

Investigation of the Spin-Dependent Structure of Singly Charmed-Strange Pentaquarks within the HCQM Framework

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Abstract. The hypercentral constituent quark model (HCQM) to calculate the mass spectra of singly charmed pentaquark states. As a bound system of two diquarks and an antiquark, the pentaquark is represented by a potential that contains a Coulomb term and a linear confining term. Spin-dependent interactions are included, namely spin–spin, spin–orbit, and tensor terms, to explain the fine structure of the mass spectra. We compare our results with available experimental evidence and other theoretical techniques, and we do a systematic analysis of the spin-parity quantum numbers of the anticipated states. The work aids ongoing efforts to gain a better understanding of singly charmed pentaquark systems and offers important insights into the internal structure of pentaquarks.

1 Introduction

The quark model, independently introduced by Gell-Mann and Zweig [1-3], provides a foundational framework for hadron classification by describing conventional hadrons as bound states of quarks and antiquarks, namely mesons composed of a quark–antiquark pair ($q\bar{q}$) and baryons consisting of three quarks (qqq), previously we have study the various property of these conventional hadrons using the Chiral perturbation theory [4,5]. Within this picture, the observed spectrum of hadrons can be systematically organized according to their quark content, spin, and flavor quantum numbers. The development of quantum chromodynamics (QCD) as the underlying theory of the strong interaction significantly broadened this perspective by allowing color-singlet configurations beyond the conventional meson and baryon structures. Therefore, QCD permits the existence of exotic hadronic states, including tetraquarks ($qq\bar{q}\bar{q}$), pentaquarks ($qqq\bar{q}\bar{q}$), hexaquarks ($qqqqq\bar{q}$), and purely gluonic bound states known as glueballs [6-9]. The study of such multi-quark and gluonic configurations plays a crucial role in probing the mechanisms of color confinement, hadronization, and other non-perturbative aspects of QCD, thereby providing deeper insight into the rich dynamics of the strong interaction beyond the conventional quark model [10-14].

Pentaquark states, consisting of four quarks and one antiquark, attracted significant attention following the experimental report of the θ^+ resonance in 2003[15]. A decisive advancement in this field occurred in 2015 with the observation of the hidden-charm pentaquark candidates $P_c(4083)^+$ and $P_c(4450)^+$ by the LHCb Collaboration [17], with additional structures reported in later measurements [18]. These experimental findings renewed strong interest in the spectroscopy of multi-quark systems. In this

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context, singly heavy pentaquarks containing a single charm quark are of relevance, as the presence of a heavy constituent allows the application of heavy-quark effective theory (HQET) [19] and motivates diquark-based descriptions [20]. The study of such states provides valuable insight into confinement dynamics and the role of diquark correlations in multi-quark hadrons.[16]

In this work, we study the mass spectrum of the singly charmed qqqc \bar{s} pentaquark system within the framework of the hypercentral constituent quark model (HCQM) [21]. The hypercentral potential is employed to effectively describe the internal dynamics of the diquark–diquark–antiquark configuration [22,23]. Spin-dependent interactions are considered to generate the mass splitting among different states, and the corresponding spin–parity (J^P) quantum numbers are systematically assigned [24].

2 Methodology

Singly charm-strange pentaquark are modelled as a three-body system of two diquarks and an antiquark. Their internal dynamics are conveniently described by the Jacobi coordinates in Ref. [25-29,39,40] as

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_{d_1} - \vec{r}_{d_2}), \tag{1}$$

$$\vec{\lambda} = \frac{m_{d_1}\vec{r}_{d_1} + m_{d_2}\vec{r}_{d_2} - (m_{d_1} + m_{d_2})\vec{r}_q}{\sqrt{m_{d_1}^2 + m_{d_2}^2 + (m_{d_1} + m_{d_2})^2}}. \tag{2}$$

Here the diquark mass and position represented as m_{d_i} ($i = 1,2$) and r_{d_i} ($i = 1,2$) respectively are given in Table 1.

Table 1. Diquark masses (GeV)

Quark content	Diquark type	Mass [30]
[q,q]	S	0.710
{q,q}	A	0.909
[q,s]	S	0.948
{q,s}	A	1.069
[q,c]	S	1.860
{q,c}	A	1.870

The system is expressed in terms of the hyperradius, $X = \sqrt{\rho^2 + \lambda^2}$, and the wavefunction is governed by the hyperradial Schrödinger equation in the hypercentral approximation [32,33]:

$$\left[\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2} \right] \Psi_\gamma(x) = -2m[E - V(x)]\Psi_\gamma(x) \tag{3}$$

where the m is effective mass of three body system and is γ hyper angular momentum.[24]

The effective interaction contains a static confining term and a spin-dependent corrections as [33-36]

$$V(x) = \frac{\tau}{x} + \beta x + V_0 + V_{SD}(x). \tag{4}$$

Here, $\tau = k\alpha_s$ is the Coulomb color strength, $\beta = GeV^2$ is the string tension, V_0 is scaling constant, and $V_{SD}(x)$ accounts for the spin-spin, spin-orbit, and tensor effects as [37,41]

$$V_{SD}(x) = V_{SS}(x)(\vec{S}_\rho \cdot \vec{S}_\lambda) + V_{\gamma D}(x)(\vec{\gamma} \cdot \vec{S}) + V_T(x)[S^2 - 3\frac{(\vec{S}\cdot\vec{x})^2}{x^2}]. \tag{5}$$

Where, $V_{SS}(x)$ denotes the spin-spin interaction term, $V_{\gamma D}(x)$ the spin-orbit interaction, and $V_T(x)$ the tensor potential is written as [31,33,38]

$$V_{LS}(x) = \frac{1}{2m_\rho m_\lambda} \left(3 \frac{dV_V}{dx} - \frac{dV_S}{dx} \right), \tag{6}$$

$$V_{SS}(x) = \frac{1}{3m_\rho m_\lambda} \nabla^2 V_V, \tag{7}$$

$$V_T(x) = \frac{1}{6m_\rho m_\lambda} \left(\frac{d^2 V_V}{dx^2} - \frac{1}{x} \frac{dV_V}{dx} \right). \tag{8}$$

Here the linear term can be written as $V_s = \alpha x$ and Coulomb term as $V_V = \frac{\tau}{x}$.

Instead of the six-dimensional delta function in Eq. (7), we use a smeared function [39,40] is given by,

$$V_{SS}(x) = \frac{-A}{6m_\rho m_\lambda} \frac{e^{-x/x_0}}{xx_0^2}. \tag{9}$$

where, $A = \frac{A_0}{(n+\gamma+3/2)^2}$, where A_0 is an arbitrary parameter, and x_0 is the hyperfine parameter.

Finally, the spin averaged mass is obtained as $M = \sum_i m_i + B.E.$, where m_i represents the mass of the constituents [42].

3 Result and discussion

The mass spectrum of the $qqq\bar{c}\bar{s}$ pentaquark system has been systematically investigated within the framework of the Hypercentral Constituent Quark Model by adopting a diquark–diquark–antiquark configuration. In this approach, the $qqq\bar{c}\bar{s}$ system is treated as an effective three-body system composed of two light diquarks (qq and qc) and a strange antiquark (\bar{s}), where the antiquark spin ($S_{\bar{s}} = 1/2$) couples with the spins of the two diquarks, which may exist in scalar ($S_d=0$) or axial-vector ($S_d=1$) configurations. The resulting states are classified as $|S_{d_1}, S_{d_2}, S_{\bar{q}}\rangle$, leading to ground-state pentaquark configurations with negative parity and total spin–parity assignments. Spin-dependent interactions among the constituents generate characteristic mass splittings between different diquark combinations, giving rise to a rich spectrum of low-lying states.

The calculated masses for the $qqq\bar{c}\bar{s}$ pentaquark states are summarized in Table 2 and compared with existing theoretical predictions available in the Ref. [43]. A generally consistent mass spectra is observed, supporting the robustness of the present model. To assess the possible stability and decay behaviour of these states, the predicted masses are examined relative to relevant baryon–meson thresholds, which are evaluated using experimentally measured hadron masses reported by the Particle Data Group [1]. States lying close to or below the lowest accessible thresholds are expected to be weakly bound or narrow resonances, while those above the thresholds are likely to decay strongly into open baryon–meson channels. Consequently, the present analysis provides useful insights into the structure and decay properties of $qqq\bar{c}\bar{s}$ pentaquarks and offers guidance for future experimental searches in the strange–charm pentaquark sector.

The calculated mass spectrum of the $qqq\bar{c}\bar{s}$ pentaquark system exhibits a clear and consistent agreement with the results of Ref. [43] across all possible J^P assignments, demonstrating the compatibility of the two theoretical approaches. For the lowest $J^P = 1/2^-$ states, the present masses of 3.0690 GeV and 2.9726 GeV closely follow the corresponding values (3.0352 ± 0.0201) GeV and (2.9564 ± 0.0196) GeV reported in Ref. [43], indicating that both models describe the ground-state dynamics in a mutually consistent manner. In the $J^P = 3/2^-$ sector, the present predictions of 3.1854 GeV and 3.2323 GeV preserve the same level ordering and lie in reasonable proximity to the Ref. [43] results (3.0524 ± 0.0200) GeV and (3.2124 ± 0.0205) GeV, reflecting a comparable treatment of spin-dependent interactions. For the highest-spin configuration with $J^P = 5/2^-$, the calculated mass of 3.4705 GeV is somewhat higher than the value (3.2814 ± 0.0227) GeV obtained in Ref. [43], which can be understood because of different implementations of spin–orbit and hyperfine contributions rather than a qualitative discrepancy. Overall, the present model reproduces the mass hierarchy and general scale reported in Ref. [43] while providing a complementary description of the $qqq\bar{c}\bar{s}$ pentaquark spectrum, thereby reinforcing the robustness of the theoretical understanding of these states rather than contradicting earlier findings.

Quark rearrangement mechanisms enable the $qqq\bar{c}\bar{s}$ pentaquark system to hadronized into conventional baryon–meson final states while preserving color, flavour, and spin–parity quantum numbers. In one possible rearrangement, the three light quarks (qqq) cluster to form a light baryon, such as a nucleon (N) or a Delta (Δ), whereas the remaining charm quark and strange antiquark ($\bar{c}\bar{s}$) combine to produce a charmed–strange meson, namely \bar{D}_s or \bar{D}_s^* . In an alternative rearrangement scheme, two light quarks together with the charm quark (qqc) form a charmed baryon, such as Λ_c or Σ_c , while the remaining light quark and strange antiquark ($q\bar{s}$) generate a strange meson, such as K or K^* . Consequently, the dominant strong decay channels of the $qqq\bar{c}\bar{s}$ pentaquark are expected to proceed through $N\bar{D}_s$, $N\bar{D}_s^*$, $\Delta\bar{D}_s$, and $\Delta\bar{D}_s^*$ modes, as well as $\Lambda_c\bar{K}$, and $\Sigma_c\bar{K}$ final states, which represent the most natural baryon–meson decay pathways for this system.

Table 2. Calculated masses of $qqqc\bar{s}$ pentaquark states and decay thresholds.

State $ S_{d_1}, S_{d_2}, S_q; J \rangle$	J^P	Present [GeV]	\overline{ND}_s [GeV]	\overline{ND}_s^* [GeV]	$\overline{\Delta D}_s$ [GeV]	$\overline{\Delta D}_s^*$ [GeV]	$\Lambda_c \overline{K}$ [GeV]	$\Sigma_c \overline{K}$ [GeV]	Ref.[43] GeV
$ 0,0,1/2; 1/2 \rangle$	$1/2^-$	3.0690	2.9065	3.0504			2.7801	2.9483	3.0352 ± 0.0201
$ 1,0,1/2; 1/2 \rangle$	$1/2^-$	2.9726	2.9065	3.0504			2.7801	2.9483	2.9564 ± 0.0196
$ 1,0,1/2; 3/2 \rangle$	$3/2^-$	3.1854		3.0504	3.1783			3.0121	3.0524 ± 0.0200
$ 0,1,1/2; 1/2 \rangle$	$1/2^-$	3.1744	2.9065	3.0504			2.7801	2.9483	
$ 0,1,1/2; 3/2 \rangle$	$3/2^-$	3.3655		3.0504	3.1783	3.3222	3.3498	3.3505	
$ 1,1,1/2; 1/2 \rangle$	$1/2^-$	3.0894	2.9065	3.0504			2.7801		
$ 1,1,1/2; 3/2 \rangle$	$3/2^-$	3.2323		3.0504	3.1783				3.2124 ± 0.0205
$ 1,1,1/2; 5/2 \rangle$	$5/2^-$	3.4705				3.3222	3.3753		3.2814 ± 0.0227

For the scalar–scalar diquark configuration, the state lies well above the \overline{ND}_s , $\Lambda_c \overline{K}$, and $\Sigma_c \overline{K}$ thresholds indicating that these strong decay channels are open. In contrast, \overline{ND}_s^* threshold places this state close to the decay threshold, suggesting a suppressed phase space and the possibility of a comparatively narrow, near-threshold resonance dominated by the \overline{ND}_s^* channel. For the scalar–axial diquark configurations, four low-lying states are obtained, each exhibiting distinct decay characteristics depending on their proximity to the relevant baryon–meson thresholds. The $|1,0,1/2; 1/2 \rangle$ state lies

moderately above the \overline{ND}_s and \overline{ND}_s^* thresholds, while remaining relatively close to the $\Sigma_c \overline{K}$ channel, indicating that this mode is likely to play an important role in its strong decay pattern. The $|1,0,1/2; 3/2 \rangle$ state is located very close to the $\overline{\Delta D}_s$ threshold, suggesting a near-threshold behavior with a suppressed decay phase space and the possible formation of a comparatively narrow resonance. In contrast, the $|0,1,1/2; 1/2 \rangle$ configuration is situated well above all considered thresholds, implying that multiple strong decay channels are kinematically allowed and that this state is expected to be relatively broad. Finally, the $|0,1,1/2; 3/2 \rangle$ state lies close to the $\Lambda_c \overline{K}$ and $\Sigma_c \overline{K}$ thresholds, pointing to limited available phase space and indicating that these channels are likely to dominate its decay dynamics and potentially lead to a narrow resonance structure. For the axial–axial diquark configuration, three possible states are obtained with total spin–parity assignments $|1,1,1/2; 1/2 \rangle$, $|1,1,1/2; 3/2 \rangle$, and $|1,1,1/2; 5/2 \rangle$. The $|1,1,1/2; 1/2 \rangle$ state is found to lie well above the \overline{ND}_s and $\Lambda_c \overline{K}$ thresholds, while remaining relatively close to the \overline{ND}_s^* channel, suggesting that this mode may play a significant role in its strong decay and that the state could exhibit moderate width. The $|1,1,1/2; 3/2 \rangle$ configuration lies near the $\overline{\Delta D}_s$ threshold and above the \overline{ND}_s^* channel, indicating a near-threshold behavior in the $\overline{\Delta D}_s$ decay mode with a potentially suppressed phase space. Finally, the highest-spin state $|1,1,1/2; 5/2 \rangle$ is located close to the $\overline{\Delta D}_s^*$ and $\Lambda_c \overline{K}$ thresholds, implying limited available phase space for strong decays and suggesting that this configuration may manifest as a relatively narrow resonance dominated by these channels.

4 Conclusion

The present work provides independent predictions for the mass spectrum of the $qqqc\bar{s}$ pentaquark system within a consistent theoretical framework. The obtained results exhibit a well-defined mass spectrum and a systematic spin–parity structure, reflecting an improved treatment of the underlying dynamics, particularly the spin-dependent interactions. The predicted masses offer a refined description of strange–charmed pentaquark states and serve as a valuable theoretical reference for future studies and experimental searches, thereby advancing the understanding of exotic multiquark spectroscopy. Overall, the threshold analysis indicates that the most promising $qqqc\bar{s}$ pentaquark candidates for experimental observation are those lying close to dominant baryon–meson thresholds, where the available phase space for strong decays is limited. States near the \overline{ND}_s , $\overline{\Delta D}_s$, $\overline{\Delta D}_s^*$ and $\Lambda_c \overline{K}$ thresholds are expected to exhibit suppressed decay widths and may appear as weakly bound or narrow resonant structures. In contrast, configurations located well above all relevant thresholds are likely to be broad due to multiple open decay channels, making them less favorable for detection. Consequently, near-threshold states, especially those with higher total spin or mixed diquark configurations, emerge as the most stable and experimentally accessible candidates in the strange–charmed pentaquark sector.

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