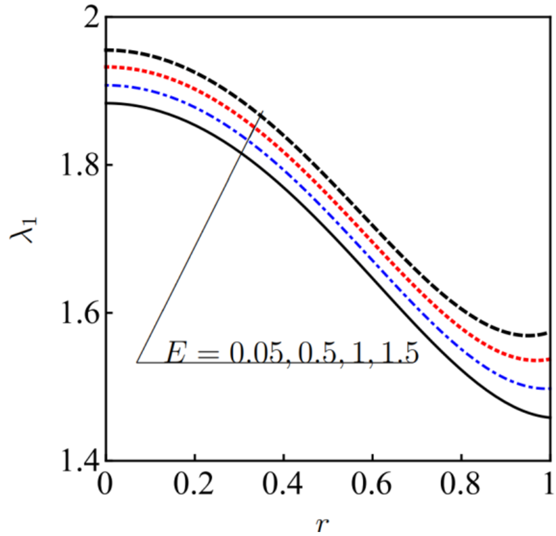

Fig. 3. Variation of meridional stretch λ_1 v/s radius r

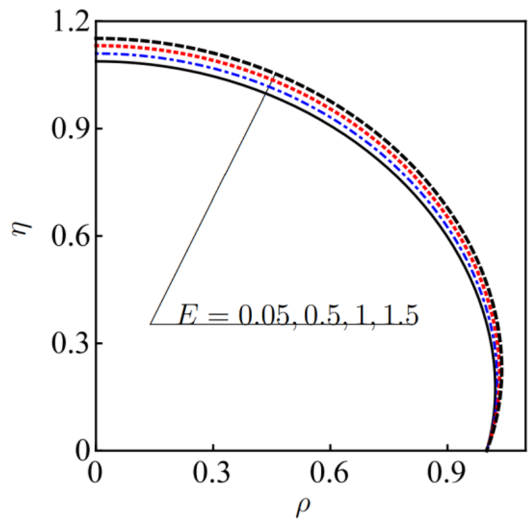

Fig. 4. Deformed profile

Before proceeding further, we introduce the following set of normalized quantities:

$\bar{p} = pR/C_1 h_0$ (nondimensionalized pressure), $\bar{E} = E\sqrt{\frac{R}{c_1 h_0}}$ (nondimensionalized plate Electric field), $\bar{\rho} = \rho/R$ (nondimensionalized deformed radius), $\bar{r} = r/R$ (nondimensionalized deformed radius), and $\bar{\eta} = \eta/R$ (nondimensionalized vertical displacement). The electric potential which leads to electric field is applied across the thickness.

4 Result and discussion

For the sake of simplicity, henceforth, we will drop the bars from the normalized variable notations. We chose material constant $\alpha = 0.1$, $\alpha_1 = 1$, $\alpha_2 = 0.1$. The membrane is then inflated with a pressure We solved the problem for different values of E and plotted the results.

Figure 2 illustrates the radial variation of the circumferential stretch in an inflated membrane under constant pressure p for various electric fields. The solid line represents the baseline case with only pressure and no electric field. The circumferential stretch varies from maximum at the center to zero at the circumference, which is obvious as the membrane is fixed circumferentially.

Figure 3 shows the corresponding variation of the meridional stretch along the membrane radius. Both the circumferential and meridional stretch ratios increase with the applied electric field.

Figure 4 depicts the deformed membrane shape under combined pressure and electric field. The increase in the deformed coordinates η and ρ with higher electric fields indicates greater inflation, a result of the membrane relaxing due to the electromechanical Maxwell stress.

5 Conclusions

In this work This study established a nonlinear, coupled electromechanical framework to model the finite deformation of a pressurized dielectric elastomer membrane under an applied electric field. While demonstrated using a Mooney-Rivlin material, the model's versatility allows for the incorporation of various hyper-elastic constitutive laws. Following are the key highlights of the work

- Our results quantitatively demonstrate that the application of an electric field induces significant additional deformation in the pre-inflated membrane
- The circumferential stretch variation is more than the meridional stretch.
- A variable dielectric permittivity of the membrane is taken to model the non linear electromechanical coupling of the DE membrane.
- This predictive model provides a critical design tool for optimizing the shape, performance and reliability of soft robotic actuators, sensors, and other soft electroactive devices.

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