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Integrating Grover's Quantum Search with Ant Colony Optimization for Efficient TSP Solving

Doddi Sreenija Reddy[1,*] and *Sountharrajan* S[2,**]

[1]Department of Computer Science and Engineering, Amrita School of Computing, Amrita Vishwa Vidyapeetham

[2]Chairperson, Department of Computer Science and Engineering, Amrita School of Computing, Amrita Vishwa Vidyapeetham

Abstract. The Traveling Salesman Problem (TSP) is a canonical example of an NP-hard combinatorial optimization problem that has extensive applications in the context of logistics and routing and network design, where classical methods cannot be used in practice due to the large scale of the instances. The approach includes extensive research on a hybrid quantum and classical optimisation model, termed Grover-ACO, that combines the Grover quantum amplitude amplification with the elitist selection stage of the Ant Colony Optimization (ACO). This study can be described as an algorithm that includes candidate pool formation, quantum oracle design, amplitude amplification, and pheromone update algorithms, and give a mathematical model of the hybrid algorithm. The evaluation of standard TSPLIB benchmark instances is conducted experimentally on a quantum circuit simulator, and performance is measured in terms of the best and average tour costs, as well as computational overhead. Findings indicate that GroverACO is consistently able to achieve better quality solutions and discourage premature convergence than classical ACO, especially on medium and large-scale problems, albeit at a higher computational budget due to quantum simulation.

1 Introduction

The recent advancements in the domain of quantum computing have seen a surge in the creation of quantum-assisted optimization methods for computationally intractable problems. Classical combinatorial optimization tools, including those involving NP-hard problems like the Traveling Salesman Problem (TSP), are strongly dependent on heuristic and metaheuristic algorithms in order to find near-optimal solutions within realistic time limits. Nevertheless, these classical methods tend to become prone to premature convergence and stagnation with the growth in the size and complexity of the problem. Recent developments in quantum algorithms provide new paradigms for enriching classical optimization models with the concept of superposition and probabilistic amplitude amplification.

The Traveling Salesman Problem is a well-known optimization task with broad uses in logistics, circuit design, and route planning. Its goal is to find the shortest possible tour that

*e-mail: ch.en.u4cse22016@ch.students.amrita.edu
**e-mail: s_sountharrajan@ch.amrita.edu

visits each city once and returns to the starting point. The quantum search algorithm, introduced by Grover, offers a quadratic speedup for unstructured search problems. It uses amplitude amplification to increase the likelihood of measuring marked states. While Grover's algorithm is not a direct solver for TSP, its probabilistic selection can bias solution choice in traditional metaheuristics.

2 Literature Survey

Almufti et al [1]., compared nine recent metaheuristic algorithms, including Vibrating Particles System(VPS), Lion algorithm (LA), Social Spider Optimization (SSO), Grey Wolf Optimizer(GWO), Cuckoo Search (CS), Ant Colony Optimization (ACO), Artificial Bee Colony (ABO), Bat Algorithm(BA) and Cat Swarm Optimization(CSO). They used conventional TSPLIB benchmarks. Findings show that ACO, GWO, and CSO offer the best trade-off between accuracy, robustness, and computational efficiency. The research gives guidelines for selecting the right algorithm based on performance needs. Its main strength is highlighting metaheuristics' ability to handle large TSP problems where exact methods do not succeed. Problem size affects the exploration-exploitation trade-off of various contemporary metaheuristic approaches to the TSP, proving that EVO, CryStAl, and SOA demonstrate better scalability and convergence speed, although none of the methods can find optimal solutions [2].

Manya et al [3]., provided a thorough performance analysis of heuristic and meta-heuristic algorithms for large-scale instances of the Traveling Salesman Problem (TSP). The research compares traditional heuristics such as Nearest Neighbour, Farthest Insertion, and Minimum Spanning Tree approximation with meta-heuristics such as Simulated Annealing, Genetic Algorithm, and Ant Colony Optimization. The experimental analysis reveals that while heuristic algorithms are faster in execution and often produce sub-optimal solutions, meta-heuristic algorithms produce better-quality solutions at the expense of increased computational time. Among all algorithms, Ant Colony Optimization is found to be the most scalable and efficient, producing near-optimal solutions for large problem sizes. This research brings out the trade-off between computational efficiency and solution quality, the need for careful parameter tuning and statistical analysis, and the potential benefits of hybrid or parallel optimization approaches for solving complex NP-hard problems such as the TSP. Evaluated [4] novel metaheuristic algorithms—Arctic Puffin Optimization, Human Evolutionary Optimization, and Ship Rescue Optimization—for solving the TSP, demonstrating their competitive convergence behaviour and solution quality compared with traditional approaches like GA and SA.

N Tram et al [5]., presents an improved metaheuristic approach based on the Discrete Spotted Hyena Optimizer (SHO), the Grey Wolf Optimizer (GWO), and a hybrid approach named MSHOSA, which incorporates simulated annealing in the initialization phase. The proposed approach is inspired by swarm intelligence and the cooperative hunting strategy of spotted hyenas. The approach seeks to address the exploration-exploitation trade-off in metaheuristics by incorporating adaptive operators and improved initial solutions. The proposed approach is tested on more than 50 TSPLIB benchmark problems and compared with several classical and contemporary optimization algorithms, including GA, SA, ACO, PSO, and other swarm intelligence-based algorithms. The results show that MSHOSA performs better than other algorithms in finding optimal or near-optimal solutions. Statistical analysis of the results confirms the superiority of the hybrid approach over other algorithms in terms of convergence prevention and scalability on medium- and large-scale problems.

Wei Li et al [6] enhanced the classical ACO by combining the HSDACO algorithm with the addition of differential edge information and a smooth pheromone update scheme. This method enhances the balance between exploration and exploitation and decreases premature convergence. It is experimentally proven that there are superior convergence rates and quality of solutions in comparison to the traditional ACO variants on TSP. Improved and new metaheuristic architectures like colored-TSP optimizers [7], metaphor-less algorithms [8], and starfish search algorithms [9] lead to better solution robustness and exploration abilities in discrete TSP problems. Distributed ACO [10], zebra optimization algorithm [11], and hybrid algorithmic models [12] show better scalability and convergence speed by integrating cooperative search paradigms with adaptive optimization techniques.

Realistic routing problems illustrate that ACO-inspired and GA-ACO hybrid models improve path quality and real-world optimization performance in logistics and transportation problems [14] - [16]. Improved pheromone mutation strategies [17], parameter optimization, and multimodal ACO models [18] improve exploration-exploitation tradeoffs and prevent premature convergence in multi-solution variants of TSP. Deep heuristic knowledge integration [19] with ACO models extends optimization capabilities to more complex problem domains like electric vehicle routing problems while retaining high solution efficiency.

Sato et al [20]., presents a Two-Step Quantum Search (TSQS) algorithm for solving the Traveling Salesman Problem (TSP) on quantum circuits, targeting the key challenge of efficiently preparing a superposition of feasible solutions. In contrast to conventional Grover-based methods, which start from an already prepared superposition of valid tours, TSQS introduces a first quantum search step that amplifies only feasible solutions using HOBO encoding, making the approach more qubit-efficient and scalable. A second quantum search stage then amplifies the optimal solution using a cost-phase oracle and a tailored diffusion operator, achieving a total query complexity of $O(\sqrt{n!})$, which is significantly faster than brute-force methods. The authors describe the quantum circuit implementation in detail, evaluate the approach through Qiskit simulations for small TSP instances ($n = 3, 4$), and compare TSQS with SSQS and GM-QAOA algorithms, demonstrating advantages in circuit depth, robustness to noise, and query complexity. Although the work is currently limited by present-day hardware and simulation constraints, it clearly highlights the potential of multi-step quantum search strategies to enhance combinatorial optimization and presents a promising direction for near-term hybrid quantum–classical optimization.

[21] offers a comparative analysis of classical, metaheuristic, and quantum solutions to the Traveling Salesman Problem, emphasizing the role of recent developments in quantum computing in overcoming scalability issues. Saini et al [22]., presented a hybrid quantum-classical approach to solve the Traveling Salesman Problem (TSP) with a Quantum-Inspired Evolutionary Algorithm (QEA) improved by a new Sort Gray Binary Encoding for TSP (SG-BET) scheme that ensures the generation of valid solutions and enhances scalability on Noisy Intermediate-Scale Quantum (NISQ) computers. The experimental study demonstrates enhanced diversity and optimization efficiency over classical methods, which indicates that hybrid quantum-classical systems can effectively overcome NISQ barriers and offer a feasible solution path to scalable quantum-assisted combinatorial optimization.

[23] suggested the use of a quantum wave function-based optimization method that utilizes quantum state representation to enhance the efficiency of solution search for TSP problems. A noise-robust quantum reinforcement learning method with adaptive error correction strategies [24], improves the stability and quality of solutions for TSP problems on noisy quantum computers. A quantum self-organizing map method [26] that utilizes quantum-inspired clustering models is better for route optimization in Euclidean TSP. Ansatz optimization with simulated annealing [27] in variational quantum algorithms is for better parameter optimization in TSP solutions.

Kate et al [28]., offered a thorough review of the state of the art in the characterisation of optimisation problem hardness and the improvement of empirical algorithm evaluation through better testing methodologies. With the Travelling Salesperson Problem (TSP) as a paradigm problem, it traces the development from traditional exact and heuristic solution methods to more recent nature-inspired and quantum approaches, pointing out how the focus of research has moved towards understanding the role of problem instance properties in determining algorithm performance. One of the main topics reviewed is Instance Space Analysis (ISA), a methodology for interpreting algorithmic strengths and weaknesses, as well as instance diversity, to guarantee rigorous and impartial benchmarking.

Stenger et al [29]., presented an amplitude encoding framework for solving the Traveling Salesperson Problem (TSP) on quantum computers, which provides a substantial improvement in qubit overhead by making it possible to scale the encoding size logarithmically with the number of cities. The new approach utilizes cost estimation based on probability distributions and nonlinear combinations of quantum expectation values, providing an alternative to the traditional summation-based method. The algorithm is developed in a variational quantum eigensolver (VQE) framework and tested on four- and five-node graphs, in which the simulations show precise convergence to optimal paths for a wide range of hyperparameters. The performance on randomly generated graph instances is also tested, and the results show robustness and efficiency, indicating the effectiveness of amplitude encoding in variational quantum algorithms. This work indicates that some instances of the TSP problem can be solved efficiently by using amplitude-encoded quantum optimization algorithms.

The Travelling Salesman Problem (TSP) is a canonical NP-hard combinatorial optimization problem, where exact algorithms require computational power as the size of the problem grows. This has resulted in the extensive use of heuristic and metaheuristic algorithms like Ant Colony Optimization (ACO), Genetic Algorithm (GA), Particle Swarm Optimization (PSO) and Simulated Annealing (SA) algorithm to achieve near-optimal solutions within a reasonable time of computation. According to several studies, metaheuristic methods offer a solid option between quality of solutions and computational efficiency, especially when dealing with large-scale TSP problems. It is always observed that ACO and its variations are more successful in comparison to most of their competing algorithms because they possess a pheromone-directed learning process that allows them to achieve successful exploration-exploitation trade-offs. However, classical ACO algorithms are subject to early convergence and stagnation, particularly when the initial reinforcement of the pheromones leads to a search bias in favor of suboptimal tours.

To cope with such constraints, various hybrid ACO models have been suggested with respect to incorporating local search heuristics, adaptive parameter tuning, or even evolutionary operators. These hybrid methods enhance the convergence rate and solution quality but are inherently limited to classical elitist selection processes, which can still support bad solutions in the presence of stochastic fluctuations. Similar to classical methods, quantum and quantum-inspired optimization methods have become potentially viable solutions to combinatorics. The search algorithm by Grover provides a quadratic acceleration to unstructured search and has been implemented in TSP models with amplitude amplification, quantum heuristics, and hybrid quantum-classical models. Although these schemes have been shown to be more efficient in searches on small and medium-scale instances, they do not directly integrate quantum algorithms into classical metaheuristic schemes; they typically use them as standalone solvers.

Although hybrid ACO techniques and quantum TSP solvers have been studied intensively, few studies have been conducted on integrating quantum amplitude amplification directly into the elitist selection step of swarm intelligence algorithms. Specifically, the addition of Grover's probabilistic amplification mechanism to bias pheromone reinforcement to high-quality solutions in ACO is not studied thoroughly.

Table 1. Comparison of Optimization Approaches for TSP

Category	Representative Works	Core Idea	Key Limitation
Classical Metaheuristics	GA, SA, PSO, ACO	Stochastic search with population-based exploration	Premature convergence; scalability limits
Hybrid ACO	ACO + 2-opt, ACO + SA, ACO + GA	Combines global search with local refinement or evolutionary operators	Still relies on classical elitist selection
Quantum TSP	Grover-based search, Quantum Annealing	Quadratic speedup via amplitude amplification or tunneling	Limited scalability; hardware constraints
Quantum-Inspired	QIEDA, Quantum-inspired GA	Uses quantum principles without quantum hardware	No theoretical quantum speedup

3 The Travelling Salesman Problem

3.1 Overview of the Traveling Salesman Problem

The Traveling Salesman Problem (TSP) can be formulated as a combinatorial optimization problem on a complete weighted graph $G = (V, E)$, where $V = \{1, 2, \dots, n\}$ represents the set of cities and c_{ij} denotes the distance (or cost) of traveling from city i to city j.

Define the binary decision variable:

$$x_{ij} = \begin{cases} 1, & \text{if the tour goes directly from city } i \text{ to city } j, \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

The objective is to minimize the total travel cost:

$$\min \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} c_{ij}\, x_{ij} \tag{2}$$

subject to the following constraints:

$$\sum_{\substack{j=1 \\ j \neq i}}^{n} x_{ij} = 1, \quad \forall i \in V \tag{3}$$

$$\sum_{\substack{i=1 \\ i \neq j}}^{n} x_{ij} = 1, \quad \forall j \in V \tag{4}$$

Constraints ensure that each city is visited exactly once and departed exactly once.

To eliminate subtours, the Miller–Tucker–Zemlin (MTZ) constraints are used:

$$u_i - u_j + n x_{ij} \leq n - 1, \quad \forall i \neq j,\ i, j \in \{2, \dots, n\} \tag{5}$$

where the auxiliary variables satisfy:

$$1 \leq u_i \leq n, \quad \forall i \in \{2, \dots, n\} \tag{6}$$

Finally, the decision variables are restricted to:

$$x_{ij} \in \{0, 1\}, \quad \forall i, j \in V \tag{7}$$

3.2 Ant Colony Optimization

The Ant Colony Optimization (ACO) algorithm can be described formally in the Problems Agent Goal Environment (PAGE) framework, which gives a conceptualization of the optimization process:

Problem: ACO is a metaheuristic algorithm useful in optimization problems like the Traveling Salesman Problem (TSP) and other routing and scheduling problems where the aim is to find an optimal configuration of a large discrete search space.

Agent: Artificial ants are the agents that construct candidate solutions incrementally using problem-specific heuristic information and pheromone trail intensities in a probabilistic manner.

Goal To obtain an optimal or near-optimal solution by minimizing or maximizing a predefined objective function, such as the total tour length in the TSP.

Environment: The environment can be defined as a graphical search space where the pheromone trails can be viewed as a shared memory that provides the means of indirect communication between agents and cooperation.

3.3 TSPLIB Dataset

In many researches the travelling salesman problem is generally solved using the TSPLIB dataset. It has instances of problems of different scales and structural properties that allow consistent and reproducible performance comparisons. Here, such representative instances, as those of Berlin52, Eil75, and Pr239, are taken; they vary in size and difficulty, with small Euclidean problems at one end, and a bigger and more difficult problem at the other. The literature often use these datasets to determine the quality of solutions, convergence, and scalability of optimization algorithms, and it is therefore appropriate to use them to validate hybrid and metaheuristic algorithms.

3.4 Circuit Implementation

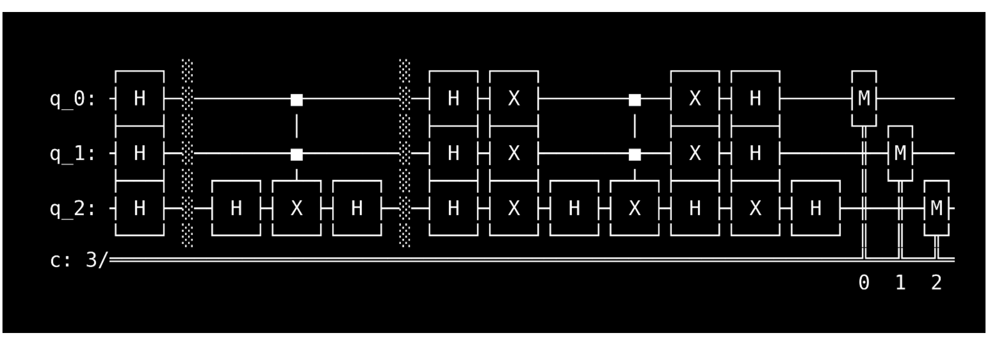

Figure 1. Quantum circuit for the Grover-based selection module

Figure 1 is a graphical representation of the quantum circuit that is used to execute the selection module based on Grover in the proposed Grover-ACO system. The first circuit is a layer of Hadamard (H) gates, which initializes all qubits in an equal superposition, modeled by candidate tour indices. The following sequence of gates uses the oracle operation, in which Pauli-X and controlled gates are used to label the solution of interest with phase inversion. Then, the Grover diffusion operator is added and consists of Pauli-X and Hadamard gates, which increase the probability amplitude of the marked state. Lastly, the measurement is conducted on all qubits, which in turn is used to probabilistically choose the amplified candidate, which is then utilized to drive elitist pheromone reinforcement in the classical ACO process.

4 Methodology

4.1 Simulation Environment

Qiskit is an open-source system of quantum computing using IBM, which assists in designing, simulating, and executing quantum circuits in hybrid quantum-classical workflows. This approach uses Qiskit to model the Grover-based amplitude amplification module, which is incorporated into the elitist selection step of the Ant Colony Optimization. Qiskit primitives are used to create quantum circuits that represent the oracle and diffusion operators and run them on the Qiskit AerSimulator, which is a classical analog of quantum hardware. It is a simulation-based environment that allows the controlled experimentation of quantum-assisted selection in idealized conditions, reproducible analysis of convergence behavior, amplification of probabilistic solutions, and computational overhead of the proposed Grover-ACO model.

4.2 Experimental Setup

An experimental demonstration of the proposed GroverACO framework was done in Python and Qiskit, with quantum circuit simulation being done on the Qiskit AerSimulator as a classical approximation to quantum hardware. The main steps involved in the experimental setup are summarized below:

1. **Setup and Imports:** Importing the required Python modules, including NumPy, Pandas, and Qiskit, and configuring the *Qiskit AerSimulator* backend for simulating quantum circuits.

2. **Problem Encoding and Candidate Selection:** Classical ACO is applied to create a family of candidate tours, and a few of the high-quality solutions that are evaluated are put together to create the input candidate pool to be fed to quantum-assisted selection.

3. **Oracle Construction:** The Grover oracle is built to find the best candidate tour index based on the binary encoding using multi-controlled quantum gates, applying the phase inversion module of the selected solution.

4. **Grover Circuit and Amplitude Amplification:** The grover circuit is implemented by using the Hadamard and Pauli-X gates by combining the oracle and diffusion operator in order to increase the probability of the marked candidate.

5. **Circuit Execution and Result Processing:** The *AerSimulator* is used to execute the quantum circuit on a predetermined number of shots and the results of the measurements are processed to determine the most likely candidate index that is used to initiate the process of elitist pheromone reinforcement in the ACO algorithm.

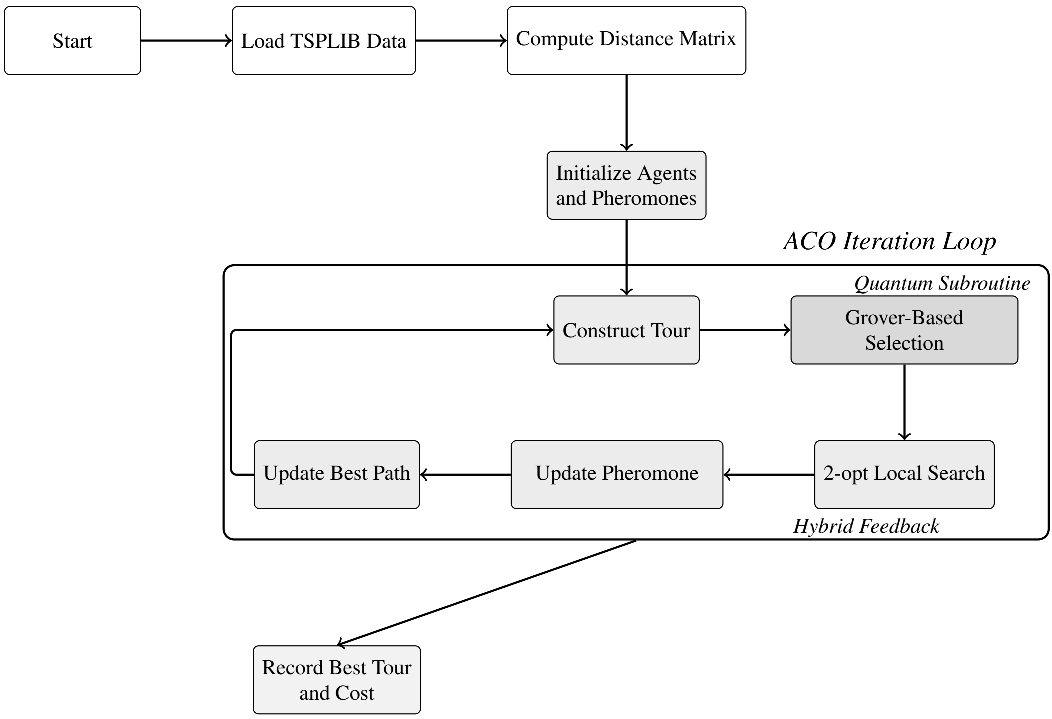

Figure 2. Architecture of the proposed hybrid Grover–ACO framework

4.3 Components of the Grover–ACO Framework

The suggested Grover-ACO framework will consist of the following main functional modules:

1. **GROVER_ACO(**$D, n_{\mathbf{agents}}, n_{\mathbf{iter}}, k$**):** Implementation of the hybrid functions of Grover-ACO optimization cycle, in which Traveling Salesman Problem tours are created with the help of Ant Colony Optimization, and Grover-based selection among the top-k candidate solutions is applied, and pheromone trails are updated to receive the best tour and its cost.

2. **CONSTRUCT_SOLUTION(**τ, D**):** This method constructs a feasible tour for ants. An ant chooses a path through probabilistic transitions based on the heuristic and pheromone trail. The path consists of a sequence of cities that have been visited once.

3. **GROVER_SELECT(***distances***):** Grover's amplitude amplification algorithm is applied to choose an elite solution for the top-k solutions. The measured quantum state is used to get a corresponding classical index that identifies the solution.

4. **GROVER_CIRCUIT(**$oracle, n_{\mathbf{qubits}}$**):** It is used to build a Grover quantum circuit with qubits in a uniform superposition, oracle operator, diffusion operator, and measurement for finding the amplified solution state.

Algorithm 1 presents the proposed Grover-based probabilistic selection for detecting eligible solutions. Quantum-enabled selection among candidates within the top-k tours enhances the convergence property through the reinforcement of high-quality solutions within the pheromone updating process.

Algorithm 1 Grover–ACO for the Traveling Salesman Problem

```
function GROVER_ACO(D, n_agents, n_iter, k)
    Initialize pheromone matrix τ
    Initialize ants with random tours
    best_path ← NULL
    best_cost ← ∞
    for iter = 1 to n_iter do
        all_paths ← ∅
        for each ant do
            path ← CONSTRUCT_SOLUTION(τ, D)
            cost ← PATH_DISTANCE(path, D)
            Add (cost, path) to all_paths
        end for
        Sort all_paths in ascending order of cost
        top_k_paths ← first k elements of all_paths
        distances ← costs of top_k_paths
        selected_index ← GROVER_SELECT(distances)
        selected_path ← top_k_paths[selected_index]
        τ ← (1 − ρ)τ
        UPDATE_PHEROMONE(τ, selected_path)
        if PATH_DISTANCE(selected_path, D) < best_cost then
            best_cost ← PATH_DISTANCE(selected_path, D)
            best_path ← selected_path
        end if
    end for
    return best_path, best_cost
end function
```

4.4 Mathematical formulation of the proposed methodology

Let $G = (V, E)$ be a complete weighted graph representing a symmetric Travelling Salesman Problem (TSP) with node set $V = \{1, \dots, n\}$ and Euclidean distances

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, \quad \forall i, j \in V. \tag{8}$$

A feasible tour is defined by a permutation $\pi = (\pi_1, \dots, \pi_n)$ with total length

$$L(\pi) = \sum_{i=1}^{n} d_{\pi_i, \pi_{i+1}}, \qquad \pi_{n+1} \equiv \pi_1. \tag{9}$$

In Ant Colony Optimization (ACO), ants construct solutions using pheromone trails $\tau_{ij}(t)$ and heuristic visibility $\eta_{ij} = 1/(d_{ij} + \varepsilon)$. The transition probability is

$$P_{ij}^{(k)}(t) = \frac{(\tau_{ij}(t))^{\alpha} (\eta_{ij})^{\beta}}{\sum_{\ell \notin V_k} (\tau_{i\ell}(t))^{\alpha} (\eta_{i\ell})^{\beta}}, \tag{10}$$

where V_k denotes the set of visited nodes.

After constructing N_a tours at iteration t, the best subset $\mathcal{P}_k(t)$ of size k is selected. Its index space $\{0, \ldots, k-1\}$ is embedded into a Hilbert space of size $N = 2^{n_q}$ with $n_q = \lceil \log_2 k \rceil$. Let $\mathcal{M}$ denote the set of marked elite tours. The Grover operator is defined as

$$G = D\,O, \qquad D = 2|\psi\rangle\langle\psi| - I, \tag{11}$$

where O performs phase inversion and $|\psi\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$ is the uniform superposition.

After $r = \left\lfloor \frac{\pi}{4} \sqrt{\frac{N}{m}} \right\rfloor$ iterations, the success probability of selecting an elite tour becomes

$$P_{\text{selected}}(r) = \sin^2((2r+1)\theta) \approx 1, \qquad \sin\theta = \sqrt{\frac{m}{N}}. \tag{12}$$

Let $\widetilde{\pi}(t)$ denote the measured tour. The pheromone update rule is modified as

$$\tau_{ij}(t+1) = (1-\rho)\tau_{ij}(t) + \begin{cases} \dfrac{1}{\widetilde{L}(t)}, & (i,j) \in \widetilde{\pi}(t), \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

The Grover–ACO mechanism replaces deterministic elitist selection with a quantum-amplified probabilistic bias, yielding an expected selection advantage of $O(\sqrt{N/m})$ while preserving exploration diversity and reducing premature convergence.

5 Experimental Analysis

5.1 Results

The results of the experiments presented in Table 2 show the relative performance of the classical Ant Colony Optimization (ACO) and the suggested Grover based on the ACO (Grover-ACO) framework on traditional TSPLIB benchmark problems. The most important observations can be summed up as follows:

- **BERLIN52:** In the case of BERLIN52, GroverACO had a lower best tour cost (7544.36) and better average tour cost than classical ACO (7870.67), meaning it has better convergence behavior and more exploration power.
- **EIL76:** EIL76: Grover-ACO performed better than classical ACO in the best and average tour cost in the EIL76 benchmark, which supports the success of quantum-assisted elitist selection of competitive candidate solutions.
- **PR264:** In the case of the larger PR264, GroverACO has shown a significant decline in the best and average tour cost, as a demonstration that it can alleviate premature convergence in a large and high-dimensional search space.
- **Computational Overhead:** In all instances of a benchmark, Grover–ACO took a longer time to execute than classical ACO because of the extra overhead incurred in the construction of the quantum circuit, amplitude amplification, and simulation on the Qiskit *AerSimulator*.

On the whole, the experimental outcomes have confirmed the positive effect of Grover-based probabilistic selection on solution quality and consistency of convergence achieved through collaboration with Ant Colony Optimization, despite the elevated complexity of simulation on a quantum processor.

Table 2. Performance comparison of classical ACO and Grover–ACO on TSPLIB benchmarks

Instance	Algorithm	Best	Avg	Time (s)
BERLIN52	ACO	7870.67	7883.41	4.77
	Grover–ACO	**7544.36**	**7582.56**	9.47
EIL76	ACO	569.73	575.93	9.55
	Grover–ACO	**552.54**	**555.52**	25.54
PR264	ACO	55141.23	55604.28	73.34
	Grover–ACO	**52374.47**	**52805.92**	1205.85

5.2 Discussion and Implications

The obtained experimental findings highlight some key points of the suggested Grover-ACO framework, and they are as follows:

- **Effectiveness of Quantum-Assisted Selection:** The addition of the amplitude amplification of Grover to the elitist selection stage increases the likelihood of selection biases towards high-quality solutions. This algorithm can be useful in avoiding premature convergence, which is a frequent weakness of classical Ant Colony Optimization.
- **Implications for Hybrid Optimization:** Despite the fact that the present method is based on classical quantum circuit simulation, the findings show that quantum-assisted probabilistic selection may have a positive impact on convergence behavior. This observation confirms the practicability of implementing quantum subroutines into classical metaheuristic optimization models.
- **Practical Considerations:** The present experiment involves a few qubits and has idealized all the conditions of quantum simulation. In realistic quantum hardware, the mechanical effects such as gate noise, decoherence, and limited qubit connectivity have to be carefully considered, since these may affect oracle fidelity and the precision of amplitude amplification.
- **Future Quantum Hardware Implementation:** With the ever-advancing development in quantum hardware, running the Grover-based selection module on real quantum hardware may help facilitate more efficient selection over time. Future research endeavors include designing more effective circuit architectures leveraging the capabilities of near-term quantum processors for large-scale combinatorial problems with hybrid approaches to scheduling on noisy quantum devices.

6 Conclusion

This approach has offered a thorough examination of the proposed Grover-ACO hybrid optimization framework, covering its conceptual foundation, algorithmic design, and empirical analysis using typical TSPLIB test problems. The proposed approach combines Grover's quantum amplitude amplification with the elitist selection step of Ant Colony Optimization, allowing for the probabilistic strengthening of good candidate solutions. Empirical analysis has shown that Grover-ACO is able to effectively enhance the best and average tour costs and prevent premature convergence compared to standard ACO, especially for medium- and large-scale test problems. Although the current implementation of Grover-ACO involves quantum circuit simulation and additional computational overhead, the results of this study indicate the promise of quantum-assisted selection for improving the performance of classical metaheuristics. As quantum computing hardware advances, the combination of efficient

Grover algorithms with scalable optimization heuristics, such as ACO, is likely to contribute significantly to the development of hybrid quantum-classical methods for solving difficult combinatorial optimization problems.

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