Microscopic study of ^{117}Te isotope using projected shell model

Amit Kumar[1*], *Meena Sharma*[2], *Aneeqa Basheer*[2], *Manvi Rajput*[2], *Suram Singh*[2] and *Arun Bharti*[3]

[1]Department of Physics, Government Degree College Akhnoor, Jammu-181201, India
[2]Department of Physics and Astronomical Sciences, Central University of Jammu, Samba-181143, India
[3]Department of Physics, University of Jammu, Jammu-180006, India

Abstract. A microscopic analysis of the nuclear structure of the odd-mass ^{117}Te has been carried out using the theoretical framework of the Projected Shell Model (PSM). The tellurium isotopes lie in a transitional region near the closed proton shell at Z=50, where the coupling between individual particle motion and collective nuclear dynamics plays a crucial role. The present study focuses on the description of negative-parity yrast band structures, energy spectra, back-bending behavior, and reduced electric quadrupole transition probabilities *B(E2)*. The calculated results, show close consistency when compared with the experimental observations, successfully reproducing the band-head spins, yrast energies, and the observed back-bending at spin $19/2^{-}$. The calculated *B(E2)* values exhibit an rising trend with spin, indicating enhanced collectivity and nuclear deformation. Present work demonstrates the efficacy of the PSM in modeling the structure of deformed odd-A isotopes.

1 Introduction

Extensive investigations of odd-A nuclei have led to a relatively clear picture of how a single unpaired valence nucleon couples to a well-defined even-even core, particularly in the spherical and well-deformed limits. However, this coupling mechanism becomes significantly more complex and remains poorly understood in transitional nuclei, where the core exhibits moderate deformation and strong shape fluctuations and neither the pure shell-model nor the rigid rotor descriptions are fully applicable. In such nuclei, the interplay between single-particle degrees of freedom and collective motion leads to complex and sometimes unforeseen features in nuclear structure.

The odd mass tellurium isotopes (Z = 52) occupy a particularly interesting region of the nuclear chart, lying just above the closed proton shell at Z = 50. In this mass region, the competition between spherical shell effects and emerging collectivity leads to complex excitation spectra, making the interpretation of experimental level schemes a persistent challenge for existing theoretical models. This difficulty is further amplified in neutron-

* Corresponding author: akbcw2@gmail.com

deficient nuclei, where subtle changes in valence nucleon configurations can strongly influence nuclear deformation and collective behavior.

Recent high-precision lifetime measurements of excited states in nuclei near the Z = 50 shell closures have provided stringent tests for shell-model-based theoretical approaches. These studies highlight the crucial role of specific single-particle orbitals namely the $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$ in shaping the observed level structures of odd-A systems [1-3]. The occupation and alignment of these orbitals are known to drive changes in deformation and rotational behavior, thereby influencing band structures and electromagnetic transition strengths.

In neutron-deficient tellurium isotopes, rotational bands built on the above mentioned orbitals exhibit clear signatures of the gradual onset of nuclear collectivity in systems containing few valence nucleons outside closed shells [4-7]. The observed band crossings, signature splittings, and enhanced *B(E2)* transition probabilities reflect the delicate balance between single-particle motion and collective rotation, providing valuable insights into the evolution of nuclear structure in this transitional mass region.

The primary objective of present work is performing detailed analysis of the nuclear structure of ^{117}Te using the Projected Shell Model (PSM). The particularly well suited PSM for describing deformed and transitional nuclei, as it combines the advantages of a deformed mean-field approach with the restoration of good angular momentum through projection techniques. This framework allows for a unified and self-consistent description of energy spectra, rotational band structures, back-bending phenomena arising from quasiparticle alignments, and electromagnetic transition probabilities such as *B(E2)* values. By applying the PSM to ^{117}Te, the present study seeks to gain deeper insight into the microscopic processes that control the interaction between individual quasiparticle configurations and collective behavior in neutron-deficient odd-A nuclei located near shell closures.

2 Methodology

The Projected Shell Model [8] is based on a deformed Nilsson mean-field approach with pairing correlations treated within the BCS approximation. The rotationally invariant Hamiltonian employed in the PSM consists of spherical single-particle energies along with quadrupole-quadrupole, monopole pairing, and quadrupole pairing interactions and is given as

$$\hat{H} = \hat{H}_0 - \frac{1}{2}\chi \sum_\mu \hat{Q}_\mu \hat{Q}_\mu^{+} - G_M \hat{P}^+ \hat{P} - G_Q \sum_\mu \hat{P}_\mu^+ \hat{P}_\mu \quad (1)$$

The quadrupole-quadrupole interaction strength is evaluated self-consistently with quadrupole deformation parameter ε_2. To further improve agreement with experimental energies, the Nilsson potential also incorporates hexadecapole deformation ε_4. The deformation parameters adopted in the current analysis have been summarized in Table 1. The monopole pairing strength

$$G_M = (G_1 \mp G_2 \frac{N-Z}{A}) \frac{1}{A} (MeV) \quad (2)$$

is adjusted to reproduce experimental odd-even mass differences, with coupling constants G_1 = 18.00 and G_2 = 11.12. The quadrupole pairing strength is taken as a fixed fraction (0.22) of the monopole pairing strength. The model space comprises major oscillator shells for both protons and neutrons, restricted to states within 3.5 MeV of the Fermi surface.

Since the intrinsic quasiparticle states do not possess good angular momentum, angular-momentum projection is applied to restore rotational symmetry. Configuration mixing is achieved by diagonalizing the Hamiltonian in the projected basis.

Table 1. Parameters ε_2 and ε_4 used in calculations.

	ε_2	ε_4
^{117}Te	0.139	-0.025

3 Results and Discussions

In these calculations, the valence space is limited to states within 3.5 MeV of the Fermi surface for both protons and neutrons, setting the basis dimension. The selected states are projected onto good angular momentum to diagonalize the shell model Hamiltonian, producing yrast spectra. The resulting wavefunctions are subsequently used to calculate various nuclear structure observables, and the outcomes are then compared with available experimental measurements [6,7] in the next subsection.

3.1 Band Diagram

A band diagram is a visual representation used to track the energy levels of various quasiparticle (qp) configurations. These configurations are projected into states of specific angular momentum and are plotted as a function of spin.

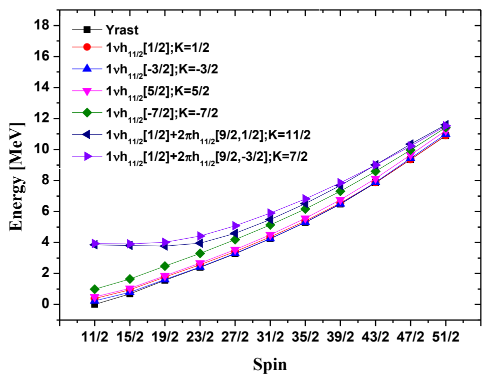

Fig. 1. Negative parity band structure of ^{117}Te**.**

The Projected Shell Model (PSM) starts with the construction of an intrinsic basis using a deformed Nilsson mean-field, while pairing correlations are introduced through the BCS formalism, converting particles into quasiparticles. From this foundation, various multi-quasiparticle configurations are developed to represent the internal structure of the nucleus. Because these deformed states do not inherently possess well-defined angular momentum, the model employs a projection operator to restore rotational symmetry. Each projected configuration generates a rotational band, and the corresponding projected energies plotted against spin constitute the band diagram. The band diagram obtained from PSM calculations for negative-parity for ^{117}Te is presented in Fig. 1. The yrast states up to spin $39/2^-$ are primarily built on one-quasiparticle (1-qp) bands with the configurations $1\nu h_{11/2}[1/2]$, K=1/2, $1\nu h_{11/2}[-3/2]$, K=-3/2, $1\nu h_{11/2}[5/2]$, K=5/2 and $1\nu h_{11/2}[-7/2]$, K=-7/2. Beyond this spin, 3-qp bands $1\nu h_{11/2}[1/2]+2\pi\, h_{11/2}\,[9/2, 1/2]$, K=11/2 and $1\nu h_{11/2}[1/2]+2\pi h_{11/2}\,[9/2, -3/2]$, K=7/2 also dive down in energy and begin to contribute significantly to the yrast sequence. Further, from the analysis of band diagrams, it is predicted that the ground-state yrast band in this nucleus is found to originate from the $\nu h_{11/2}$ orbital, exhibiting prolate quadrupole deformation which confirmed the experimental observation.

3.2 Yrast Spectra

In nuclear structure studies, the set of states possessing the minimum excitation energy at each value of the total angular momentum *I* is collectively referred to as the yrast sequence or yrast band. This sequence plays a central role in understanding rotational behavior and shape evolution in nuclei. Within the present framework, the intrinsic quasiparticle configurations are first projected onto states with good angular momentum, generating a set of projected basis states that form the foundation of the band diagram.

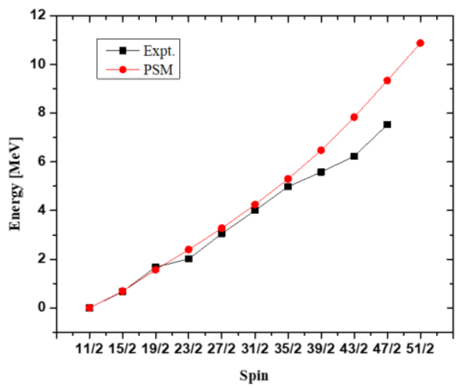

Fig. 2. Comparison between the calculated and experimentally available negative parity yrast band spectra for ^{117}Te.

These projected states are subsequently allowed to interact through the diagonalization of a Hamiltonian that incorporates both pairing correlations and quadrupole-quadrupole interactions. This configuration mixing is essential for capturing the collective and single-particle aspects of nuclear motion. For a given spin *I*, the lowest eigen energy resulting from this diagonalization process is identified as the yrast state, ensuring that the energetically favored configuration at each angular momentum is correctly determined.
The obtained calculations reproduce the experimentally observed band-head spin of $11/2^-$, which is a crucial test of the model's validity. Moreover, the theoretical yrast energies closely follow the experimental trend across the entire spin range, as illustrated in Fig. 2. This strong correspondence demonstrates that the PSM provides an accurate description of the rotational structure of the nucleus.
The agreement further indicates that the model effectively accounts for the intricate coupling between nuclear deformation and specific quasiparticle excitations. In particular, the dominant role of the neutron $h_{11/2}$ orbital is clearly reflected in the calculated spectrum, underscoring its importance in governing the rotational dynamics of this tellurium isotope.

3.3 Back-bending phenomena

The back-bending phenomenon in the Projected Shell Model is explained as a microscopic effect arising from the crossing and mixing of angular-momentum projected qp configurations. At low spins, 1-qp bands dominates the yrast band, leading to smooth collective rotation with a relatively small moment of inertia. As the rotational frequency increases, the coriolis force weakens pairing correlations and favours the alignment of a pair of nucleons in high-j intruder orbitals. The resulting aligned 2-quasiparticle band approaches and crosses the ground band, becoming energetically favoured.

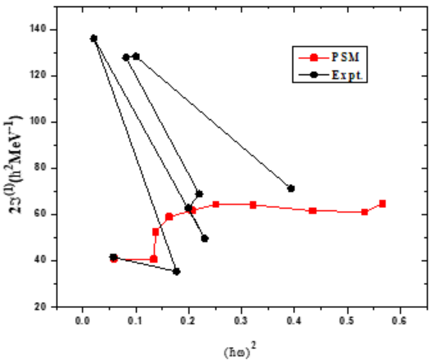

Fig. 3. Plots showing twice the Kinetic Moment of inertia as a function of square of the rotational frequency $(\hbar\omega)^2$ for negative parity band of ^{117}Te.

This sudden change in the dominant configuration of the yrast band produces an abrupt increase in the moment of inertia, observed as back-bending in rotational spectra, and provides a clear microscopic interpretation within the PSM framework. The rotational frequencies $(\hbar\omega)^2$ and kinetic moment of inertia ($2\Im^{(1)}$) are calculated using the following formulae [9]:

$$\hbar\omega = \frac{E_\gamma}{\sqrt{(I+1)(I+2)-K^2} - \sqrt{(I-1)I-K^2}} \tag{3}$$

$$2\Im^{(1)} = \frac{(2I-1)}{\omega} \tag{4}$$

$$\text{where, } E_\gamma = E(I) - E(I-2) \tag{5}$$

Experimental studies of the ^{117}Te isotope indicate the presence of back-bending feature phenomenon occurring at a spin of $19/2^-$, which corresponds to a rotational frequency of 0.1771 $(\hbar\omega)^2$. While the PSM correctly predicts the onset of this structural shift at the exact same spin state $19/2^-$, it calculates the transition at a lower rotational frequency of 0.1339 $(\hbar\omega)^2$. The observed shift in the moment of inertia with rotational frequency is primarily driven by the interaction of the 1-quasiparticle (1-qp**)** and 3-quasiparticle (3-qp) bands. As previously detailed in the band diagram analysis, these distinct configurations compete for stability as the nucleus gains angular momentum.

3.4 *B(E2)* transition probabilities

The reduced electric quadrupole transition probabilities corresponding to transitions from *B(E2)* (from an initial state ($I_i = I$) to a final state ($I_f = I - 2$)) provide important information about the collective nature, deformation, and configuration mixing of nuclear states and is given by

$$B(E2, I \to I-2) = \frac{e^2}{2I+1} \left| \langle \Psi^{I-2} || \hat{Q}_2 || \Psi^I \rangle \right|^2 \tag{6}$$

the operator $\hat{Q}_2$ is related to the quadrupole operators.

In these calculations, proton and neutron effective charges of 1.5e and 0.5e, respectively, were used, as is customary in shell-model analyses to account for core polarization effects that lie beyond the explicitly included model space. These phenomenological values effectively simulate the collective response of the inert core and provide a realistic representation of the quadrupole collectivity arising from valence nucleons. Their inclusion allows for more accurate description of the electric quadrupole operator and, consequently, of the intrinsic deformation characteristics of the nucleus under investigation.

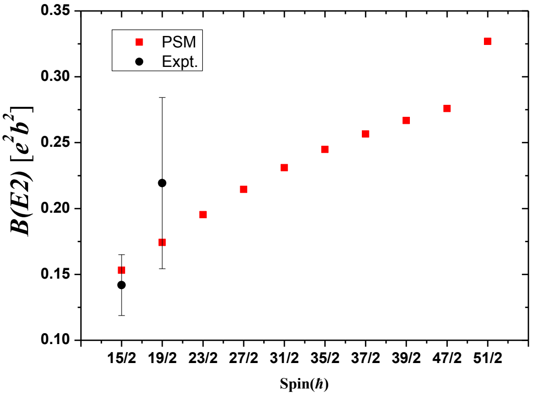

Fig. 4. *B(E2)* transition probabilities for the negative parity yrast band of ^{117}Te (Experimental data is taken from Ref. [6]).

The resulting *B(E2)* values, illustrated in Fig. 4, display a systematic increase with spin. This progressive increase suggests a growing degree of quadrupole collectivity as the angular momentum increases, which is typically associated with the development of nuclear deformation. Such behavior implies that the nucleus becomes more structurally coherent at higher spins, leading to stronger E2 transitions between successive energy levels. These transitions occur predominantly through γ-ray emission or absorption and serve as sensitive probes of the underlying nuclear shape and collective motion.
Although experimental *B(E2)* data obtained for negative-parity yrast states in ^{117}Te isotope are currently available for only limited spin values, the measurements that do exist are agree well with the theoretical predictions obtained in this work. This level of consistency reinforces the reliability of the adopted model framework and parameterization. Furthermore, the ability of the model to accurately reproduce the existing experimental systematics lends confidence to extending the theoretical predictions to states that have not yet been measured, thereby providing a reliable reference for interpreting the evsolution of quadrupole collectivity in ^{117}Te with increase in spin. As a result, the present study serves as a valuable benchmark for forthcoming experimental and theoretical studies of nuclear structure in this mass region.

4 Conclusion

The research work presented utilizes the PSM to examine the microscopic structure of the odd-mass ^{117}Te. The theoretical framework effectively replicates the observed negative-parity yrast states, identifying their primary structural origin as the $h_{11/2} \otimes h_{11/2}$ configuration. The *B(E2)* values have been calculated, although experimental data is available for only a few transitions, the existing measurements align closely with our theoretical predictions. Additionally, the study offers a clear explanation of the back-bending phenomenon in ^{117}Te isotope, supported by both band diagrams and analysis of PSM wave functions, thereby strengthening the understanding of structural changes at high angular momentum. These theoretical predictions provide important insights into the quadrupole collectivity and high-spin behaviour of the system.

Acknowledgement

One of the authors (Amit Kumar) gratefully acknowledge the financial support provided by the Anusandhan National Research Foundation (ANRF) under Teachers Associateship for Research Excellence (TARE) scheme File No: TAR/2023/000395. This funding has been pivotal in enabling the successful completion of the present research work.

References

1. H. C. Scraggs *et al.*, High-spin study of ^{113}Xe: Smooth band termination in valence space, Phys. Rev. C **61**, 064316 (2000). https://doi.org/10.1103/PhysRevC.61.064316
2. T. Ishii, A. Makishima, M. Shibata, M. Ogawa, and M. Ishii, Nuclear structure of ^{109}Sb, Phys. Rev. C **49**, 2982 (1994). https://doi.org/10.1103/PhysRevC.49.2982
3. E. S. Paul *et al.*, High-spin study of ^{111}I, Phys. Rev. C **61**, 064320 (2000). https://doi.org/10.1103/PhysRevC.61.064320
4. B. Hadinia *et al.*, First identification of γ-ray transitions in ^{107}Te, Phys. Rev. C **70**, 064314 (2004). https://doi.org/10.1103/PhysRevC.70.064314
5. A. J. Boston *et al.*, High-spin states in ^{109}Te : Competition between collective and single-particle excitations, Phys. Rev. C **61**, 064305 (2000).https://doi.org/10.1103/PhysRevC.61.064305
6. K. Starosta *et al.*, Lifetime measurements of low-lying states in odd-mass isotopes ^{117,119}Te and the nature of vibration in Te isotopes**,** Phys. Rev. C **112**, 014321 (2025). https://doi.org/10.1103/fmgd-tj4w
7. Jean Blachot, Adopted Levels, Gammas for ^{117}Te, Nuclear Data Sheets (2009). https://www.nndc.bnl.gov/nudat3/getdataset.jsp?nucleus=117Te&unc=NDS
8. K. Hara and Y. Sun, Projected shell model and high-spin spectroscopy, Int. J. Mod. Phys. E **4**, 637 (1995). https://doi.org/10.1142/S0218301395000250
9. A.Ibannez-Sandoval *et al.*, Projected shell model study of yrast states of neutron-deficient odd-mass Pr nuclei, Phys. Rev.C **83** 034308 (2011). https://doi.org/10.1103/PhysRevC.83.034308