

Quantum Simulation of the Thermal Dark Counts of the SPAD Detector for Space Applications

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Abstract. Single-Photon Avalanche Diodes (SPADs) are widely employed in space-based photon detection systems for applications such as deep-space communication, LIDAR, and radiation monitoring. However, their performance is fundamentally limited by thermal dark counts, which are spurious detection events arising from temperature-induced carrier excitations. In this work, we present a quantum-mechanical modelling framework in which the SPAD is represented as a quantized two-level system $|g\rangle$, $|e\rangle$ corresponding to the ground and excited (avalanche-triggering) states. Thermal effects are incorporated through open quantum system dynamics using the Lindblad master equation, enabling a physically consistent description of temperature-driven carrier generation and relaxation processes. To enable scalable and hardware-relevant analysis, we implement the model within a gate-based quantum simulation framework, where quantum state transitions emulate thermally induced carrier excitation events. The time evolution of the excitation probability is computed to extract the theoretical dark count rate under varying thermal conditions relevant to space environments. This approach establishes a quantum simulation platform for predicting SPAD noise behaviour in temperature-variable environments, providing a foundation for optimizing detector design and improving photon-counting reliability in space applications.

1 Introduction

Single-Photon Avalanche Diodes (SPADs) are semiconductor detectors that are used for single-photon detection. The single photon is an information source in deep-space communication, LIDAR, quantum key distribution (QKD) and radiation monitoring, ghost imaging and super-resolution microscopy, etc [1]. SPADs are integrated into satellite payloads in low Earth orbit and are exposed to high-energy protons, alpha particles, and heavy ions in the space radiation environment, including Galactic Cosmic Rays (GCRs) and Solar Particle Events (SPEs), and external thermal noise. They cause lattice displacement, creating defect states that degrade the SPAD detector and affect important characteristics such as dark counts. [2]. Fundamentally, Dark count rate (DCR) is the false detection counts due to thermal excitation rather than a real photon [3]. It causes bit errors in deep-space communication, false distance readings in LiDAR, and the breaking of security keys in quantum communication. Furthermore, previous experimental studies show that the DCR of SPAD increases and then saturates [2] and exhibits Arrhenius behaviour, with activation energy around 0.4 eV [3].

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Currently, TCAD simulation (Technology Computer-Aided Design) are used to design and simulate the SPAD detector and validate their performance under space radiation environments. These TCAD simulations are extremely helpful to model the device as a bulk. However, they do not address the fundamental quantum statistics of single-photon interactions or the quantum open-system dynamics intrinsic to SPAD operation. To simulate SPAD, in which a single photon interacts with the detector, it is essential to consider the atomic-level interactions. [4].

The Quantum simulation, which treats SPAD as a two-level system ($|g\rangle$ & $|e\rangle$), interacting with a single photon, helps to analyze and visualize this problem at an atomic level. In this work, the Jaynes-Cummings model [5] is used to simulate the interaction between a single photon and such a two-level SPAD detector. Furthermore, light is modelled as a quantum harmonic oscillator, and second quantization and Fock space are employed to truncate the photon number in the system [6]. Furthermore, to simulate thermal dark counts, it is necessary to include open-system dynamics. The Lindblad master equation [7] and IBM Qiskit's Kraus channels [8, 9] are used to simulate the dynamics of this open system on Aer simulator [10]. The motivation for this work is not to compete with classical TCAD simulations, but to develop a new framework for SPAD detectors based on Quantum simulations, which could provide an important approach to simulate this problem at the atomic level and to extract the aforementioned characteristic parameters of SPAD detectors for space applications.

2 Methodology

The SPAD detector is modelled as a two-level system with energy levels $|g\rangle$ & $|e\rangle$, which takes account of SPAD in ground and excited state respectively. Furthermore, the photon Fock space is truncated to a fixed number of photons [6] to simulate photon-SPAD interactions. The entire photon-SPAD interaction is described by the Jaynes-Cummings model [5]

2.1 Theoretical Framework

1. Two-level modelling of SPAD: SPAD is modelled as a two-level quantum system. As any physical two-level system can be regarded as a Qubit [11], so in the quantum simulation, SPAD is represented as a qubit. These two states are written as qubit computational states as the ground state ($|0\rangle$) and an excited state ($|1\rangle$). These two states represent SPAD when it is armed (ready to detect) and when an avalanche occurs, respectively. The ground-state energy (E_{gs}) is the zero-energy reference point, and the excited-state energy (E_{ex}) is 0.4 eV [3]. Figure (1) shows the energy-level diagram of such a two-level SPAD quantum system. The Hamiltonian of this quantum model is given by Eq. (1),

$$H_{\text{SPAD}} = E_{gs}|0\rangle\langle 0| + E_{ex}|1\rangle\langle 1| \quad (1)$$

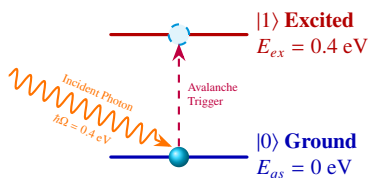


Figure 1: Compact two-level SPAD model.

2. Boson Truncation: To accurately model the interaction between a single photon and SPAD, it is necessary to quantize the light. Hence, light is modelled as a quantum harmonic oscillator. The energy of a quantum harmonic oscillator is expressed as Eq. (2) [5],

$$H_{\text{photon}} = \hbar\Omega(a^\dagger a) \quad (2)$$

where $\hbar\Omega$ is the energy of a photon and $a^\dagger a$ is known as the number operator \hat{N} whose eigenvalue is the number of photons in the system, n . The photon number n is truncated to have at most three photons in the system. For $n_{\text{max}}=3$, two qubits are required to describe photons in the simulation [6].

3. System Hamiltonian: The Jaynes-Cummings (JC) model provides a quantum-mechanical framework for understanding the interaction between a single photon and a SPAD detector. The JC hamiltonian for the photon-SPAD system is written as Eq. (3),

$$H_{\text{JCM}} = H_{\text{photon}} + H_{\text{SPAD}} + H_{\text{Interaction}} \quad (3)$$

where we have defined H_{SPAD} and H_{photon} in Eq. (1) & (2) respectively. The interaction Hamiltonian is written as $H_{\text{Interaction}} = \hbar g(a^\dagger \sigma_- + a \sigma_+)$, where g is the coupling constant between photon and SPAD, and σ_+ & σ_- are SPAD state raising and lowering operators, respectively.

2.2 Quantum simulation framework

In this section, a detailed analysis of the quantum simulation of thermal dark counts in SPADs is presented. To perform a quantum simulation of photon-SPAD interaction, both are represented as qubits. IBM Qiskit's quantum simulator [10] is used to implement the problem. Quantum simulation framework comprises the following three steps.

1. Pauli string decomposition of system Hamiltonian: The Hamiltonian of the combined photon-SPAD system described in Eq. (3) needs to be converted into Pauli strings, in order to code into the Aer-simulator. Qiskit internally converts any given matrix into the Pauli strings by Qiskit's `SparsePauliOp` [12]. The Pauli string Hamiltonian is expressed in Eq. (4),

$$H_{\text{JCM}} = \left[\frac{1}{5} I_{p2} (3I_{p1} - Z_{p1}) - \frac{2}{5} Z_{p2} I_{p1} \right] \otimes I_{s0} + \frac{1}{5} I_{p2} I_{p1} (I_{s0} - Z_{s0}) \\ + \hbar g \left[\begin{aligned} & \frac{1+\sqrt{3}}{4} I_{p2} \otimes (X_{p1} X_{s0} + Y_{p1} Y_{s0}) + \frac{\sqrt{2}}{4} X_{p2} \otimes (X_{p1} X_{s0} - Y_{p1} Y_{s0}) \\ & + \frac{\sqrt{2}}{4} Y_{p2} \otimes (X_{p1} Y_{s0} + Y_{p1} X_{s0}) - \frac{\sqrt{3}-1}{4} Z_{p2} \otimes (X_{p1} X_{s0} + Y_{p1} Y_{s0}) \end{aligned} \right] \quad (4)$$

Here, the first term of Eq. (4) is the Pauli string decomposition for the photon Hamiltonian described in Eq. (2), the second term is for the SPAD Hamiltonian described in Eq. (1), and the third term is the interaction Hamiltonian. The subscript of each Pauli matrix of Eq. (4) is applied to the respective qubit. The quantum state of the system is written as $|p_2 \otimes p_1 \otimes s_0\rangle$, where p_2 and p_1 represent photon qubits, while s_0 represents the SPAD qubit in the quantum circuit.

2. Initializing the Quantum circuit: The quantum circuit with the photon and SPAD is described as qubits, is shown in Figure (2).

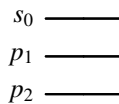


Figure 2: Quantum Circuit for simulating SPAD as a two-level system.

the quantum circuit is initialized in a state with no photon in the system and SPAD in the ground state, i.e., $|p_1 \otimes p_o \otimes s_o\rangle = |000\rangle$. IBM Qiskit's `QuantumCircuit` [13] is used to initialize the problem, with three quantum registers.

The observable state is set to be $|p_1 \otimes p_o \otimes s_o\rangle = |001\rangle$, which denotes SPAD detector in the excited state $|1\rangle$.

3. System evolution under noisy environment: The Pauli string Hamiltonian is now evolved to measure the probability of getting observed state $|001\rangle$, when the system is initialized in $|000\rangle$ state. To simulate the thermal dark counts, noise is incorporated in the quantum simulation with the help of the Lindblad master equation [7] and Qiskit's gate-based method `thermal_relaxation_error(t_1, t_2, excited_state_population, gate time)` [9]. Furthermore, it is observed that thermal dark counts follow Arrhenius law [3]. Hence, in this simulation, the rate of thermal excitation is derived from this law and expressed as Eq. (5),

$$R_{DCR}(T) = A_{eff} \exp(-E_a/k_B T) \quad (5)$$

Dark counts are simulated using both the Lindblad master equation and Qiskit's gate-based method for comparative study. The evolution of the system for both methods differs slightly. Lie Trotter algorithm [14] is used to evolve the ideal system Hamiltonian in the presence of Qiskit's noise model `thermal_relaxation_error`. Furthermore, another way is the Lindblad master equation, which provides the density-matrix evolution of an open quantum system and is defined by Eq. (6),

$$\frac{d\rho}{dt} = -i[H, \rho] + \sum_k \gamma_k \left(L_k \rho L_k^\dagger - \frac{1}{2} \{L_k^\dagger L_k, \rho\} \right) \quad (6)$$

The first term of this equation governs the evolution of an ideal/closed system, while the second term accounts for all non-unitary/dissipative phenomena, such as thermal dark counts. The Lindblad dissipator $L_{th} = \sqrt{\gamma_{th}(T)}(I \otimes \sigma_+)$ is jump operator, where $\sigma_+ = |e\rangle\langle g|$ is the raising operator. This operator projects the ground state $|000\rangle$ to the excited state $|001\rangle$ at a rate $\gamma_{th}(T)$ derived directly from Eq. (15). The simulation parameters and information regarding these noise models are listed in Table 1.

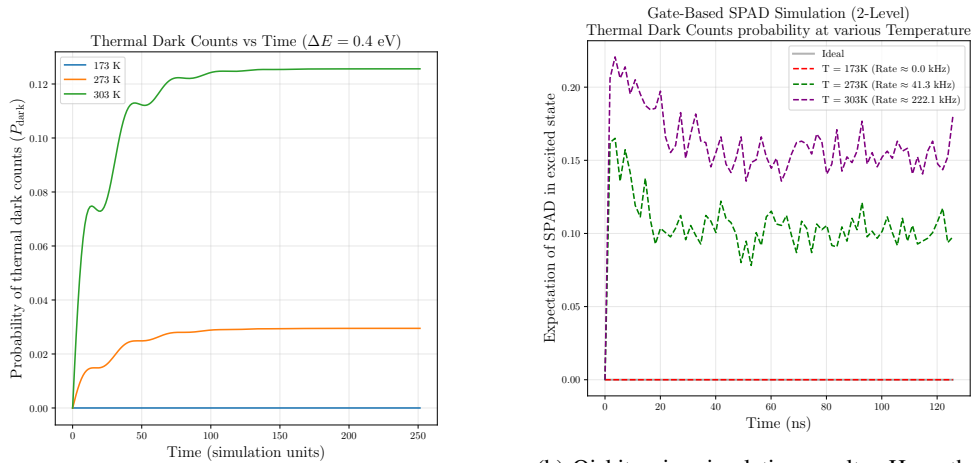
Table 1: Parameters for photon-SPAD simulation in Qiskit.

Parameter	Physical Meaning	Value & Ref.
Coupling (g)	Interaction strength between photon and SPAD	0.1 [15]
t_1	Relaxation time constant for state $ 1\rangle$	Inverse Arrhenius rate
t_2	Dephasing time for superposition state	$\approx 2T_1$ [9]
p_{th}	<code>excited_state_population</code>	Fermi-Dirac statistics.
<code>gate_time</code>	Time lag to process between gates	5×10^{-8} s

3 Results and discussion

Thermal dark counts are the primary noise source in SPAD detectors, arising from thermal generation of charge carriers rather than photon absorption. In this simulation, this phenomenon is modelled on the Shockley-Read-Hall (SRH) mechanism [3], where the Arrhenius law governs the generation rate. Following the experimental characterization in previous

studies [2, 3], an activation energy of $E_a \approx 0.40$ eV and temperature values of 173 K, 273 K and 303 K are used. To capture this noise dynamics, the Lindblad master equation and Qiskit's gate-based noise method are used. The quantum simulation result for the thermal dark counts using the Lindblad master equation is shown in Figure 3(a).



(a) Two-Level SPAD Thermal effects dynamics using Lindblad master equation.

(b) Qiskit noise simulation results. Here, the initial state is $|000\rangle$ and the observable state is $|001\rangle$.

Figure 3: Comparison of SPAD thermal effect simulations for the two-level system: (a) continuous-time dynamics showing the relaxation toward thermal equilibrium, and (b) discrete gate-based approximation across various temperatures

In this simulation, the initial state is $|000\rangle$, while the observable state is $|001\rangle$. Qiskit's Estimator [16] is used for calculating expectation values. At 173 K, the probability of thermal dark counts is negligible. When the temperature is increased further from 273 K to 303 K, a significant increase in thermal dark count probability is observed, as the charge carriers get enough external thermal energy to break the covalent bonds and become free. The thermal dark count probability saturates as the system evolves in time, which is in agreement with previous work [2, 3].

Another important approach to modelling SPAD dark counts is based on Qiskit's circuit-based noise model [9]. To simulate thermal noise in a Qiskit circuit model, we use Qiskit's `thermal_relaxation_error`, which introduces noise in the system. Figure 3(b) shows the Qiskit gate-based noise simulation for the SPAD detector. The probability of thermal dark counts at 173 K is flat, indicating zero probability of dark counts at low temperatures. As we increase the temperature, we observe a non-zero probability of dark counts. At room temperature, from 273 K to 303 K, a consistent nonzero probability of observing thermal dark counts is observed, indicating that the SPAD will always have a finite probability of thermal dark counts. The simulation results show close agreement with the previous Lindblad noise simulation and with observations from previous studies [2, 3]. As it can be seen from the quantum simulation results of thermal dark counts are in the range of 40 to 50 kHz at 273 K and around 222 kHz at 303 K, which is in agreement with previous results, targeting LEO missions. This simulation work will be very useful input for future space mission target-

ing LEO region as well as for deep space missions for single photon detection for quantum communication.

4 Summary and Future Work

A single-photon avalanche diode is modelled as a two-level system, and thermal dark counts are simulated for the first time using quantum computing via Qiskit. This approach helps to mimic the quantum nature of SPAD and photon interaction at the atomic scale. Dark counts are observed as the quantum-state expectation value. It is further observed that the dark count simulation shows the same behaviour observed in previous experimental works.

Furthermore, a more sophisticated approach would be to simulate this SPAD as a three-level system, examine its dynamics under radiation damage, accounting for afterpulsing and radiation-induced defects, and run the exact simulation on IBM's real Quantum hardware.

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