

Mechanical Stabilization of Magneto-Gravitational Instability in Rotating Viscoelastic Media through a Second Order Generalized Hydrodynamic Framework

Rajni Sharma^{1*}, Syed Mohd. Arif², Shalom Akhai³, and Sumit Bhardwaj⁴

¹Department of Mathematics, University Institute of Sciences, Chandigarh University, Mohali, Punjab, India

²Department of Civil Engineering, School of Engineering and Technology, CGC University, Jhanjeri, Mohali, Punjab, India

³Department of Mechanical Engineering, Maharishi Markandeshwar Engineering College, Maharishi Markandeshwar (Deemed to be University), Mullana, Ambala, India

⁴Department of Physics, University Institute of Sciences, Chandigarh University, Mohali, Punjab, India

Abstract. A theoretical study of magneto-gravitational instability of a rotating, finitely conducting viscoelastic substance is presented in this paper by a second-order generalized hydrodynamic (GH) system. The given model integrates the effect of the viscoelastic relaxation time, shear viscosity, finite electrical resistivity, constant magnetic field, self-gravity and Coriolis forces because of rotation. Normal-mode analysis is used to derive and analyze linearized perturbation equations of both strongly coupled plasma (SCP) and weakly coupled plasma (WCP) regimes. The generalized complex dispersion relation is acquired and analysed on the basis of the Hurwitz stability criteria and dimensionless parameters. These findings reveal that rotation and finite resistivity have no effect on the modified threshold of the Jeans instability but greatly slow down the growth rate of unstable modes. Rotational forces make the system stable in the presence of Coriolis forces and the viscoelastic relaxation adds some elastic memory that slows the gravitational collapse. Higher viscosity and resistivity also inhibit more the short wavelength perturbations. The relative comparison shows that SCP media have lower growth rates compared to WCP regimes.

1 Introduction

The gravitational instability (GI) is a situation in which the force of self-gravitation in a given medium is greater than the repelling force of pressure, rigidity or other stabilizing forces and the medium collapses and structures are formed. Sir James Jeans was the first to formulate the classical theory of gravitational instability, and he studied the stability of a homogeneous, infinite, and at rest medium and showed that the perturbations increase as

* Corresponding author: rajni.math22@gmail.com

the wavenumber decreases. The wavenumber of k is less than a critical number. That condition, often known as the Jeans instability, has been used since as an important paradigm to explain the star formation, molecular cloud collapse, and large-scale structure formation of astrophysical systems [1-4].

The ubiquity and dynamical importance of rotation is observed in observational studies of star-forming molecular cloud cores. Rotational movement of dense cores was common, and was reported by Goodman et al. [5] and Burkert and Bodenheimer [6], and is commonly believed to arise through turbulent processes in molecular clouds. Larson [7] went on to point out that the angular momentum of the cores well in advance of the star is much larger than that which can be stored in an individual star, suggesting that redistribution or loss of angular momentum during gravitational collapse is necessary. These results point out the relevance of the rotational dynamics in altering the behavior of gravitational instability.

Rotating self-gravitating systems have been widely studied in both analytical and numerical works, as to their stability. The studies of rotating disks by Miyama et al. [8], Larson [9] and Binney and Tremaine [10] all showed that rotation tends to have a stabilizing effect, slowing the growth rates of unstable modes and decreasing the range of unstable wavelengths. In the case of rigid rotation, as well as axisymmetric perturbations of short wavelengths, the most unstable wavelength is near to twice the Jeans length, so rotation does not change the very onset of instability but only changes the time dependence of instability.

Stabilizing effect Magnetic fields are also important in gravitational instability. The early works by Chandrasekhar and Fermi [11], Pacholczyk [12], Stodolkiewicz [13], Nakano and Nakamura [14], and Nakamura et al. [15] demonstrated that the magnetic tension tends to suppress the short wavelength perturbation of magnetized sheets, filaments, and disks. In linear theory, it was shown by Nakamura [16] that the stabilizing effects of rotation and magnetic fields are quite additive. Chandrasekhar [17] also discovered that both uniform rotation and uniform magnetic fields do not cause any change to the classical Jeans criterion in a homogeneous medium (however, both do have a large effect on unstable mode growth rates).

Strongly coupled plasmas (SCP) have also garnered great interest due to their application in small-scale astrophysical environments and planetary interiors, including white dwarf stars, the crusts of neutron stars, and the cores of giant planets, and in laboratory plasmas generated by laser compression [18]. The systems are usually described in terms of viscoelasticity that has both viscous dissipation and elastic memory properties. Rosenberg and Shukla [18] demonstrated that such media cannot be properly described by conventional hydrodynamic models only. Rather, generalized hydrodynamic (GH) framework offers an appropriate description with the inclusion of viscoelastic relaxation effects, which were initially described by Kaw and Sen [23] and Frenkel [24].

Later researchers have used the GH model to viscoelastic media gravitational instability problems. Dhiman and Sharma [19] observed the effects of rotation on magneto-gravitational instability in a viscoelastic material and discovered that the growth rate is changed by rotation without changing the threshold of instability. The extensions to non-uniform geometries of cylindrical shape and with non-uniform magnetic fields were reported in [20,21], and the influence of the dust temperature and the radiative heat-loss processes were studied in [22]. All these studies indicate that viscoelasticity provides elastic memory able to defer collapse and alter instability dynamics.

Similar physical and engineering systems have also been analyzed using optimization based methods like the Taguchi method at this stage in order to examine parametric sensitivity and stability improvement in a systematic way [26–28].

Following the multiple significance of rotation, magnetic fields and viscoelastic in astrophysical and laboratory plasmas, the current paper explores the magneto-gravitational

instability in a rotating, finitely conducting viscoelastic fluid with a second-order generalized hydrodynamic model. It will be analyzed how the viscosity, viscoelastic relaxation and resistivity and Coriolis forces collectively contribute to the nature of the instability threshold and growth-rate nature in both the strongly and weakly coupled plasma regimes.

2 Mathematical Modelling and Stability Analysis (Second-Order Formulation)

The above-described physical problem is mathematically formulated as following; The dynamical behavior of a rotating, finitely conducting viscoelastic continuum influenced by uniform magnetic field $\vec{H}_0 = (0, 0, H_z)$ and self-gravitational fields can be represented by the generalized hydrodynamic (GH) framework. This approach couples classical conservation laws with a viscoelastic relaxation operator to represent both the elastic memory and dissipative response of the medium. An appropriate extension of the GH equation into the nonlinear regime has been proposed Kaw and Sen (1998) and Frenkel (1946) by using the convective derivative relation $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ [20, 21]. The convective derivative is responsible for obtaining a nonlinear GH equation (Janaki and Chakrabarti (2010) Kaw and Sen (1998)) [20, 22]. The governing set of equations is expressed as follows:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) &= 0 & (1) \\ \left(1 + \tau \frac{\partial}{\partial t}\right) \left[\rho \frac{D\mathbf{u}}{Dt} + 2\rho (\boldsymbol{\Omega} \times \mathbf{u}) \right] &= -\nabla p + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \rho \nabla \phi + \mu \nabla^2 \mathbf{u} + \left(\xi + \frac{\mu}{3}\right) \nabla (\nabla \cdot \mathbf{u}) & (2) \\ \frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} & (3) \\ \nabla^2 \phi &= 4\pi G \rho & (4) \end{aligned}$$

To study the effect of infinitesimal small disturbances, all physical quantities are decomposed into steady-state and perturbation components as:

$$\begin{aligned} \rho &= \rho_0 + \rho', \\ \mathbf{u} &= \mathbf{0} + \mathbf{u}', \\ \mathbf{B} &= \mathbf{B}_0 + \mathbf{B}', \\ \phi &= \phi_0 + \phi'. \end{aligned} \tag{5}$$

The equilibrium configuration is assumed to be homogeneous and static with uniform background magnetic field $\mathbf{B}_0 = (0, 0, B_0)$ and uniform rotation $\boldsymbol{\Omega} = (0, 0, \Omega_z)$. Substituting equation (5) into the set (1)–(4) and neglecting higher-order perturbation terms yields the linearized system:

$$\begin{aligned} \frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{u}' &= 0 & (6) \\ \rho_0 \left(1 + \tau \frac{\partial}{\partial t}\right) \frac{\partial \mathbf{u}'}{\partial t} + 2\rho_0 \left(1 + \tau \frac{\partial}{\partial t}\right) (\boldsymbol{\Omega} \times \mathbf{u}') &= \\ -\nabla p' + \frac{1}{\mu_0} (\nabla \times \mathbf{B}') \times \mathbf{B}_0 + \rho' \nabla \phi_0 + \rho_0 \nabla \phi' + \mu \nabla^2 \mathbf{u}' + \left(\xi + \frac{\mu}{3}\right) \nabla \left(\xi + \frac{\mu}{3}\right) \nabla (\nabla \cdot \mathbf{u}') & (7) \\ \frac{\partial \mathbf{B}'}{\partial t} &= \nabla \times (\mathbf{u}' \times \mathbf{B}_0) + \eta \nabla^2 \mathbf{B}' & (8) \\ \nabla^2 \phi' &= 4\pi G \rho'. & (9) \end{aligned}$$

Assuming plane-wave perturbations of the form $\exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$, and eliminating the dependent variables ρ' , \mathbf{B}' , and ϕ' , we obtain the linearized second-order equation governing the motion of the medium as

$$[\omega^2(1 + i\omega\tau) - (c_s^2 + V_A^2)k^2 + 4\pi G\rho_0 + i\omega(\nu k^2 + 2\Omega_z)]\mathbf{u}' = 0, \quad (10)$$

where

$c_s = \sqrt{\frac{\partial p}{\partial \rho}}$ is the isothermal sound speed,

$V_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$ the Alfvén velocity,

$\nu = \mu/\rho_0$ the kinematic viscosity, and
 τ the relaxation parameter.

Equation (10) is the **second-order complex dispersion relation** incorporating the effects of magnetic field, viscoelasticity, relaxation parameter and Coriolis effects.

2.1 Second-Order Dispersion Relation and Dimensionless Form

Introducing the following dimensionless parameters:

$$\gamma = \frac{\omega}{\omega_J}, \kappa = \frac{k}{k_J}, \omega_J = \sqrt{4\pi G\rho_0}, V_J = \sqrt{\frac{G\rho_0}{k_J^2}},$$

and normalizing all physical quantities with respect to the Jeans frequency ω_J , equation (10) can be transformed into

$$\gamma^2(1 + i\gamma\tau^*) - \kappa^2(c_s^{*2} + V_A^{*2}) + 1 + i\gamma(\nu^*\kappa^2 + 2\Omega^*) = 0, \quad (11)$$

where starred quantities denote non-dimensional counterparts.

Expanding equation (11) and separating real and imaginary components, one obtains

$$\begin{cases} (\gamma_r^2 - \gamma_i^2 - \kappa^2 C^* + 1) - \gamma_i(\nu^*\kappa^2 + 2\Omega^*) = 0, \\ 2\gamma_r\gamma_i + \gamma_r(\nu^*\kappa^2 + 2\Omega^*) + \gamma_r^3\tau^* = 0, \end{cases} \quad (12)$$

where $\gamma = \gamma_r + i\gamma_i$ and $C^* = (c_s^{*2} + V_A^{*2})$.

The growth rate of instability corresponds to the real positive value of γ_i . The onset of instability occurs when the real part of γ^2 becomes negative, yielding the modified Jeans criterion:

$$4\pi G\rho_0 > k^2(c_s^2 + V_A^2). \quad (13)$$

Hence, the critical wave number (or critical wavelength) is given by

$$k_c = \sqrt{\frac{4\pi G\rho_0}{c_s^2 + V_A^2}}. \quad (14)$$

2.1.1 Hurwitz Stability Criteria

To ensure the stability of the system described by equation (11), the Hurwitz conditions for a real polynomial are applied. Expressing the dispersion relation as a real fifth-order polynomial in γ ,

$$a_5\gamma^5 + a_4\gamma^4 + a_3\gamma^3 + a_2\gamma^2 + a_1\gamma + a_0 = 0, \quad (15)$$

the necessary and sufficient conditions for all roots to have negative real parts are $a_i > 0$ and all Hurwitz determinants positive. If any $a_i < 0$, the system admits at least one exponentially growing mode. The constant term a_0 in equation (15) yields the instability condition:

$$\omega_j^2 > (c_s^2 + V_A^2)k^2, \tag{16}$$

which is identical to the modified Jeans criterion (13). This confirms that rotation and finite electrical resistivity influence only the growth rate, not the onset of instability.

2.1.2 Dimensionless Growth-Rate Expression

In order to find the effect of various parameter on the growth rate of instability, the normalized growth rate $\Gamma = \text{Im}(\gamma)$ can be expressed from equation (11) as:

$$\Gamma = \frac{-[v^* \kappa^2 + 2\Omega^*] \pm \sqrt{(v^* \kappa^2 + 2\Omega^*)^2 - 4[(1 - \kappa^2 C^*) + \gamma_r^2 \tau^*]}}{2(1 + \tau^*)}. \tag{17}$$

The condition $\Gamma > 0$ corresponds to exponential amplification of perturbations. Numerical analysis of equation (17) typically shows that as the values of Ω^* (rotation) increasing, growth rate decreases which leads to the stabilizing effect. Whereas the increasing value of the τ^* (relaxation) delay the collapse (elastic memory). Further increasing v^* (viscosity) or η^* (resistivity) leads to damping of small-scale disturbances.

3 Graphical Interpretation and Discussion of Growth Rate

This section interprets the variation of the normalized growth rate (Γ) with the normalized wave number (κ) for different physical parameters derived from the non-dimensional dispersion relation. The influence of rotation, viscosity, resistivity, and viscoelastic coupling is analyzed for both the strongly coupled plasma (SCP) and weakly coupled plasma (WCP) regimes. All figures are plotted using the dimensionless parameters defined in Section 3, normalized with respect to the Jeans frequency (ω_j).

3.1 Effect of Rotation

Figure 1 depicts the variation of the normalized growth rate with normalized wave number for different rotational frequencies $\Omega^* = 0.1, 0.2, 0.3$ under both kinetic (SCP) and hydrodynamic (WCP) limits. The growth rate decreases progressively with increasing Ω^* , and the unstable wavelength band becomes narrower. The Coriolis force introduced by rotation reduces the effective gravitational acceleration, thereby retarding the collapse of perturbations.

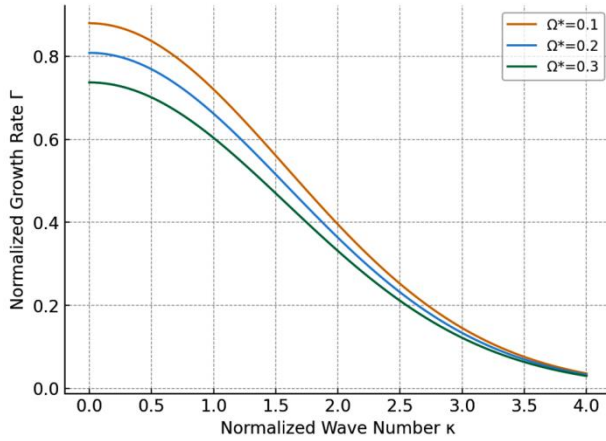


Fig. 1. Effect of Rotation on Growth Rate (Transverse Mode).

3.2 Influence of Shear Viscosity

The effect of shear viscosity on instability is presented in Figure 2 for $\xi^* = 0.0$ and 0.6 at rotational frequencies $\Omega^* = 0.0$ and 0.3 . The results indicate that an increase in viscosity lowers the amplitude of the growth rate throughout the entire wavenumber spectrum. In the absence of rotation, growth remains significant at small κ ; however, when rotation is included, viscous damping becomes dominant and the system stabilizes at shorter wavelengths.

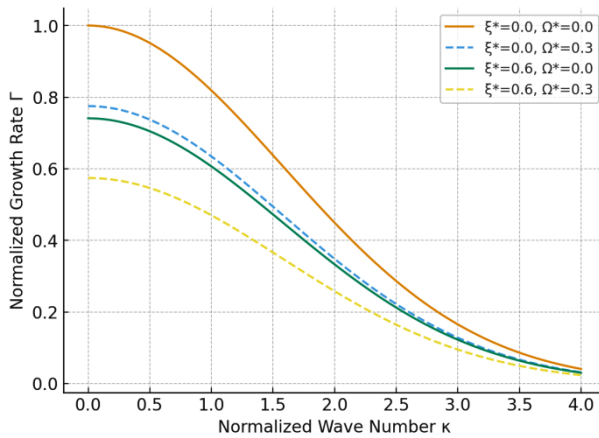


Fig. 2. Influence of Shear Viscosity on Growth Rate.

3.3 Combined Effect of Resistivity and Rotation

Figure 3 illustrates the combined influence of electrical resistivity and rotation. For $\eta^* = 1.0$, resistivity weakens magnetic coupling, slightly enhancing the growth rate in non-rotating conditions. When rotation ($\Omega^* = 0.3$) is introduced, however, a competing stabilizing influence arises. The joint presence of finite resistivity and rotation produces an overall reduction in Γ , indicating that resistive diffusion cannot overcome the Coriolis stabilization.

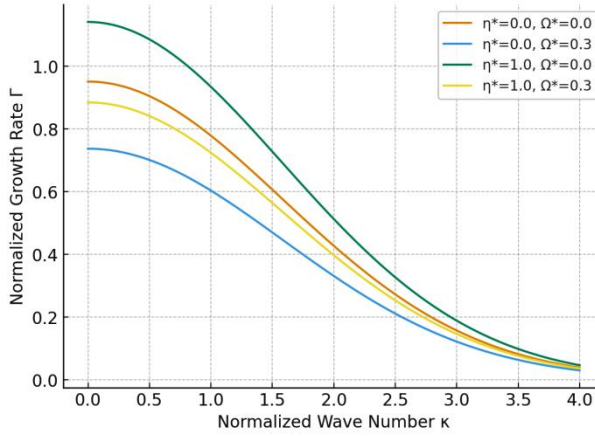


Fig. 3. Combined Effect of Resistivity and Rotation.

3.4 Comparison between SCP and WCP Regimes

Figure 4 compares the growth-rate profiles of SCP ($\sigma\tau \gg 1$) and WCP ($\sigma\tau \ll 1$) media for $\Omega^* = 0.0$ and 0.3 . The WCP regime consistently exhibits higher growth rates, reflecting its weaker elastic memory and stronger hydrodynamic response. The SCP case, dominated by viscoelastic restoring forces, shows a slower rate of amplification and earlier stabilization. Increasing Ω^* further suppresses growth in both regimes, confirming that rotational stabilization operates independently of the coupling state.

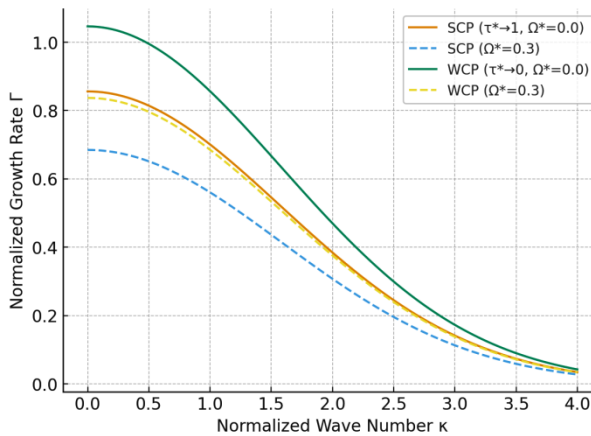


Fig. 4. Comparison of SCP and WCP Growth Rates.

Growth-rate comparison between strongly coupled (SCP) and weakly coupled (WCP) media under $\Omega^* = 0.0$ and 0.3 . Viscoelastic elasticity in SCP suppresses perturbation growth; rotation decreases the instability range in both cases.

4 Conclusions

A generalized hydrodynamic type of description has been established in form of a second-order generalized, to examine magneto-gravitational instability in a rotating, finitely

conducting viscoelastic fluid. The formulation manages to take into account the joint action of self-gravity, uniform magnetic field, rotation, viscoelastic relaxation, viscosity, and finite electrical resistivity and offers a single description of both strongly coupled plasma (SCP) and weakly coupled plasma (WCP) regimes. The obtained dispersion relation and stability analysis indicate that the gravitational instability onset is determined by a modified Jeans criterion, which is not dependent on rotation and resistivity. Nevertheless, these parameters have a decisive role in regulating the time dependent development of instability, as they greatly decrease the rate of growth of unstable modes. Coriolis-induced rotation effects induce stabilization which in turn reduces the destabilizing wavelength band, and magnetic tension and resistive diffusion further suppress the short-scale perturbations. The viscoelastic relaxation time adds the elastic memory to the system and postpones gravitational collapse and increases dynamic stability. Relative comparison reveals that SCP media have lower growth rates when compared to WCP media, because they have a higher viscoelastic coupling and elastic restoring forces. In general, a combination of rotation, magnetic diffusion, viscosity and viscoelasticity converts the classical Jeans instability into a damped and scale dependent one. The findings are of good use in studying the stability of rotating astrophysical plasmas, magnetized interstellar structures, dusty plasmas, and viscoelastic planetary interiors.

5 Future Scope

The current analysis can be expanded in some significant ways to make the model more realistic and applicable physically. First, the homogeneous magnetic field and uniform rotation can be replaced to study the impact of different rotation and homogenous magnetic field, which are closer to astrophysical conditions like accreting disks and molecular clouds. Secondly, to study the thermo-magneto-gravitational instability in viscoelastic media the existing second-order generalized hydrodynamic formulation can be intertwined with thermal effects, such as temperature-dependent transport coefficients, radiative heat-loss processes, and thermal conduction.

The proposed extensions can be made with anisotropic viscosity, Hall currents, and variability in dust charge to represent multi-component plasmas more realistically. Numbers of studies on the later phase development and saturation of unstable modes beyond the linear regime could be undertaken through nonlinear stability analysis and numerical simulations. Moreover, optimization-based methods like the Taguchi method and other design-of-experiments methods can also be applied to determine in a systematic way the superior control parameters and the most optimal stability conditions in sensitive parameter spaces. These generalizations would make the current scheme more relevant to rotating astrophysical plasmas, laboratory dusty plasma experiments, and highly coupled viscoelastic fluids.

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