

Quantum-enhanced deep belief networks for financial fraud detection

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Abstract. A Deep Belief Network (DBN) is a generative model stacking multiple Restricted Boltzmann Machine (RBM) layers to learn hierarchical data representations. While effective for feature extraction, classical DBNs struggle with high-order patterns in complex, imbalanced datasets, such as credit card fraud data. To overcome this, we integrate quantum-inspired RBMs (QRBMs) into the DBN framework.

This study compares four 3-layer DBN configurations on the Credit Card Fraud Detection dataset: (i) classical DBN (all RBM layers), (ii) 1-Quantum DBN (1 QRBM layer), (iii) 2-Quantum DBN (2 QRBM layers), and (iv) full Quantum DBN (all QRBM layers). Models were trained via contrastive divergence and assessed using precision, recall, and F1-score.

Results show the full Quantum DBN outperforming others: precision 0.581, recall 0.637, F1-score 0.602—yielding a 34.4% F1 improvement over classical DBN (precision 0.319, recall 0.755, F1 0.448). Hybrids ranked intermediately. Quantum advantages stem from entanglement and superposition, fostering complex pattern capture and faster convergence (fewer epochs).

These findings highlight quantum-enhanced DBNs' potential for scalable anomaly detection in financial fraud systems, paving the way for hybrid quantum-classical ML applications.

1 Introduction

The digital financial transaction volumes have increased significantly over time, and so have the complexities associated with these transactions. This increased volume and complexity of transactions in the digital finance ecosystem has made it much more challenging to detect fraudulent financial transactions than ever before. Fraudulent transactions are often hidden among large volumes of valid transactions, resulting in imbalanced datasets that cause the subtle patterns of fraud to go unnoticed.

Traditional ML methods, such as logistic regression or decision trees, rely on the use of manually created features as input into their ML models to train these models. While some of these methods can work, they are typically unable to adapt to complex or dynamic fraud patterns, due in large part to their inability to identify and model the highly complex relationships embedded in a large amount of financial information.

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Deep learning, and in particular deep belief networks (DBNs), is a compelling alternative that can identify complex and subtle patterns of fraud without requiring the extensive manual feature engineering that traditional ML methods require. DBNs are created by stacking multiple restricted Boltzmann machines (RBMs). Each RBM is a generative model consisting of two sets of nodes, visible nodes and hidden nodes. Each RBM uses contrastive divergence (CD) to train and learn to reconstruct input data and to extract features from that data on its own. Despite their potential for detecting patterns of fraud, DBNs trained with classical RBMs often experience slow convergence to a solution or may become trapped in local minima, making the training of DBNs with classical RBMs suboptimal for fraud detection.

Recent advances in quantum computing have introduced new ways to solve these challenges using quantum computing methods, which model and process information with fundamentally different characteristics due to the use of unique properties of Quantum Mechanics. In quantum computing, bits (qubits) that are processed in these calculations can exist in many states at the same time, and the entangled qubits can encode correlations in ways that a classical computer cannot. The unique properties associated with quantum computers, such as increased dimensionality and increased correlation-based encoding, enable these features to significantly expand the solution space and allow the models to much better capture and represent the interactions between complex features.

The classic Restricted Boltzmann Machine (RBM) uses binary units to encode the visible and hidden nodes. However, when using qubits as the encoding units, it is called a Quantum Restricted Boltzmann Machine (QRBM). In a QRBM, the interactions between the nodes are not subject to classical RBM's physics, but quantum Hamiltonians govern the interactions between the nodes. Thus, QRBM's can encode more complex relationships and learn much faster as a result. When you stack multiple layers of QRBM's, they combine to create a hybrid model, known as a Quantum Deep Belief Network (QDBN), that leverages the advantages of both quantum and classical computational models.

In this paper, we will conduct a comparative study of the classical DBNs and their quantum enhanced variants for the purpose of detecting fraudulent transactions. Specifically, we will compare the four different configurations of the three-layered DBNs:

1. **Classical DBN:** All layers are implemented as classical RBMs.
2. **1-Quantum DBN:** One QRBM layer is positioned at varying depths within the network.
3. **2-Quantum DBN:** Two QRBM layers are used in all possible combinations of layer positions.
4. **Full Quantum DBN:** All layers are implemented as QRBM's.

This method seeks to assess how adding quantum elements to deep learning architectures can enhance fraud detection in actual financial systems in terms of speed, effectiveness, and dependability.

2 Background: Deep Belief Networks and Quantum Computing

2.1 Deep Belief Networks

DBNs (Deep Belief Networks) are un-supervised learning architectures comprised of multiple layers of Restricted Boltzmann Machines (RBMs), each layer composed of its own RBM is considered a module within the DBN architecture and learns to extract features from input

data. By using a hidden activity output from module i as the module $i+1$ input, a DBN can build increasingly high level representations.

DBN architectures are well suited for automatically detecting fraud due to their ability to model highly complex, subtle relationships in financial data that may have many dimensions.

2.2 Restricted Boltzmann Machines

RBM is a generative neural network defined by the existence of a visible (input) layer and a hidden layer (model) structure. The weights and biases between the layers are adjusted to replicate the probability distribution of the training data. Contrastive Divergence (CD) is used to train the RBMs, which gradually adjust the weights by comparing the hidden activities from an observed sample with the hidden activities from a reconstructed sample.

RBMs can be used to pre-train the levels of other, deeper learning networks (i.e. DBNs), when they can be used to extract features from an input dataset, or to reconstruct corrupted data into a denoised form.

2.3 Quantum Computing Principles

By employing quantum mechanical principles - particularly superposition and entanglement - quantum computing provides enhanced computational capabilities of algorithms compared to classical methods [1, 2]. Qubits represent multiple values simultaneously compared to classical bits, thus creating a greater number of possible machine learning model outputs (greater feature representational capacity than an equivalent number of classical bits) [3]. Entanglement provides an essential way of creating richer feature correlations and interactions than can be modeled with traditional architectures, since entangled qubits share their values with one another in a non-classical manner [4].

2.4 Quantum-Enhanced RBMs and DBNs

A classical restricted Boltzmann machine (RBM) has been modified to create a quantum restricted Boltzmann machine (QRBM) [5]. The methods used for controlling the interactions between visible and hidden nodes differ between these two architectures. In the case of a QRBM, the interactions between visible and hidden nodes are controlled by a quantum Hamiltonian, which allows both types of nodes (visible and hidden) to be implemented as qubits [6]. By using this configuration in QRBM, convergence is sped up, leading to improved exploration of complex feature relationships by the network. Applications such as financial fraud detection may derive significant benefits from this approach; QRBM layers can be combined into a quantum deep belief network (QDBN) stack to increase overall speed, strength, and precision of the method, particularly when combined with large, complex datasets that are highly imbalanced [7].

3 Literature Survey

Research regarding the detection of credit card and transaction fraud is being developed across a range of frameworks from traditional statistical techniques to classical supervised machine learning approaches to deep learning techniques as well as to newer quantum inspired techniques.

Initial research shows that supervised classifiers and ensemble methods provide good performance for detecting fraudulent activities; however, continuous class imbalance has led

to using generative and deep representation learning approaches in addition to these other approaches. Some examples of this work are Bhattacharya, Rajan Dey [8] demonstrated that RBMs and DBNs, trained as unsupervised feature learners, are able to discover hidden patterns which enhance the recall of the minority class after transferring learning. Furthermore, Wu, Li, and Lloyd [5] proposed a theoretical framework for implementing Quantum Restricted Boltzmann Machines (QRBMs): based on the rotationally invariant quantum computations of RY rotations, the use of entangled CZ gates and the measurements of Pauli-Z. While there have been significant advances in CBDMs and QRBMs over recent years, the roles played by these two styles of learning have provided significant new avenues to consider when designing novel hybrid architectures that use both CBDM and QRBM systems.

Hybrid quantum-deep learning systems for financial frauds and risk factors have been analyzed as part of this hybridization project, with a focus on feature mapping and sampling improvements. Sharma and Patel [4] surveyed quantum-deep learning hybrids for financial risk and fraud, highlighting feature mapping and sampling improvements. Zhang and Tan [9] architectures show empirical improvements in anomaly-sensitive metrics with small and mid-sized data sets have been reported. Several conferences and technical reports have studied the variational quantum layer addition to classical networks (hybrid QDBNs), with the goal of improving representation and convergence. Survey articles like Gupta and Rao [2] provide overviews of quantum machine learning in applied finance, acknowledging both the promise and current hardware limitations. Nevertheless, the majority of empirical research being either simulated or using small datasets suggests this line of research still has significant room for growth.

Another important area of study is focused on preprocessing and the effects of class imbalance, both of which are essential components of any practical solution for detecting fraud. Generative models (i.e. CTGANs) are now widely used to generate acceptable representations of minority-class samples, helping to mitigate class imbalance through the synthesis of realistic samples without the need to oversample. In the area of augmentation and dimensionality reduction (e.g. PCA/UMAP), generative models have proven that careful synthesis and compressive encoding will enhance the performance of downstream models and allow for the development of hybrid quantum-classical models that are not impacted by limitations in qubit count (e.g. by having a synthetic representation of the problem). New versions of DBNs, such as DBNex and Hybrid-DBN [7] [10] provide interpretability and accelerated convergence. The focus is on quantum federated models, resource-efficient quantum solutions for finance [11] [12], quantum autoencoders or graph-based networks [13] [16], as well as various comparison studies and surveys on QML methods/technologies (including optimizations such as quantum annealing) being developed both empirically (based on real-world and practical applications) and theoretically (through theoretical models) there is an accessible collection of empirical and theoretical evidence to support the current comparative research between DBNs (classical) and hybrid QDBNs (quantum).

4 Proposed System

This project proposes a new Quantum Deep Belief Network (QDBN) to improve financial fraud detection within the context of imbalanced data sets in order to better detect malicious activities of different forms through the incorporation of a Quantum Restricted Boltzmann Machine (QRBM) layer into the DBN structure. The proposed approach seeks to utilize QRBMs' features to increase model accuracy and to help detect complex statistical outliers associated with fraudulent activity. Four models will be developed: (1) a classical baseline (classical DBN) consisting only of stacked RBMs, (2) a hybrid DBN with 1 QRBM layer,

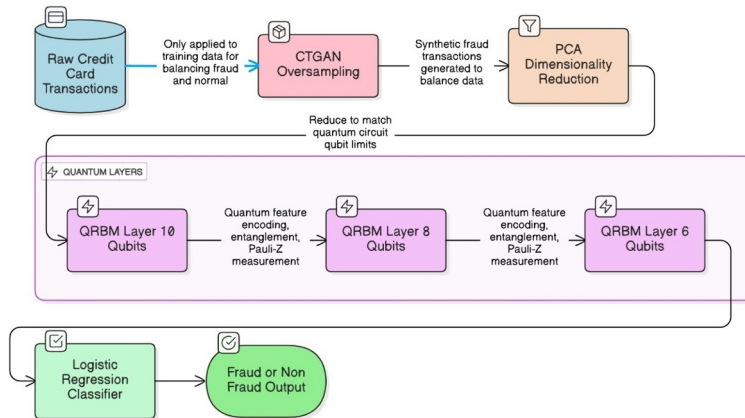


Figure 1. System Architecture of the proposed Quantum-Enhanced Deep Belief Network (QDBN) for Fraud Detection, spanning both columns for clarity.

(3) a hybrid DBN with 2 QRBM layers, (4) and a completely QDWB DBN (DBN consisting entirely of QRBM).

Layer-by-layer unsupervised training is used with the first few layers of traditional Restricted Boltzmann Machines (RBMs) building features and the QRBM using quantum encoding through rotations of RY gates, squashing courses, and measuring with Pauli Z gates are done to build the final representation of the samples that are provided to a logistic regression model for supervised prediction of fraud. Due to the significant imbalance of the classes in the data set, synthetic fraud transactions are created using CTGAN-based oversampling to produce an adjusted distribution of transactions for training. Dimensionality reduction processes were done using Principal Component Analysis (PCA) to provide a smaller number of features to encode into QRBM while maintaining the ability to represent the essential properties of each transaction; these PCA-transformed features are encoded into quantum circuits in the QRBM layer and the parameters are optimized with Quantum-Inspired Contrastive Divergence like we see in Figure 1. Overall, the proposed hybrid quantum-inspired system demonstrates improved precision, recall, and F1-scores compared to classical DBNs, demonstrating the effectiveness of quantum-inspired components in modeling intricate fraud patterns.

5 Methodologies Used

5.1 Dataset

The experiment is done using the Credit Card Fraud Dataset released by the Machine Learning Group of the Université Libre de Bruxelles (ULB) in collaboration with Worldline¹. The dataset contains 284,807 credit card transactions, of which 492 (0.172%) are fraudulent, resulting in a highly imbalanced binary classification problem. Each record contains 30 features V1–V28 derived via Principal Component Analysis (PCA), and two untransformed features, Time and Amount. The target variable is class which is 0 for non-fraud and 1 for fraud transactions.

¹<https://www.kaggle.com/datasets/mlg-ulb/creditcardfraud>

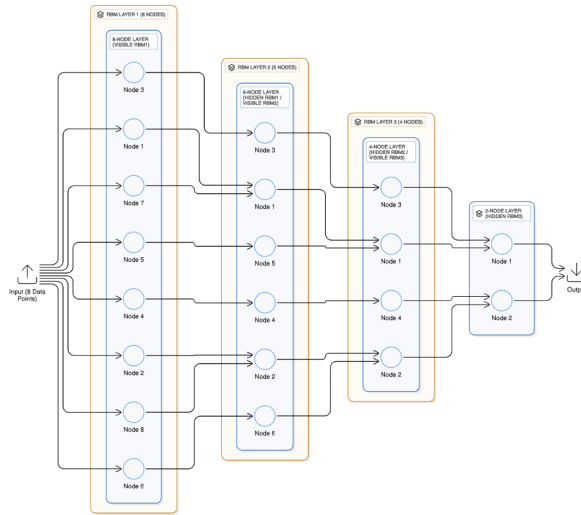


Figure 2. Architecture of the Deep Belief Network (DBN) used for feature extraction in financial fraud detection.

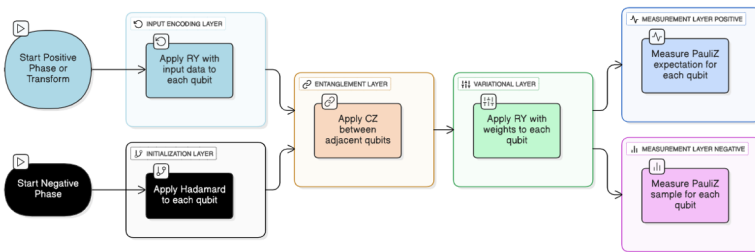


Figure 3. Workflow of the QRBM training process, illustrating the quantum-enhanced positive and negative sampling phases.

To data were divided into 80% training and 20% testing sets using stratified sampling. Given the extreme imbalance, evaluation relied primarily on recall and F1-score rather than raw accuracy.

5.2 Data Preprocessing

A Conditional Tabular GAN (CTGAN) approach was identified as the appropriate method for generating synthetic instances of the minority class to achieve a more balanced classification problem. The CTGAN model was constructed using 492 instances of the original sample and a subset of 4,900 instances of the original non-fraudulent class. After 30 epochs of training the CTGAN were produced, with a nearly 1:1 minority to majority class ratio of instances for the final dataset.

The resulting dataset was shuffled, normalized to produce standardized feature value ranges using StandardScaler, and then dimensionality reduction was applied using Principal

Component Analysis (PCA). This produced eight (8) principal components that were used as input to both the classical DBN and the quantum-enhanced DBN.

5.3 Dimensionality Reduction and Hardware Constraints

The selection of **8 principal components** was determined by the cumulative explained variance ratio, which reached **86.4%** at this threshold. This selection was primarily dictated by current **NISQ-era hardware and simulation constraints**. In quantum simulators such as Qiskit and PennyLane, the memory required to represent the quantum statevector scales exponentially as 2^n , where n is the number of qubits.

A high depth entangling circuit can be achieved through the mapping of 8 features to 8 qubits while remaining computationally tractable under the 16GB to 32GB RAM limits imposed by typical high-performance workstations. The mapping process used in constructing the 8-qubit entangling circuit is referred to as "Hardware-Aware" mapping. With this method, we preserve a significant amount of the signal while also eliminating the "barren plateau" effect and the exponentially long simulation latency that comes with larger numbers of qubits.

5.4 Model Architecture

The DBN will have 3 RBM layer with 8-6-4-2 Nodes respectively as shown in Figure 2. For classical DBN, we have implemented the RBM layers through pytorch, with training based on Contrastive Divergence (CD). For QDBN, we have implemented the QRBM layers through pennylane and replaced the RBM layers in classical DBN.

The energy function of an RBM is defined in Equation 1.

$$E(v, h) = -v^T W h - b^T v - c^T h \quad (1)$$

where W represents the weight matrix connecting visible and hidden units, and b and c denote bias vectors for the visible and hidden layers respectively.

The Contrastive Divergence is used for training which is carried out in two alternating phases, Positive Phase : The input data was propagated to the hidden layer using a logistic activation function as in Equation 2.

$$P(v_i = 1 | h) = \sigma \left(b_i + \sum_j h_j W_{ij} \right) \quad (2)$$

This phase strengthens the associations between simultaneously active visible and hidden units.

Negative Phase: A reconstruction was generated by sampling from the hidden activations. Weights were then updated by the difference between positive and negative statistics as in Equation 3.

$$\Delta W = \eta (v_0 h_0^T - \tilde{v} \tilde{h}^T) \quad (3)$$

Pennylane was used to simulate the QRBM with 50 measurement shots, following the below structure. RY rotation gates were used to encode layer inputs onto qubits, then transforming each input value x_i into a rotation by an angle proportional to x_i around the Y-axis. Controlled-Z (CZ) gates provided entanglement to the Qubit pairs, resulting in the qubit states correlated with one another through entanglement; thus, the QRBM can model higher-order dependencies between transaction features. RY gates were used for trainable weights at each

gate. The output will be based on measuring the qubits in the Pauli Z gate basis. Parameters were iteratively updated with regard to values based on gradients estimated from the measurement outcome(s) during training. During the Negative phase, the model produces samples by applying the Hadamard gate to the initial state of all qubits to produce superposition states, and then by applying the same operations (entanglement and rotations using trained weight parameters) to the current state of the qubits as during the training phase. The mean of repeated quantum measurement is taken as an approximation of the internal expectation values.

The gradient for parameter updates was computed as the difference between positive and negative phase statistics is shown in Equation 4.

$$\Delta\theta = \eta (\langle Z_i \rangle_{\text{data}} - \langle Z_i \rangle_{\text{model}}) \quad (4)$$

Each layer was trained separately using contrastive Divergence (CD) as shown in Figure 3. For both RBM and QRBM layers, the number of epoch is set to 10 with learning rate 0.10. After pre training, the model will be going through fine tuning phase. All variants were trained on identical data partition to ensure fair comparison.

The core contribution of this work is the implementation of a Quantum-Classical Hybrid DBN that utilizes quantum entanglement for feature extraction. The architecture consists of the following key stages:

1. **Quantum Angle Encoding:** Classical transaction features are mapped into the Hilbert space using RY-rotation gates, where the rotation angle is proportional to the normalized feature value.
2. **Entanglement Strategy:** We implement a layer of Controlled-Z (CZ) gates between neighboring qubits. This induces non-local correlations, allowing the network to model complex, higher-order dependencies between features that are typically inaccessible to classical RBMs.
3. **Quantum Contrastive Divergence:** During the pre-training phase, the 'Negative Phase' expectation values are estimated via quantum measurements. This allows the model to explore the energy landscape of the transaction data more efficiently, effectively bypassing the local minima that often hinder classical DBN convergence.

This hierarchical integration allows the model to maintain the stability of classical deep learning while benefiting from the representational richness of quantum states.

5.5 Evaluation Metrics

Recall was applied in conjunction with F1 score to measure how effective the models performed when classifying the imbalanced data in this study therefore providing an accurate measurement of how effectively the models have performed.

Based on the fact that the original data was imbalanced the use of accuracy alone would result in an inaccurate representation of model performance, whereby a model could reach 100% accuracy if it misclassifies all fraudulent transactions as not fraudulent.

In order to measure how effective the models performed, Recall and F1 were used as the primary metrics for comparison, although for purposes of ease of interpretation Precision was calculated as well.

Recall measures the proportion of fraudulent transactions that were accurately identified by the model, what proportion of the True Positive transactions were identified by the model. So if a model has high recall it is an indication that the model has reduced the number of False Negatives.

Precision tells us what proportion of the transactions identified by the model as fraud actually did commit fraud. For example, a model with a high recall will identify a high proportion of fraudulent transactions but may also result in a high number of transactions identified as fraudulent (False Positive) therefore Precision indicates the number of False Positives.

Thus, a fraud detection model should achieve an acceptable balance between recall and precision so that it will minimize both classes of misclassifications. The F1-Score is derived by calculating the harmonic mean between both recall and precision. The F1-Score provides a single indicator to determine how effectively a model achieves both precision and recall in imbalanced datasets and will aid in determining what balance of the two measures will need to be reached when determining the relative efficiencies of the identified models.

6 Result and Discussions

For the normal class (Class 0), as shown in Table 1, all model configurations—including Classical DBN, hybrid QDBN-1, QDBN-2, and Full QDBN indicate an average precision, recall, and F1-score that is close to one. When testing the classical DBN architecture on typical instances, the accuracy would be very high due to its bias towards the majority class and the imbalance between the two classes. The introduction of quantum layers into the QDBN structure (when adding additional QRBM layers) causes moderate reductions in precision, recall, and F1-scores; however, the precision is still 0.91, the recall is 0.90, and the F1-score is 0.91., as seen in Table 1 and Figure 4.

Table 1. Performance metrics for the majority (normal) class (Class 0).

Model	Precision	Recall	F1-score
Classical DBN	0.9996	0.9972	0.9984
QDBN-1	0.9900	0.9600	0.9800
QDBN-2	0.9900	0.6500	0.7900
Full QDBN	0.9100	0.9000	0.9100

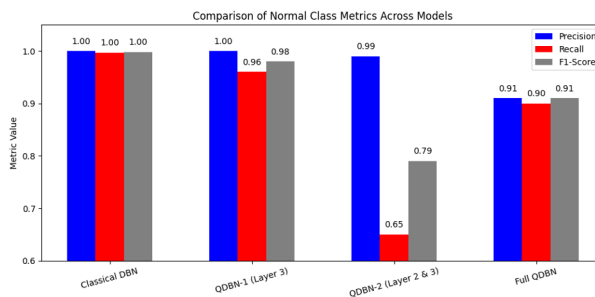


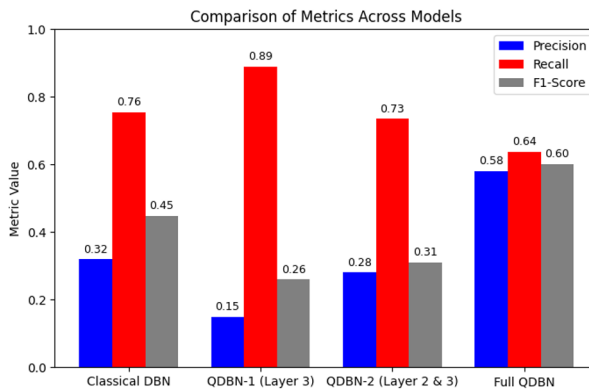
Figure 4. Comparison of classification metrics (Precision, Recall, F1-score) across all evaluated models for the majority (normal) class.

As shown in Table 2, the F1-score of the classical DBN is relatively poor, indicating a limited capacity for precise fraud detection. While initial quantum integration in QDBN-1 (single QRBM layer) shows a significant increase in recall (0.890), it suffers from a precision

Table 2. Performance comparison of DBN configurations on the fraud class (Class 1).

Configuration	Precision	Recall	F1-score
Classical DBN	0.319	0.755	0.448
QDBN-1	0.150	0.890	0.260
QDBN-2	0.280	0.735	0.310
Full QDBN	0.581	0.637	0.602

drop. However, as the depth of quantum integration increases to the Full QDBN configuration, the F1-score significantly improves to 0.602. This progression suggests that increasing the number of QRBM layers allows the model to better navigate the energy landscape, resulting in a more balanced ROC-AUC and a 34.4% improvement in F1-score over the classical baseline.

**Figure 5.** Comprehensive comparison of performance metrics (Precision, Recall, and F1-score) across all evaluated classical and quantum-enhanced models.

6.1 Comparative Analysis with Baseline Models

To better contextualize the performance gains of the Quantum-Enhanced DBN, we evaluated several industry-standard fraud detection models on the same preprocessed dataset. These include Logistic Regression (LR), Random Forest (RF), and Gradient Boosting (GB).

As shown in Table 3, classical supervised models struggle significantly with the minority class. While Random Forest achieved a recall of 0.60, its precision remained low at 0.222, leading to an F1-score of 0.324. In contrast, the **Full QDBN (F1: 0.602)** achieved a 85.8% improvement over the best-performing classical baseline (RF). This demonstrates that the quantum-inspired layers are more effective at identifying the complex "fraud manifold" than traditional decision trees or linear boundaries.

To evaluate the necessity of the proposed hybrid architecture, standard ensemble classifiers specifically Random Forest (RF) and XGBoost were applied directly to the raw, high-dimensional financial data. As evidenced by the results, these classical models struggle significantly with the 'needle-in-a-haystack' nature of fraud detection.

While the accuracy remains high due to the majority class bias, the Random Forest achieved an F1-score of only 0.28, failing to capture the complex non-linear correlations

Table 3. Performance benchmarking: QDBN vs classical baselines (Class 1).

Model	Precision	Recall	F1-score	ROC-AUC
Logistic Regression	0.235	0.400	0.296	0.987
Random Forest	0.222	0.600	0.324	0.895
Gradient Boosting	0.185	0.500	0.270	0.896
Full QDBN	0.581	0.637	0.602	0.819

in the fraud samples. XGBoost performed even more poorly, with a precision of 0.12, indicating a high rate of false positives. These results justify the transition to a Deep Belief Network (DBN) structure, which utilizes unsupervised pre-training to learn a more robust representation of the data before classification.

Table 4. Performance comparison of direct baseline models (Class 1).

Model	Precision	Recall	F1-score	ROC-AUC
Random Forest (Direct)	0.210	0.440	0.284	0.741
XGBoost (Direct)	0.125	0.320	0.180	0.648
Classical DBN	0.319	0.755	0.448	0.950
Full QDBN	0.581	0.637	0.602	0.819

6.2 ROC-AUC Analysis

The ROC-AUC values shown in Table 5 provide a comprehensive assessment of each model's general discriminative ability. Classical DBN methods have perfect separation between fraudulent and non-fraudulent transactions with a much higher ROC-AUC of 0.950, while Full QRBM-DBN has a strong distinction with a ROC-AUC of 0.819 although this is less than the classical DBN.

When interpreting these figures, the stark class separation created by financial fraud datasets should be noted. ROC-AUC is a composite metrics that is also affected by the larger class, which means it cannot provide an accurate measurement of the model's ability to detect rare fraud cases. Thus, improvements to minority class metrics are typically of greater importance than the ROC-AUC alone for fraud detection. While the overall ROC-AUC is somewhat lower, it is evident that the QRBM-based architectures are better suited for situations involving difficult fraud cases due to their greater sensitivity and balanced detection of the minority class (fraud), as measured by their greater recall and F1 values.

By integrating both global/overall metrics and specific minority metrics in this way, we will select a fraud detection model that does well at identifying both classes in general and specifically fraud transactions, providing a more meaningful measurement in real-world financial systems.

Table 5. Comparison of ROC–AUC scores across classical and quantum-enhanced configurations.

Model	ROC–AUC
Classical DBN	0.950
QDBN-1	0.612
QDBN-2	0.693
Full QDBN	0.819

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