EPJ Web of Conferences **360**, 01024 (2026) https://doi.org/10.1051/epjconf/202636001024
QUICK'26

Quantum blind image watermarking with qdct-qim and bb84 key distribution

Budati Jaya Lakshmi Narayana[1], Bala Sumanth Reddy *Manda*[1*], *Koppuravuri* Sathvik[1] and *Koduru* Balavignesh Reddy[1]

[1]School of Computing, Amrita Vishwa Vidyapeetham, Amaravathi, India

Abstract: The protection of multimedia information against copying and tampering attacks by unauthorized parties is a challenge nowadays because of the tremendous increase in digital communications. Based on this, this paper presents a quantum blind image watermarking scheme incorporating Improved Novel Enhanced Representation (INEQR) for efficient image encoding, two-dimensional Quantum Discrete Cosine Transform (QDCT) frequency domain processing, and adaptive QIM for watermark embedding. The selection of watermarking parameters in this framework is carried out in a completely secure manner using Quantum Key Distribution (QKD) based on BB84. Simulation work on 30 standard grayscale images shows the strong imperceptibility, achieving PSNR values in the range of 52–58 dB and SSIM values between 0.98 and 0.999. and a more than 95 percent accuracy level in watermark recovery under attacks such as noise, compression attacks, and geometric attacks, thus authenticating PSNR-52–58 dB and SSIM-0.98–0.999. Robustness analysis performed against signal processing and geometric attacks indicates reliable extraction of watermarks, where the proposed framework attains an extraction accuracy of more than 95% under additive noise and filtering scenarios and moderate robust- ness against geometric attacks such as rotation and scaling. These indicate that the proposed framework is effective and provides imperceptible blind image watermarks based on the quantum approach.

1 Introduction

With the increasing creation and dissemination of digital image content, the copying, modifying, and misrepresenting of the images have also increased. The authenticity of the image and its rightful ownership have thus become critical parts of multimedia forensic analysis and commercial security studies. Watermarking digital images has been proposed as a solution to the problem of image copying and misrepresenting.

Despite all this research being done in this field, there are still some inherent basic challenges that continue to impede classical watermarking. These

* Corresponding author: sumanthreddy202039401@gmail.com

watermark schemes, which are being implemented in both spatial and other domains such as DCT, DWT, and SVD, pro- vide inherent trade-offs between perfect perceptibility and appropriate robustness. Higher embedding capacity better resists attacks and hence worsens perceptibility and makes them sensitive to compression attacks, noise, and geometric attacks when maximized for perceptibility. Additionally, in all conventional watermark schemes, embedding points are primarily deterministic and depend on basic key security and are sensitive to statistical attacks, water- mark forgeries, and unauthorized extraction. Current developments in the area of quantum information science create a push to tackle the challenges. Quantum watermarking schemes exploit the representations amenable to the quantum domain, along with its randomness, to provide the benefits of enhanced unpredictability and security. The roles of Quantum Key Distribution (QKD) emerge to develop uniformly random and secure embedding factors resilient to the challenges of deterministic watermarked embedding factors. Conversely, the parallel and reversible representations of images based on the benefits of quantum representations are applicable in the domain of frequency watermarking.

In this work, an image watermarking technique based on an improved novel enhanced quantum representation, a 2-dimensional quantum discrete cosine transform, and an adaptive quantization index modulation as a quantum method for robust frequency domain watermarking is presented. The positions and values of the embedding parameters for blind watermark embedding are safely established by utilizing a BB84 quantum key distribution method. This proposed system enables blind extraction of the watermarking technique without requiring the original image.

In the case of grayscale images, there are highly successful experiments to demonstrate that the proposed scheme has high imperceptibility and robustness, as it has a value of 52–58 dB and an SSIM index greater than 0.98, even under attacks by signal processing and geo- metric attacks.

The key contributions of this work are as follows:

- The framework for blind watermarking, which is compatible with quantum computers and is based on INEQR, supports reversible processing in the frequency domain in parallel.
- An efficient QDCT-QIM watermarked image should have high CBR-based imperceptibility and resistance against distorted image attacks.
- Integration with the BB84 quantum key distribution protocol for secure and unpredictable choices for parameters during the embedding of the watermark.
- Full-blind watermark extraction technique improved with the assistance of parity checks, without any need to access the source image.
- The technique has undergone comprehensive experimental validation, which has demon- strated strong robustness and accurate watermark recovery under significant attacks.

The paper is structured as follows. Section 2 is dedicated to the detailed literature review, covering classical and quantum algorithms of image watermarking, the quantum models of image representation, and outlining the gaps in the research found in the previous studies. We talk about the proposed watermarking scheme in Section 3. This section includes the process of embedding, the extraction algorithm, and how it is different from other watermarking methods. In Section 4, you can see the results of the experiments and the analysis of how well they work for imperceptibility, robustness, removing watermarks, and being able to handle different types of attacks.

2 Literature Review

2.1 Foundations of Digital and Quantum Watermarking

Watermarking of digital images is associated with embedding data invisibly, but this is ac- companied by the traditional watermark tradeoff of invisibility and robustness, which traditional watermarks cannot address. More robustness translates to visibility, while increased invisibility translates to reduced robustness. Moreover, traditional watermarks rely on fixed embedding points and standard key exchanges, making them vulnerable to brute force and statistical attacks by unauthorized individuals. Quantum watermarking is thus an extension of this requirement that uses quantum image patterns and methods of secure key exchanges using quantum mechanisms. Application of BB84-coded quantum key exchanges helps in enabling the completely invisible and random selection of parameters in watermark embed- ding, and quantum image representation expresses image pixels in qubits to enable blind watermark recovery.

2.2 Existing Studies

Recent studies have explored the transition from classical to quantum watermarking. Dhar and Sahu (2024) [1] provided a comprehensive theoretical review highlighting how quantum principles such as superposition and entanglement can enhance robustness and extraction accuracy compared to classical methods, though without practical implementation. Khan (2019) [2] introduced the Flexible Representation of Quantum Images (FRQI), which encodes pixel information in qubit phases; however, its phase-dependent nature makes it sensitive to noise. To address this limitation, Iranmanesh et al. (2022) [3] proposed a NEQR-based quantum watermarking scheme and demonstrated its feasibility on IBM Quantum Experience, but the approach suffers from scalability issues due to high circuit depth and gate complexity.

Subsequent works focused on improving robustness and imperceptibility through transform- domain and adaptive techniques. Refiyanti et al. (2024) [4] enhanced robustness using rotation-gate-based embedding on FRQI images, achieving strong imperceptibility at the cost of increased decoherence noise. Zhou et al. (2024) [5] proposed a QDCT- and Baker Map- based dual watermarking framework that outperformed classical DWT methods with PSNR values exceeding 50 dB, although it relied on classical key generation. Yu et al. (2023) [6] introduced an adaptive LSB quantum watermarking algorithm using the Haar Wavelet Trans- form, achieving PSNR values between 48–58 dB but with limited resistance to geometric distortions. Singh et al. (2025) [7] proposed a quantum and compressive sensing–based im- age authentication framework that ensured secure watermarking and robustness, yet did not evaluate blind extraction performance or resilience to geometric attacks.

Overall, these studies demonstrate significant progress from quantum image representations such as FRQI and NEQR toward transform-based quantum watermarking frame- works incorporating QDCT and Quantum Haar Wavelet Transform(QHWT). While these approaches improve fidelity and robustness, many still suffer from high circuit complexity, dependence on classical key exchange mechanisms, and limited robustness against combined attacks.

2.3 Research Gap

Although there has been significant progress in the development of Quantum Frequency- domain Watermarking(QFWs), the current state of the art is still impeded by three significant drawbacks of existing schemes: high circuit complexity, inadequate resistance to joint geo- metric and signal processing attacks, and the use of classical or deterministic methods of key management. There are only a few reports that talk about adaptive embedding and quantum-secure key generation at the same time. It is therefore clear that a significant research gap exists in developing an adaptive and scalable QFW scheme that incorporates adaptive embedding in the frequency domain along with secure key management. With this background, the need to propose the integration of QFW and the BB84 QKD protocol is evident.

Differing from the existing methods of quantum and classical watermarking, the proposed scheme combines the quantum embedding in the frequency domain, the secure key generation of the BB84 scheme, and the blind extraction based on the parity check in a unique way.

3 Proposed Methodology

In this section, the proposed quantum blind image watermarking system and process are described. The proposed system is blind in that extracting a watermark does not require any knowledge of or need for the original host image but rather is accomplished in a manner that utilizes a secret key and runs on parity calculations for predetermined frequency components. The method uses secure key-driven watermarking and quantum processing in the frequency domain. In particular, the transformation of the host image is conducted by Quantum Discrete Cosine Transform (QDCT), the embedding of watermarks follows the principles of parity-based Quantization Index Modulation (QIM), and the decision of where to place the watermarks and the choice of quantizers to apply the watermarks follow the principles of BB84-based Quantum Key Distribution (QKD).

The proposed watermarking scheme is defined using the INEQR-based quantum image representation; however, due to current hardware limitations, all quantum operations are evaluated through classical simulation that emulates quantum state evolution and measurement outcomes.

3.1 System Overview

Fig. 1 below depicts the workflow of the proposed blind watermarking scheme. To begin with, the grayscale image of resolution 128×128 is divided into non-overlapping blocks of resolution 8×8, and each block is converted to the frequency domain via QDCT. In cases of embedding, we use the mid-frequency coefficients and the adaptive quantization step size based on the key stream from the QKD process aided by the BB84 protocol.

The watermark, which comprises a 4-bit watermark with fixed bits of authentication as well as parity bits, is embedded in each block through parity-based QIM. The inverse QDCT is used in the watermark embedding process to obtain the watermarked image. However, upon extraction, the keystream is recreated to determine the embedded coefficients, and the extraction of the watermark bits does not need the original image.

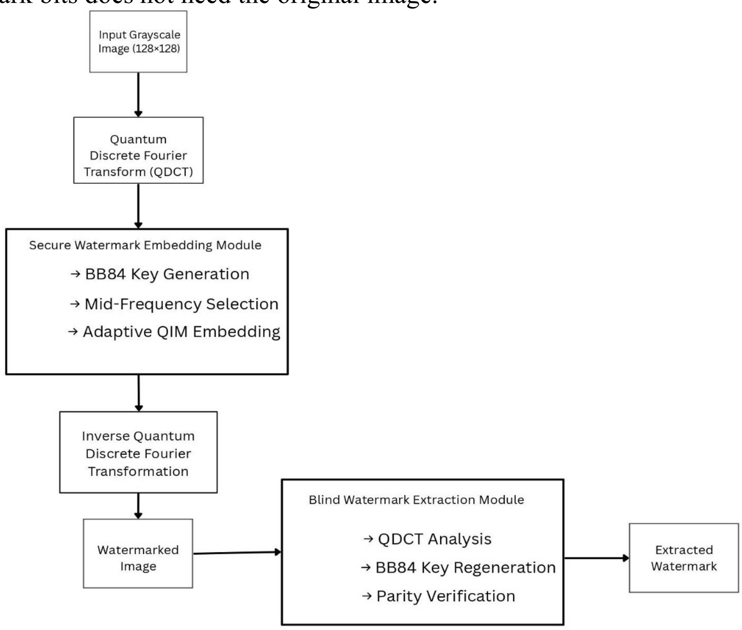


Fig. 1. High-level system overview of the proposed quantum blind image watermarking framework, illustrating the major processing modules including quantum frequency transformation, secure key-driven watermark embedding, inverse reconstruction, and blind water- mark extraction.

3.2 Watermark Embedding

Before frequency-domain processing, the host image is first encoded with the Improved Novel Enhanced Quantum Representation (INEQR) to make it reversible and parallel operation- compatible that can be realized by quantum transforms. These are then split into 256 non- overlapping 8×8 blocks, each of which has a 4-bit watermark that includes fixed authentication bits and parity bits for checking the integrity of the data. In the embedding process of the watermark, QDCT is performed on each block to get the frequency coefficients. After undergoing a two-dimensional QDCT transformation, each 8x8 block is broken down into low, mid, and high frequency coefficients is calculated using (1).

Mathematically, the QDCT for a block is expressed as:

$$F(u,v) = \alpha(u)\alpha(v)\sum_{x=0}^{7}\sum_{y=0}^{7} f(x,y)\cos\left[\frac{\pi(2x+1)u}{16}\right]\cos\left[\frac{\pi(2y+1)v}{16}\right] \quad (1)$$

Where:

$$\alpha(k) = \begin{cases} \frac{1}{\sqrt{2}}, & k = 0, \\ 1, & \text{otherwise.} \end{cases}$$

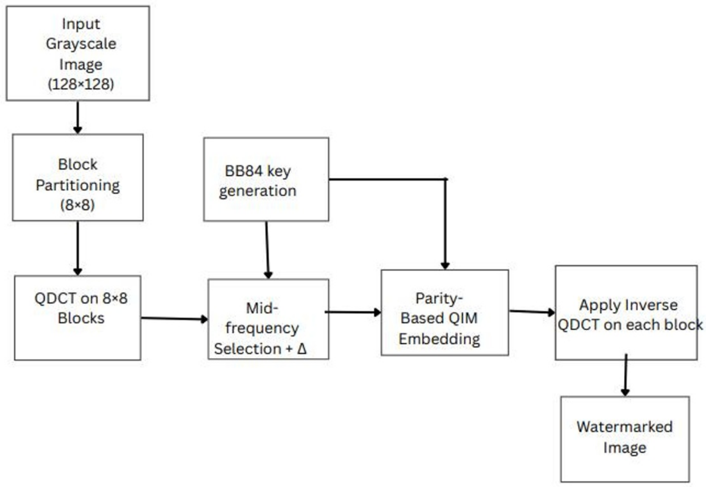


Fig. 2. Flowchart of the watermark embedding process using QDCT-based frequency transformation and BB84 key-driven parity-based QIM embedding

Each block consists of four bits [1, 0, 1, 0], two of which are fixed authentication bits [1, 0], and the remaining two are error-correcting parity bits [1, 1]. The repetitive bits in each block provide consistency. A simulated BB84-based QKD protocol generates a secure and random key stream that governs the selection of mid-frequency coefficients and the adaptive quantization factor for each block. Parity-Aware Quantization Index Modulation (QIM) embeds watermark bits into selected frequency coefficients.

The quantization step size is adaptively derived from the BB84 key, and the modified mid-frequency coefficient is obtained using the QIM rule. shown in (2):

$$c' = Q(c, b, \Delta_i) = \text{round}\left(\frac{c}{\Delta_i}\right) \tag{2}$$

Where: c is the original coefficient.
Δ_i is the adaptive quantization factor.

Where in the parity of the selected quantized coefficients will be flipped based on the water- mark bits. The underlying strategy lets you embed images without damaging them and keeps the host image's visual quality. Finally, the inverse QDCT is used to make the watermarked image after all the blocks have been processed.

Algorithm 1 Quantum Key-Driven Image Watermark Embedding Algorithm

Require: Grayscale image I of size 128×128, watermark bits W, QKD key stream K
Ensure: Watermarked image I_w

Divide I into 8×8 blocks $\{B_i\}$, $i = 1, \ldots, 256$
for each block B_i **do**
    Apply 2D QDCT to obtain coefficients C_i
    Set watermark bits $W_i = [1, 0, 1, 0]$ using parity encoding
    Derive key subset K_i from K to select 4 mid-frequency coefficients and determine Δ_i
    for each selected coefficient c_{ij} **do**
        Embed bit w_{ij} using QIM:
            $c'_{ij} = Q(c_{ij}, w_{ij}, \Delta_i)$
    end for
    Apply inverse QDCT to obtain modified block $\hat{B}_i$
end for
Combine all modified blocks $\hat{B}_i$ to form the watermarked image I_w

3.3 Watermark Extraction

During watermark extraction, the received image is divided into 8×8 blocks and QDCT- transformed. To find out where the embedded coefficients are, the BB84-generated key stream is made again.

The receiver uses the regenerated BB84 key to determine which QDCT coefficients in each block were modified during embedding. The parity of the quantized coefficient indices is used to recover the embedded bits. The retrieved data bit is approximated as the extracted watermark bit, given in (3):

$$\hat{b} = \operatorname{mod}\left(\operatorname{round}\left(\frac{c'}{\Delta_i}\right), 2\right) \tag{3}$$

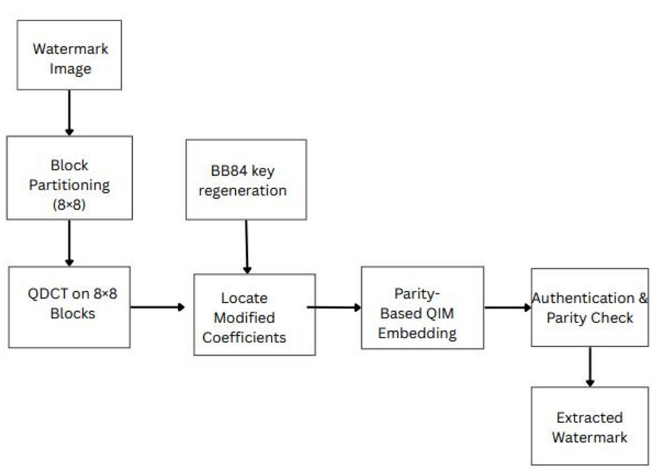


Fig. 3. Flowchart of the blind watermark extraction process using QDCT analysis and BB84 key regeneration, enabling parity-based watermark recovery without access to the original image.

Algorithm 2 Quantum Watermark Extraction Algorithm

Require: Watermarked image I_w, BB84 key generation parameters
Ensure: Extracted watermark bits $\hat{W}$

function EXTRACTION(I_w)
  Divide I_w into 8×8 blocks $\{B'_i\}$
  Regenerate BB84 key stream $K = \{K_1, K_2, \ldots, K_{256}\}$
  for each block B'_i **do**
    Apply 2D QDCT to obtain frequency coefficients $F'_{i,j}$
    Use key segment K_i to select mid-frequency coefficients and determine Δ_i
    for each selected coefficient c'_{ij} **do**
      Extract embedded bit $\hat{b}_{ij} = \mathrm{mod}\left(\mathrm{round}\left(\frac{c'_{ij}}{\Delta_i}\right), 2\right)$
    end for
    Form the 4-bit watermark $\hat{W}_i = [\hat{b}_{i1}, \hat{b}_{i2}, \hat{b}_{i3}, \hat{b}_{i4}]$
    Perform parity check on $\hat{W}_i$
    Verify fixed authentication pattern $[1, 0]$
  end for
  return complete extracted watermark sequence $\hat{W}$
end function

3.4 Comparison with Classical Watermarking Techniques

The traditional watermarking schemes can easily be deceived and affected by statistical attacks and signal processing attacks. Based on a concept that combines QKD and QIM watermarking schemes in QDCT domain transform, the proposed system is better in terms of security and robustness achieved through quantum signal processing in the frequency domain. Table 1 compares the proposed system with traditional watermarking schemes, demonstrating its stealthiness and robustness against key extraction attacks.

Table 1. Comparison of classical watermarking techniques and the proposed framework in terms of PSNR, robustness, and computational complexity.

Method	PSNR	Robustness	Complexity
Classical DCT	Medium	Medium	Medium–High
Classical DWT	Medium–High	High	High
Classical SVD	Medium	Medium	High
Proposed	High (48–58 dB)	High	Medium

4 Results and Inferences

In this section, it is demonstrated how the experimental scenario for evaluating the performance of the quantum watermarking system based on the QDCT-QIM scheme has been con- ducted. In this experiment, peak signal-to-noise ratio (PSNR), mean square error (MSE), and structural similarity index (SSIM) measures are used as performance measures to define the imperceptibility and robustness of the watermarked image. Experiments are carried out using different values of the quantum density factors (Q=1.0, 2.0, 3.0) and various geometric and signal processing attacks.

4.1 Experimental Setup and Metrics

4.1.1 Testing Dataset

The experimental data set employed in this research work is a customized grayscale image data set developed by the authors. The data set includes 30 images of size 128 × 128 pixels, which exhibit different visual properties like smooth areas, textures, edges, shadows, and natural scenes. The images were collected from publicly accessible open-source image repositories and converted to 8-bit grayscale images. Since there is no standardized benchmark data set available for the evaluation of quantum watermarking algorithms, a controlled customized data set has been developed for this research work to provide a uniform experimental setup. The data set is available on request for validation.

All experiments are conducted using a Python-based quantum circuit simulator executed on Google Colab. The simulator implements the prescribed INEQR-based quantum circuits and emulates quantum state evolution and measurement outcomes in a classical computing environment.

In this experiment, the proposed method has been tested on an image set comprising 30 grayscale images of size 128 × 128. The imperceptibility test of the watermark has been performed by PSNR and SSIM values, and the correctness of the extraction method has been evaluated by the accuracy in watermark extraction, which is calculated by the ratio of correctly extracted watermark bits to the total number of watermark bits.

PSNR and MSE estimate the distortion caused by adding watermarks and the attack distortion. SSIM, on the other hand, looks at how people see things by looking at differences in brightness, contrast, and structure. The accuracy of extraction indicates how robust an embedded watermark is against distortion.
The PSNR is computed using:

$$\mathrm{PSNR} = 10\log_{10}\left(\frac{MAX}{\mathrm{MSE}}\right) \quad (4)$$

- MAX_i is the maximum possible pixel value (255 for 8-bit images)
- MSE is the Mean Squared Error, which is computed using (5). This shows the difference between the original and attacked watermarked images

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N}[I(i,j) - K(i,j)]^2 \tag{5}$$

$$SSIM(x,y) = [l(x,y)]^{\alpha} \cdot [c(x,y)]^{\beta} \cdot [s(x,y)]^{\gamma} \tag{6}$$

4.2 Imperceptibility Analysis

Table 2 shows the PSNR and SSIM values for different quantization density factors. The results obtained prove that the proposed method performs very well in terms of maintaining visual quality for all the tested settings. The watermark is very difficult to see at Q=1.0, with PSNR values over 58 dB and SSIM values close to 1. As the quantization factor goes up, the strength of the embedding goes up, but the quality of the image slowly goes down. Even at Q=3.0, though, PSNR values stay above 52 dB, and the image is still very difficult to see.

Table 2. PSNR and SSIM values for different density factors (Q = 1.0, 2.0, and 3.0).

Densities	Q=1.0	Q=2.0	Q=3.0
PSNR	58.8	55.9	52.8
SSIM	0.999	0.998	0.983

The visual comparisons between the original and watermarked images also confirm that only a very little perceptual distortion has been introduced by the proposed embedding strategy even at higher embedding densities.

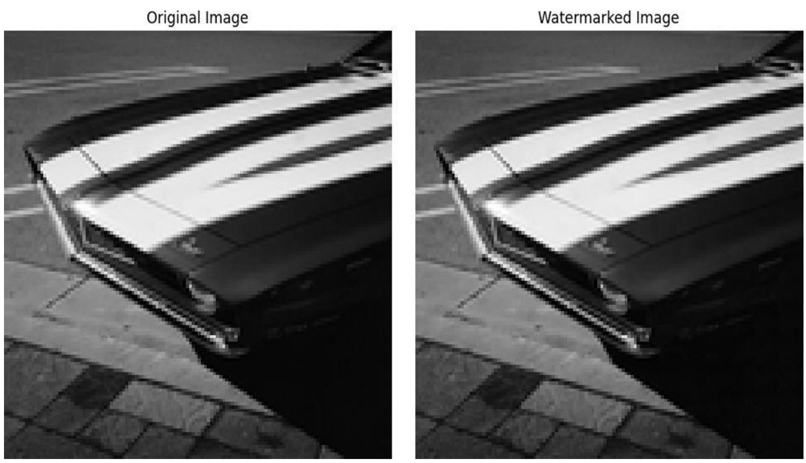


Fig. 4. The image on the left is the original, and the image on the right is the watermarked version. Both look very similar, showing that the watermark does not noticeably change the appearance of the image.

4.3 Robustness Analysis

The resilience of the proposed QDCT-QIM solution is tested using a number of typical signal processing attacks as well as geometric attacks. These signal processing attacks include noise addition, filtering, content manipulation, cropping, rotations, and scaling. Figure 5 illustrates the average values of the metrics measured for the proposed solution based on the quantization factors.

Analysis of the results shows that the proposed scheme has excellent resistance to noise- based attacks and filtering attacks, and the visual quality and recovery of the watermark are acceptable. The proposed scheme exhibits moderate robustness against geometric attacks such as rotation and scaling, and this is due to the phenomenon of block mismatch; nevertheless, partial recovery of the watermark has been attained in most cases.

4.4 Embedding Capacity and Comparative Evaluati

The proposed system will put 1024 bits of watermarks into a 128x128 image, making sure that the embedding ratio is 0.0625 bits per pixel. A comparison of the embedding ratio and security analysis of the proposed system and existing quantum image watermarking techniques will be presented in Table 3.

The experimental results demonstrate that the suggested scheme attains a proficient balance among embedding, imperceptibility, and robustness. The proposed QDCT-QIM-QKD system offers the advantages of secure key-driven embedding and frequency domain optimization, unlike the prevailing schemes that rely on traditional key exchange and may involve complex circuitry.

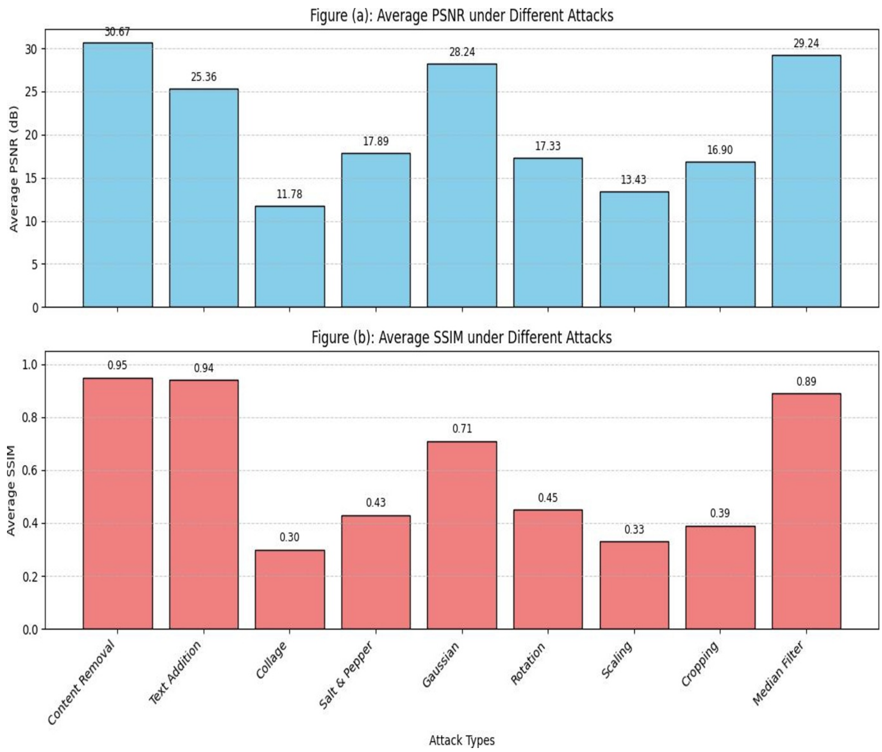
4.5 Discussion and Limitations

The results clearly show that the PSNR values are above 50 dB, and the SSIM values are close to 1 for all quantization factors. When Q=2.0, a satisfactory tradeoff between imperceptibility and robustness is achieved; robustness improves slightly with large values of Q.

Table 3. Embedding capacity comparison of representative quantum image watermarking techniques.

Method	Image Model	Capacity (bpp)
TMQIR-based [25]	TMQIR	1.00
FRQI-based [9]	FRQI	0.25
NEQR-based [7]	NEQR	0.50
INQW/NEQR [14]	INQW / NEQR	0.375
GTA-QIR / NEQR [17]	GTA-QIR / NEQR	0.125
Proposed	**INEQR**	**0.0625**

Although the proposed scheme performs very well, it is still vulnerable to serious geometric transformations like rotation and scaling, while its robustness decreases with high levels of noise. These issues demonstrate the necessity for future improvements, particularly in geometric synchronization functions and adaptive embedding processes.



4.6 Computational Complexity Analysis

The computational complexity of the proposed QDCT-QIM watermarking scheme is analyzed and compared with classical DCT-based watermarking methods.

For a 128×128 grayscale image divided into 8×8 blocks, the total number of blocks is:

$$\frac{128 \times 128}{8 \times 8} = 256 \tag{7}$$

For each block:

- Classical 2D DCT complexity is approximately $O(N^2 \log N)$.
- For an 8×8 block, the computation cost is constant and small.
- Therefore, the total classical DCT complexity becomes:

$$O(256 \times 8^2 \log 8) \tag{8}$$

The proposed QDCT implementation is simulated classically. Hence, its computational complexity remains comparable to classical DCT:

$$O(256 \times 8^2 \log 8) \tag{9}$$

Additional computational overhead is introduced by:

- BB84 key generation: $O(n)$
- QIM embedding: $O(1)$ per coefficient

Thus, the overall computational complexity of the proposed watermarking framework remains:

$$O(N^2 \log N) \tag{10}$$

which is asymptotically comparable to classical DCT-based watermarking.

Runtime Comparison: In runtime evaluation using Python (Google Colab), the average execution time per image was:

- Classical DCT watermarking: ~ 0.18 s
- Proposed QDCT-QIM-QKD scheme: ~ 0.24 s

This indicates a moderate computational overhead of approximately 33% due to secure key generation and adaptive embedding, while maintaining practical feasibility for real-world applications.

5 Conclusion

This paper has described an innovative blind image watermarking scheme that combines QDCT, adaptive QIM, and BB84-based QKD. The designed scheme achieved high imperceptibility, providing robust resistance to attacks based on keys as well as computational attacks, in which the image bits were embedded using key-based positions in the QDCT mid-frequency coefficients.

Evaluation for the experiment was conducted using 30 greyscale images, achieving high fidelity levels for PSNR, ranging from 52 to 58 dB, with a high SSIM measure of more than 0.98, which implies a negligible distortion level. The extraction results confirm that the reliability of recovered watermarks is of uniformly high accuracy with regard to different quantization factors under non-attack conditions. The specific results of this work can be summarized as:

- Parity-based QIM embedding helps in resisting fluctuations. This framework is very resilient to a variety of attacks such as noise, filtering, cropping, collage, and geometric attacks.
- Adaptive QIM embedding makes it possible to find a good balance between how well something is hidden and how accurate the extraction is.

Future Work: For the future, the proposed framework can be extended towards the implementation on real quantum hardware, with a focus on the optimization of circuit depth and complexity for NISQ-era quantum hardware. The proposed framework can also be generalized to RGB color images and video sequences by using multi-channel and temporal embedding strategies. The size and diversity of the dataset can also be increased for further improvement in the experimental validation and generalization performance. Hybrid quantum-classical models can also be explored for further improvement in the robustness and efficiency of the proposed framework. These would be towards the development of secure quantum-inspired multimedia authentication systems.

References

[1] S. Dhar and A.K. Sahu, “Digital to quantum watermarking: A journey from past to present and into the future,” *Computer Science Review*, vol. **54**, p. 100679, (2024). [Online].

Available: https://www.sciencedirect.com/science/article/pii/S1574013724000637

[2] R. A. Khan, “A flexible representation of quantum images for polynomial preparation, image compression, and processing operations,” *Quantum Information Processing*, vol. **18**, no. 7, p. 201, (2019). [Online]. Available: https://doi.org/10.1007/s11128-019-2306-6

[3] S. Iranmanesh, R. Atta, and M. Ghanbari, “Implementation of a quantum image watermarking scheme using neqr on ibm quantum experience,” *Quantum Information Processing*, vol. **21**, no. 6, p. 194, (2022). [Online]. Available: https://doi.org/10.1007/s11128-022-03530-9

[4] M. Refiyanti, G. Budiman, and L. Novamizanti, “Quantum image water- marking based on quantum state modification using rotation gate,” in *2024 International Conference on Artificial Intelligence, Blockchain, Cloud Comput- ing, and Data Analytics (ICoABCD)*, (2024), pp. 178–183. [Online]. Available: https://ieeexplore.ieee.org/document/10704405

[5] N.-R. Zhou, J.-W. Wu, M.-X. Chen, and M.-M. Wang, “A quantum image encryption and watermarking algorithm based on qdct and baker map,” *International Journal of Theoretical Physics*, vol. **63**, no. 4, p. 100, (2024). Available: https://doi.org/10.1007/s10773-024-05630-x

[6] Y. Yu, J. Gao, X. Mu, and S. Wang, “Adaptive lsb quantum image watermarking algorithm based on haar wavelet transforms,” *Quantum Information Processing*, vol. **22**, no. 5, p. 180, (2023). Available: https://doi.org/10.1007/s11128-023-03926-1

[7] R. K. Singh, K. N. Tripathi, G. Kaur, and R. Agrawal, “Qmcs-ias: Quantum mechanism and compression sensing-based image authentication and security technique,” *Multimedia Tools and Applications*, vol. **84**, no. 27, pp. 32 209–32 235, (2025). [Online]. Available: https://doi.org/10.1007/s11042-024-20474-

[8] Zhu, X. Chen, and Y. Yang, “A blind watermarking scheme for tmqir images,” *Multimedia Tools and Applications*, vol. **83**, no. 8, pp. 23 109–23 127, (2024). [Online]. Available: https://doi.org/10.1007/s11042-023-15368-z

[9] M.-X. Wang, H.-M. Yang, D.-H. Jiang, B. Yan, J.-S. Pan, and T. Wang, “A novel quantum image watermarking scheme for tamper localization and self-recovery,” *Quantum Information Processing*, vol. **21**, no. 8, p. 277, (2022). [Online]. Available: https://doi.org/10.1007/s11128-022-03619-1

[10] Z. Xing, C.-T. Lam, X. Yuan, G. Huang, and P. Machado, “A novel geometrically invariant quantum watermarking scheme utilizing quantum error correction,” *Journal of King Saud University - Computer and Infor- mation Sciences*, vol. **35**, no. 10, p. 101818, (2023). [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1319157823003725

[11] X. Mu, H. Wang, R. Bao, S. Wang, and H. Ma, “An improved quantum watermarking using quantum haar wavelet transform and qsobel edge

detection," *Quantum Information Processing*, vol. **22**, no. 5, p. 223, (2023). [Online]. Available: https://doi.org/10.1007/s11128-023-03964-9

[12] Z. Xing, X. Yuan, and C.-T. Lam, "A quantum watermarking scheme using new enhanced quantum image representation," in *2023 8th International Conference on Signal and Image Processing (ICSIP)*, (2023), pp. 326–330. [Online]. Available: https://ieeexplore.ieee.org/document/10270930

[13] R. Gorle and A. Guttavelli, "A novel dynamic image watermarking technique with features inspired by quantum computing principles," *AIP Advances*, vol. **14**, no. 4, p. 045024, (2024). [Online]. Available: https://doi.org/10.1063/5.0209417

[14] J. Su, X. Guo, C. Liu, and L. Li, "A new trend of quantum image representations," *IEEE Access*, vol**. 8**, pp. 214 520–214 537, (2020). [Online]. Available: https://ieeexplore.ieee.org/document/9268129

[15] P. Easom-McCaldin, A. Bouridane, A. Belatreche, R. Jiang, and S. Al-Maadeed, "Efficient quantum image classification using single qubit encoding," *IEEE Transactions on Neural Networks and Learning Systems*, vol. **35,** no. 2, pp. 1472–1486, (2024). [Online]. Available: https://ieeexplore.ieee.org/document/9798852

[16] Z. Xing, C.-T. Lam, X. Yuan, S.-K. Im, and P. Machado, "Mmqw: Multi- modal quantum watermarking scheme," *IEEE Transactions on Information Forensics and Security*, vol. **19**, pp. 5181–5195, (2024). [Online]. Available: https://ieeexplore.ieee.org/document/10509727

[17] Z. Xing, X. Yuan, C.-T. Lam, and P. Machado, "Ngqr: A novel gener- alized quantum image representation," *IEEE Transactions on Emerging Top- ics in Computing*, vol. **13**, no. 3, pp. 591–603, (2025). [Online]. Available: https://ieeexplore.ieee.org/document/10706768

[18] Z. Xing, X. Yuan, and C.-T. Lam, "Pattern-based quantum text wa- termarking: Securing digital content with next-gen quantum techniques," *iScience*, vol. **27**, no. 12, p. 111364, (2024). [Online].

Available: https://www.sciencedirect.com/science/article/pii/S2589004224025896

[19] "Dwt-dct image watermarking with quantum-inspired optimization," *International Journal of Intelligent Engineering and Systems*, (2025). [Online]. Available: https://api.semanticscholar.org/CorpusID:275059671

[20] J.-W. Jiang, T. Zhang, W. Li, and S.-M. Wang, "A quantum image watermarking scheme based on quantum hilbert scrambling and steganography about the moire´ fringe," *Quantum Engineering*, vol. **2023**, no. 1, p. 3746357, (2023). [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1155/2023/3746357

[21] Z. Wei, M. Han, H. Yin, and G. Hu, "A quantum color image watermarking protocol based on least significant bit," in *2024 5th International*

Conference on Computer Vision, Image and Deep Learning (CVIDL), (2024), pp. 174–179. [Online]. Available: https://ieeexplore.ieee.org/document/10603657

[22] F. Yan and S. E. Venegas-Andraca, *Quantum Image Security*. Singapore: Springer Singapore, (2020), pp. 79–104. [Online]. Available: https://doi.org/10.1007/978-981-32- 9331-14

[23] Z. Wang, M. Xu, and Y. Zhang, “Review of quantum image processing,” *Archives of Computational Methods in Engineering*, vol. **29,** no. 2, pp. 737–761, (2022). [Online]. Available: https://doi.org/10.1007/s11831-021-09599-2

[24] A. Anand and A. K. Singh, “An overview on quantum image watermarking for security of covid-19 patient record,” *IETE Journal of Research*, vol. **0**, no. 0, pp. 1–15, (2021). [Online]. Available: https://ieeexplore.ieee.org/document/9778153

[25] W.-W. Hu, R.-G. Zhou, A. El-Rafei, and S.-X. Jiang, “Quantum Image Watermarking Algorithm Based on Haar Wavelet Transform,” *IEEE Access*, vol. 7, pp. 121 303–121 320, 2019. [Online]. Available: https://inspirehep.net/literature/2706722