

Mass Estimates of Ω_{bc} Baryons in a Quark–Diquark Rotating String Model

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Abstract. We present a comprehensive study of the mass spectra of Ω_{bc} baryons within a quark–diquark rotating string description. In this framework, the baryon is treated as a bottom–charm diquark core coupled to a strange quark via a relativistically rotating flux tube. Spin–dependent interactions are included to capture finer details of the spectrum. Employing this formulation, we predict the masses of both ground and excited Ω_{bc} states. The ground-state predictions show close consistency with earlier theoretical work, supporting the validity of the model. Furthermore, we provide extensive results for radial and orbital excitations across the spectrum. These theoretical insights offer important guidance for experimental programs aimed at detecting Ω_{bc} baryons, furnishing a timely benchmark for interpreting possible signals.

1 Introduction

The spectroscopy of heavy hadrons with distinct heavy flavours – in particular the bottom–charm Ω_{bc} baryon – provides a crucial window into the nonperturbative dynamics of quantum chromodynamics (QCD). This system contains two heavy quarks (b and c) bound to a light quark, situating them at the interface of heavy–quark physics and light–quark confinement. Precise knowledge of their mass spectra is not only important for understanding hadronic structure, but also for refining theoretical models of the strong interaction in the heavy–quark sector.

Like their doubly–bottom counterparts, the predicted bottom–charm baryons (with a bc diquark core) offer a unique probe of diquark dynamics in QCD. In these systems the two different heavy quarks act almost as static colour sources, albeit with unequal masses, providing a scenario akin to a miniature nucleus bound to a light quark. Studying such configurations can illuminate how a heavy diquark subsystem behaves inside a baryon and how this compares to a single heavy antiquark in a meson, or to a doubly heavy diquark of identical quarks, thereby deepening our understanding of colour confinement for heavy quarks in different configurations. The observation of a bc baryon would thus enrich our knowledge of baryon spectroscopy and improve our understanding of the quark structure inside baryons.

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Over the past two decades, significant experimental progress has been made in heavy-hadron spectroscopy. In the meson sector, the B_c^+ meson (containing a $b\bar{c}$ pair) was first observed at the Fermilab Tevatron in 1998, providing the first example of a bound state with two different heavy flavours. More recently, several excited B_c states have been reported at the LHCb, underscoring that hadrons with both b and c can be produced and studied. In the baryon sector, the major breakthrough was the discovery of the doubly charmed baryon Ξ_{cc}^{++} (with two c quarks) by the LHCb Collaboration in 2017 – the first confirmed baryon containing two heavy quarks [1]. This milestone strongly suggests that the analogous bottom–charm and double-bottom baryons should also exist if produced in sufficient numbers. To date, however, no baryons containing a b and a c quark (nor those with two b quarks) have been observed experimentally.

Their non-observation so far is not surprising – producing a b and c together in the same baryon is rare, and detecting their decays amid large backgrounds is experimentally challenging. Such a finding would provide the first direct handle on baryons with two different heavy quarks and a stringent test of our theoretical understanding.

In fact, dedicated searches have already been undertaken: for example, LHCb has searched for Ξ_{bc}^0 in the $D^0 p K^-$ decay mode but found no significant signal in the expected mass range [2]. Continued experimental efforts at the LHCb and future colliders will depend crucially on theoretical input to guide the selection criteria and signal characterization. It is therefore imperative that theory provides sharp predictions for the masses and quantum levels of Ξ_{bc} and Ω_{bc} , to assist in identifying potential signals against background.

On the theory side, heavy-hadron spectroscopy is developed by a spectrum of phenomenological tools ranging from effective quark-model descriptions to first-principles lattice computations. Relativistic and semi-relativistic quark–antiquark potential models have provided a consistent description of charmonium, bottomonium, and heavy–light meson spectra, including fine and hyperfine splitting [3,4,5,6,7]. Extensions of these approaches within relativistic Dirac formalisms and heavy quark effective theory have enabled systematic treatments of open-flavour charm and bottom mesons [8–10]. Further, analyses based on HQET-guided potential models and variational schemes have further clarified the higher orbital and radial excitations [11–12].

For baryons, three-body constituent quark models and quark–diquark approximations, often formulated within the hyper central framework, have been extensively applied to heavy baryons [13–16]. Further, the Regge phenomenology have also been carried out by Oudichhya et al. to explore the mass spectra of singly, doubly, and triply heavy baryons [17–21]. Phenomenological studies are frequently cross-validated with QCD sum rules, lattice QCD simulations, and bag-model estimates, allowing quantum number assignments and mass predictions to be constrained from multiple perspectives [4,7].

On the doubly heavy baryon side, a variety of approaches – ranging from quark model calculations [22–25] and QCD sum rules [26–27] to lattice QCD studies – have been employed to predict the masses of the ground and excited states. Since no experimental data exist yet for calibration, different models sometimes lead to differing level ordering or splitting predictions – underscoring the need for further theoretical scrutiny. In particular, whether the spectrum exhibits any unexpected features (such as inverted level ordering for certain orbitally excited states due to the intermediate mass of the c quark) is an open question that various models address differently. Continued theoretical studies are essential to firm up these predictions and to provide a robust framework for interpreting future discoveries.

In this context, we pursue a theoretical determination of the Ω_{bc} baryon mass spectra using the relativistic flux-tube model, which has proven successful in the description of other heavy–light mesons and baryons. In our approach, the baryon is treated in a quark–diquark picture within a rotating flux-tube (string) framework, and we derive a mass formula that incorporates both the confining potential and relativistic kinematics. Crucially, we include

spin-dependent interactions (spin–spin, spin–orbit, etc.) in a jj coupling scheme appropriate for heavy–light systems, allowing us to predict the hyperfine and fine-structure splitting around the spin-averaged masses. This methodology has been validated in our previous studies by successfully reproducing the known spectra of singly heavy baryons [28-30] and heavy–light mesons, as well as the doubly charmed baryon and doubly bottom baryon sector [31-32]. In the present work, we apply this model to the bottom–charm baryons, aiming to generate a comprehensive theoretical spectrum for Ω_{bc} .

The paper is organized as follows. In Section 2, we outline the relativistic flux-tube model as applied to bottom–charm baryons and detail the methodology used to calculate their mass spectra, including the treatment of spin-dependent interactions. Section 3 presents our results for the Ω_{bc} spectra and compares them with existing theoretical predictions. Finally, Section 4 offers our conclusion.

2 Theoretical Framework

We consider doubly heavy baryons as systems composed of a heavy diquark and a light quark, bound together by a confining colour field visualized as a straight, rotating flux tube (or string). This flux tube is characterized by a constant string tension, σ , analogous to the energy per unit length of a string. Within this framework, the constituent quarks of the heavy diquark are assumed to remain in their ground state configuration. The combined system of the heavy diquark, the light quark, and the flux tube rotates around its centre of mass.

Within the relativistic flux tube model, a relationship between the mass (\bar{M}) and the angular momentum quantum number (L) of doubly heavy baryons can be expressed as [33, 28]

$$(\bar{M} - m_d)^2 = \frac{\sigma}{2} (L + \lambda n_r) + (m_q + m_d v_d^2)^2, \quad (1)$$

where, $\sigma = 2 \pi T$. The dynamical masses for the rotating heavy diquark and the light quark are given by $m_d = m_{0d}/\sqrt{1 - v_d^2}$ and $m_q = m_{0q}/\sqrt{1 - v_q^2}$, respectively. Given the ultra-relativistic nature of the light quark rotation, we adopt the assumption $v_q \approx 1$, while v_d denotes the speed of the diquark. The parameter λ is a model-dependent quantity determined by fitting to experimental data, and $n_r = n - 1$, where n is the principal quantum number for the baryonic state. Notably, λ is treated as universal across a particular class of hadrons and can be reliably extracted from known spectral data.

The separation between the heavy and light components is given by [33, 28]

$$r = (v_q + v_d) \sqrt{\frac{8(L + \lambda n_r)}{\sigma}}. \quad (2)$$

Up to this point, the flux-tube model neglects spin interactions. The mass \bar{M} derived corresponds to a spin-averaged mass. To obtain realistic physical masses, we introduce perturbative spin-dependent interactions, including standard spin–orbit, spin–spin, and tensor interaction terms given explicitly by [34]:

$$H_{so} = \left[\left(\frac{2\alpha}{3r^3} - \frac{b_0}{2r} \right) \frac{1}{m_q^2} + \frac{4\alpha}{3r^3} \frac{1}{m_d m_q} \right] L \cdot S_q + \left[\left(\frac{2\alpha}{3r^3} - \frac{b_0}{2r} \right) \frac{1}{m_d^2} + \frac{4\alpha}{3r^3} \frac{1}{m_d m_q} \right] L \cdot S_d \quad (3)$$

$$H_t = \frac{4\alpha}{3r^3 m_d m_q} \left[\frac{3(S_q \cdot r)(S_d \cdot r)}{r^2} - S_q \cdot S_d \right] = \frac{4\alpha}{3r^3 m_d m_q} \hat{B}, \quad (4)$$

$$H_{ss} = \frac{32\pi\alpha\sigma_0^3}{9\sqrt{\pi}m_d m_q} S_q \cdot S_d, \quad (5)$$

where α denotes the strong coupling constant, b_0 characterizes the long-range confinement potential, and σ_0 represents the width parameter associated with spin–spin interactions.

We have evaluated expectation values for the operators $L \cdot S_q$, $L \cdot S_d$, $S_q \cdot S_d$, and \hat{B} within the $|J, j\rangle$ basis suited to heavy–light hadrons. In the case of doubly charmed baryons, adopting a quark–diquark picture, one typically couples the spin of the light quark S_q with the orbital momentum L to form an intermediate angular momentum $j = L + S_q$. This intermediate j then couples with the heavy quark spin S_h to yield the total angular momentum J . The Ω_{bc} baryons possess a spin-one (vector) diquark configuration bc .

We assume an empirical power-law scaling relationship that connects the string tension to the mass of the diquark. This scaling is expressed through the following equations [35, 31, 32]:

$$\frac{\sigma_{B_s}}{\sigma_{\Omega_{cb}}} = \left(\frac{m_b}{m_d}\right)^Q. \quad (6)$$

The values of model parameters employed here were obtained previously by analyzing heavy-light meson spectra. Our earlier study [31, 32] yielded best-fit values with corresponding statistical uncertainties, specifically: the dynamical mass of the bottom quark as $m_b = 4431.08 \pm 0.16$ MeV and the strange quark mass $m_s = 484.29 \pm 0.06$ MeV. Additional fitted parameters include $\lambda = 1.479 \pm 0.015$, the confining interaction strength $b_0 = (36.2 \pm 1.2) \times 10^3$ MeV², the width parameter $\sigma_0 = 502.8 \pm 2.8$ MeV, and the string tension of B_s meson: $\sigma_{B_s} = (1.9931 \pm 0.0006) \times 10^6$ MeV², along with the scaling exponent $Q = 0.0927 \pm 0.0017$.

Given the limited experimental data currently available for doubly heavy baryons—except for the confirmed observation of the Ξ_{cc}^{++} —it is necessary to incorporate theoretical mass predictions for the $1S$ -wave states to constrain the model parameters further. A significant theoretical estimate by Ebert et al. [22] placed the ground-state mass of the Ξ_{cc} baryon at 3620 MeV, remarkably close to the subsequent experimental measurement by the LHCb Collaboration, which reported a mass of 3621.55 MeV [36]. Inspired by this successful prediction, we have utilized theoretical mass values from Ref. [22] for the $1S$ -states. This allows us to estimate the dynamical diquark mass as $m_d = m_{\{bc\}} = 5833.61$ MeV and to determine the strong coupling constant as $\alpha = 0.2867$.

By substituting this diquark mass into the scaling relations Eq. (6), we derive the string tension as $\sigma_{\Omega_{bc}} = (2.044 \pm 0.005) \times 10^6$ MeV². The intrinsic mass of the heavy diquark in the $1S$ state is approximated simply by summing the individual bottom and charm quark masses, giving $m_{0d} = m_{0b} + m_{0c} = 5456 \pm 8$ MeV [37]. Finally, the diquark speed is

computed from the relation $v_d = \sqrt{1 - (m_{0d}/m_d)^2}$, yielding a value of 0.354 ± 0.004 .

3 Results and Discussion

In this section, we present and discuss the theoretical results obtained for the mass spectra for the Ω_{bc} baryon. The calculated masses are listed in Table I. We denote baryon states by $nL(J^P, j)$, where $n = 1, 2, 3, \dots$ is the radial quantum number of the lighter component, $L = S, P, D, \dots$ denotes its orbital angular momentum, followed by the state's total angular momentum J , parity P , and the angular momentum of the light quark j . The calculated masses are systematically compared predictions from other theoretical studies [22, 38]. The first radial excitations in the $2S$ sector is found around 7.56 GeV, while the lowest P -wave states

are concentrated in the mass window 7.37–7.43 GeV. Broadly, our predictions for the ground-state masses fall within the range of existing theoretical estimates.

Table I. Masses of Ω_{bc} baryons (in MeV).

States $nL(J^P, j)$	This work	Ref. [22]	Ref. [38]
1S (1/2 ⁺ , 1/2)	7023±17	7088	7060
1S (3/2 ⁺ , 1/2)	7062±17	7130	7094
2S (1/2 ⁺ , 1/2)	7561±13	-	7480
2S (3/2 ⁺ , 1/2)	7563±12	-	7497
1P (1/2 ⁻ , 1/2)	7376±13	-	7266
1P (1/2 ⁻ , 3/2)	7412±13	-	7296
1P (3/2 ⁻ , 1/2)	7403±13	-	7304
1P (3/2 ⁻ , 3/2)	7420±13	-	7326
1P (5/2 ⁻ , 3/2)	7433±12	-	7361
1D (1/2 ⁺ , 3/2)	7705±11	-	-
1D (3/2 ⁺ , 3/2)	7711±11	-	7520
1D (3/2 ⁺ , 5/2)	7701±11	-	7619
1D (5/2 ⁺ , 3/2)	7720±11	-	7541
1D (5/2 ⁺ , 5/2)	7705±11	-	7501
1D (7/2 ⁺ , 5/2)	7712±11	-	7619

Our mass predictions provide useful guidance for ongoing experimental searches. It can inform selection cuts and reconstruction strategies in future analyses at LHCb or other facilities. Ultimately, the discovery of a bottom–charm baryon (or an excited state thereof) would not only fill in a missing piece of the doubly heavy baryon puzzle but also provide a critical test of the theoretical approaches – including the flux-tube model used here – that underlie our understanding of QCD in the two-heavy-quark regime. The modest differences between models underscore the importance of such experimental input: even a single observed mass can help discriminate among predictions and calibrate the dynamics of the bc diquark system.

4 Conclusion

In this study, we have calculated the mass spectra of the doubly heavy Ω_{bc} baryon using the relativistic flux-tube model. Within our framework, these baryons are treated as bound systems consisting of a heavy bc diquark connected by a confining gluonic flux tube to a light quark, with spin-dependent corrections included perturbatively in a jj coupling scheme. Our computed ground-state and excited-state masses align closely with existing theoretical estimates. The predictions provided here, especially for low-lying states, offer essential benchmarks to guide experimental searches at current and future facilities like LHCb, where observation of even one Ω_{bc} resonance would significantly advance our understanding of heavy-diquark dynamics and confinement in QCD.

Acknowledgment

Ms. Pooja Jakhad gratefully acknowledges the financial support provided by the Council of Scientific and Industrial Research (CSIR) through the JRF Fellowship program, under file number 09/1007(13321)/2022-EMR-I.

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