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Constraining heavy-light Tetraquark Masses: A Regge Approach

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Abstract. In this work we investigate mass bounds of doubly strange–doubly charm tetraquark states ($ss\bar{c}\bar{c}$) within the framework of Regge phenomenology. Using a quasi-linear Regge-trajectory ansatz, we derive model-independent linear and quadratic mass inequalities that constrain the allowed mass ranges of $ss\bar{c}\bar{c}$ ground states. Regge slope and intercept parameters are fixed by fitting well-established hadronic trajectories and by exploiting heavy-quark symmetry relations; these parameters are then employed to place upper and lower bounds on masses in the (J, M^2) plane for orbital excitations. Where possible, we contrast the derived bounds with existing theoretical predictions. Finally, we discuss experimental consequences: which mass regions are most promising for searches, how our inequalities can assist in spin–parity assignment, and which observed structures (if any) could be compatible with an $ss\bar{c}\bar{c}$ interpretation. The mass bounds presented here provide robust, phenomenological benchmarks to guide future experimental and theoretical studies of strange–charm multiquark spectroscopy.

1 Introduction

In 1964, Gell-Mann and Zweig independently proposed the quark model [1, 2], which transformed hadron spectroscopy by describing hadrons as composites of quarks. Recently, many new hadronic bound states have been reported by LHCb [3], Belle [4], and BESIII [5], and their masses and properties are broadly consistent with modern theoretical predictions [6]. Since Belle's 2003 observation of $X(3872)$ [7], searches for exotic hadrons - especially tetraquarks and pentaquarks - have accelerated, yielding many non-standard candidates. Notable examples include $T_{c\bar{c}1}(3900)$ [8], $T_{b\bar{b}1}(10610)$ [9], and the LHCb pentaquark signals $P_c(4380)^+$ and $P_c(4450)^+$ [10].

Studying the mass spectra of exotic hadrons such as tetraquarks helps probe strong-interaction dynamics in QCD, especially its non-perturbative regime, and sheds light on color confinement. Tetraquark spectroscopy is explored using lattice QCD, QCD sum rules, effective field theories, and phenomenological; lattice QCD is particularly fundamental because it simulates QCD from first principles on a spacetime lattice [11]. Phenomenological models, often calibrated to data, are also widely used to interpret and predict tetraquark masses, typically via diquark-anti diquark approach [12-14].

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In this work, we study the doubly strange- doubly charm ($ss\bar{c}\bar{c}$) tetraquark mass spectra using Regge phenomenology. Inspired by Wei et al. [15], who derived mass equalities and inequalities from quasi-linear Regge trajectories, we adopt a similar Regge-based strategy and extend it to excited tetraquark levels under the assumption of linear trajectories. By relating Regge slopes and intercepts to tetraquark masses in the $((J, M^2))$ plane, we estimate allowed mass windows for ground and excited states across different (J^P) assignments.
The remainder of the paper is organized as follows: Section 2 outlines the theoretical formalism, Section 3 discusses the method of getting tetraquark mass spectra, Section 4 presents and discusses the results, and the last section lists the references cited.

2 Theoretical Formalism

Regge theory offers a simple and effective phenomenological approach to hadron spectroscopy. An especially intuitive interpretation is provided by Nambu's string picture, using which we can get the following linear relationship for hadron mass M and its angular momentum J [16,17].

$$J = \frac{M^2}{2\pi\sigma} + c'' \ldots (1)$$

Hence, within the quasi-linear Regge framework, the relationship between the hadron mass (M) and its total angular momentum quantum number (J) can be written as:

$$J = \beta(M) = \beta(0) + \beta' M^2 \ldots (2)$$

The Regge trajectory parameters - namely the slopes (β') and intercepts $(\beta(0))$ - for mesons with given spin-parity (J^P) and different quark compositions are connected by the following relations [18]:

$$\beta_{i\bar{\imath}}(0) + \beta_{j\bar{\jmath}}(0) = 2\beta_{i\bar{\jmath}}(0) \ldots (3)$$

$$\frac{1}{\beta'_{i\bar{\imath}}} + \frac{1}{\beta'_{j\bar{\jmath}}} = \frac{2}{\beta'_{i\bar{\jmath}}} \ldots (4)$$

Here, (i) and (j) label the quark flavors. By, solving the above quadratic equation we get the following relations which relates the hadrons mass with slopes in (J, M^2) plane,

$$\frac{\beta'_{j\bar{\jmath}}}{\beta'_{i\bar{\imath}}} = \frac{1}{2M^2_{j\bar{\jmath}}} \times \left[\left(4M^2_{i\bar{\jmath}} - M^2_{i\bar{\imath}} - M^2_{j\bar{\jmath}}\right) + \sqrt{\left(4M^2_{i\bar{\jmath}} - M^2_{i\bar{\imath}} - M^2_{j\bar{\jmath}}\right)^2 - 4M^2_{i\bar{\imath}}M^2_{j\bar{\jmath}}}\right] \ldots (5)$$

$$\frac{\beta'_{i\bar{\jmath}}}{\beta'_{i\bar{\imath}}} = \frac{1}{4M^2_{i\bar{\jmath}}} \times \left[\left(4M^2_{i\bar{\jmath}} + M^2_{i\bar{\imath}} - M^2_{j\bar{\jmath}}\right) + \sqrt{\left(4M^2_{i\bar{\jmath}} - M^2_{i\bar{\imath}} - M^2_{j\bar{\jmath}}\right)^2 - 4M^2_{i\bar{\imath}}M^2_{j\bar{\jmath}}}\right] \ldots (6)$$

2.1 Linear mass inequalities and quadratic mass inequalities

Since the Regge slopes $\beta'_{j\bar{\jmath}}$ and $\beta'_{i\bar{\imath}}$ are positive real numbers, their ratio $\beta'_{j\bar{\jmath}}/\beta'_{i\bar{\imath}}$ is also real. Thus, from Eq. (5) we obtain

$$\left|4M^2_{i\bar{\jmath}} - M^2_{i\bar{\imath}} - M^2_{j\bar{\jmath}}\right| \geq 2M_{i\bar{\imath}}M_{j\bar{\jmath}} \ldots (7)$$

By performing a series of mathematical manipulations on the above equation, we obtain the following inequality.

$$2M_{i\bar{\jmath}} > M_{j\bar{\jmath}} + M_{i\bar{\imath}} \ldots (8)$$

Studies have shown that Regge trajectory slopes decrease with increasing quark mass [15]. Thus, for $m_j > m_i$, we have $\beta'_{j\bar{\jmath}}/\beta'_{i\bar{\imath}} < 1$. From Eq. (5), it then follows that,

$$\frac{1}{2M^2{}_{j\bar{j}}} \times \left[\left(4M_{i\bar{j}}^2 - M_{i\bar{i}}^2 - M_{j\bar{j}}^2\right) + \sqrt{\left(4M_{i\bar{j}}^2 - M_{i\bar{i}}^2 - M_{j\bar{j}}^2\right)^2 - 4M_{i\bar{i}}^2 M_{j\bar{j}}^2}\right] < 1 \;\ldots(9)$$

As the square root term in the Eq. (9) is positive, we can conclude that

$$2M_{j\bar{j}} - \left(4M_{i\bar{j}}^2 - M_{i\bar{i}}^2 - M_{j\bar{j}}^2\right) > 0 \;\ldots(10)$$

By, equations (9) and (10) we get,

$$\left(4M_{i\bar{j}}^2 - M_{i\bar{i}}^2 - M_{j\bar{j}}^2\right)^2 - 4M_{i\bar{i}}^2 M_{j\bar{j}}^2 < \left[2M_{j\bar{j}}^2 - \left(4M_{i\bar{j}}^2 - M_{i\bar{i}}^2 - M_{j\bar{j}}^2\right)\right]^2 \;\ldots(11)$$

The last two equations can be used to get the following inequality:

$$2M_{i\bar{j}}^2 < M_{j\bar{j}}^2 + M_{i\bar{i}}^2 \ldots(12)$$

Combining equations (8) and (12) we get the following mass bound for $M_{j\bar{j}}$

$$\frac{M_{j\bar{j}} + M_{i\bar{i}}}{2} < M_{i\bar{j}} < \sqrt{\frac{M_{j\bar{j}}^2 + M_{i\bar{i}}^2}{2}} \ldots(13)$$

3 Tetraquark mass spectra

In this section we calculate the mass spectra of doubly strange- doubly charm ($ss\bar{c}\bar{c}$) tetraquark while considering them as the bound states of two clusters (diquark and anti-diquark). The diquark can only be discovered contained within hadrons and employed as an effective degree of freedom since a pair of quarks cannot be a color singlet.

In eq (13), if we take $i = [ss]$, $j = [cc]$, we get the following relation:

$$\frac{M_{cc\bar{c}\bar{c}} + M_{ss\bar{s}\bar{s}}}{2} < M_{ss\bar{c}\bar{c}} < \sqrt{\frac{M_{cc\bar{c}\bar{c}}^2 + M_{ss\bar{c}\bar{c}}^2}{2}} \;\ldots(14)$$

By inserting the suitable values of i and j in equation (5), we can get,

$$\beta'{}_{ss\bar{c}\bar{c}} = \frac{\beta'{}_{ss\bar{s}\bar{s}}}{4M^2{}_{ss\bar{c}\bar{c}}} \times \left[(4M_{ss\bar{c}\bar{c}}^2 + M_{ss\bar{s}\bar{s}}^2 - M_{cc\bar{c}\bar{c}}^2) + \sqrt{(4M_{ss\bar{c}\bar{c}}^2 - M_{ss\bar{s}\bar{s}}^2 - M_{cc\bar{c}\bar{c}}^2)^2 - 4M_{ss\bar{s}\bar{s}}^2 M_{cc\bar{c}\bar{c}}^2}\right] \;\ldots(15)$$

By inserting the values of $M_{ss\bar{s}\bar{s}}$, $M_{cc\bar{c}\bar{c}}$, and $\beta'{}_{ss\bar{s}\bar{s}}$ (which can be calculated using masses of 0^+ and 1^- states of $ss\bar{s}\bar{s}$ tetraquark) into the above equation, $\beta'{}_{ss\bar{b}\bar{b}}$ can be represented as a function of $M_{ss\bar{b}\bar{b}}$.

Furthermore, using below equation, the mass of the excited $ss\bar{c}\bar{c}$ tetraquark state can be expressed as :

$$M_{J+k(ss\bar{c}\bar{c})} = \sqrt{M_{J(ss\bar{c}\bar{c})}^2 + \frac{k}{\beta'{}_{ss\bar{c}\bar{c}}}} \;\ldots(16)$$

4 Results and Discussion

The ground and excited state masses of fully $ss\bar{c}\bar{c}$ for $J^P = 0^+, 1^+$ and 2^+ is calculated using Eqs. (14) and (16) and by taking ground state mass of ($ss\bar{s}\bar{s}$) and ground and excited state masses of ($cc\bar{c}\bar{c}$) as an input from Refs. [19] and [20] respectively, which are shown in table 1. The calculated values are compared with Ref. [21].

Table 1. Mass spectra of $ss\bar{c}\bar{c}$ tetraquark (in $J(M^2)$ plane).

State	J^P	Predicted Mass (in GeV)	Ref. [21]
1^1S_0	0^+	4.126-4.515	4.352
1^1P_1	1^-	4.449-4.698	4.556
1^3S_1	1^+	4.166-4.555	4.374
1^3P_2	2^-	4.496-4.743	4.581
1^5S_2	2^+	4.239-4.629	4.418
1^5P_3	3^-	4.561-4.813	4.612

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