Improvement of Aerosol Extinction Profiles from Raman Lidar

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Abstract: In this work, we revise the precise retrieval of extinction from Raman Lidar data. This is traditionally challenging, as the derivative of a function with height-increasing noise is sought. Hence, we apply different regularization techniques for an example of the N2 387 nm profile in the Arctic under daylight conditions. Due to the decreasing lidar data quality, we split the retrieval into different intervals. This allows each interval to be smoothed individually. Although the final extinction profile depends on the chosen method, the interval selection and numerical parameters, we show that a trustful solution can be obtained even in altitudes in which a non-regularized solution will fail.

1. Introduction

The solution according to Ansmann [1] for the aerosol extinction coefficient α_0 is

$$\alpha_0(R) = \frac{\frac{\mathrm{d}}{\mathrm{d}R}\left[\ln \frac{N_{\mathrm{Mol}}(R)}{S_{\mathrm{Ra}}(R)}\right] - \alpha_{\lambda_0}^{\mathrm{Ray}}(R) - \alpha_{\mathrm{Ra}}^{\mathrm{Ray}}(R)}{1 + \left(\frac{\lambda_0}{\lambda_{\mathrm{Ra}}}\right)^{å(R)}} \quad (1)$$

where S_{Ra} ($=P_{\mathrm{Ra}}\ R^2$) is the range corrected Raman lidar signal, N_{Mol} the air number density and λ_0, λ_{Ra} the wavelengths of emission and Raman scattering. We know that the method [1] allows the determination of the extinction and backscatter coefficients without any assumptions of the Lidar ratio. Furthermore, uncertainties concerning the Ångström exponent (å) hardly affect the error of Eqn. 1. Ansmann [1] observed that a deviation from the Ångström exponent of less than 1 results in a relative error of < 5 %.

However, when applying method [1], we have to face other difficulties. The main problem is that the inelastic Lidar signal is significantly lower than the elastic Lidar signal. By spectral theory, on average, only 1 out of 1000 scattering is inelastic [2]. This is especially a problem in regions like the Arctic. Here, we have to handle noise from daylight in every measurement between March and October.

For the evaluation of Eqn. 1, we need to differentiate this weak and noisy Lidar signal. An example of such a signal is shown in Fig.1(a). In this case, ordinary numerical differentiation cannot give adequate results. Furthermore, smoothing of the Lidar signal has to be applied carefully, as over-smoothing would cancel out any information about the gradient. Smoothing is especially difficult since the signal-to-noise ratio is not constant and decreases with height.

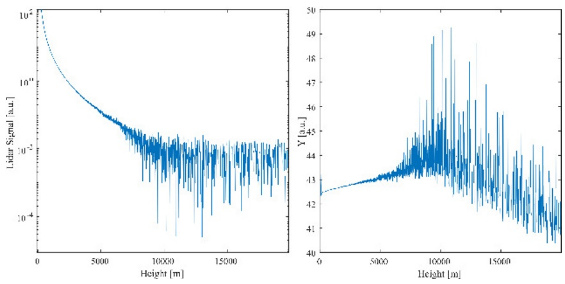

Figure 1. (a) Left: Signal P_{387} from 02.08.2019, 16:02 UT. (b) Right: Corresponding term Y.

Hence, for measurement by daylight, the method [1] is often not applicable or can only be applied for low altitudes, typically (700-2500 m) in our case.

We observe that the signal-to-noise ratio of every signal decrease with height. This effect also transmits into the term

$$Y(R) = \ln \frac{N_{\mathrm{Mol}}(R)}{P_{\mathrm{Ra}}(R)\cdot R^2} = -\ln \frac{P_{\mathrm{Ra}}(R)\cdot R^2}{N_{\mathrm{Mol}}(R)} \quad (2)$$

that we need to differentiate where $X(R) = \frac{d}{dR}Y(R)$ is the derivative. The term Y evaluated for our example from Arctic summer can be seen in Fig. 1(b). We suggest using regularization technique with a regularization parameter, which depends on the noise level, for the differentiation of Eqn. 2. Hence, if the noise level is not constant, the regularization parameter cannot be optimal for every part of the solution. Thus, we consider splitting the data *Y* into smaller pieces (splitting of interval (SI)), differentiating them piecewise, and sticking the solution together.

Finally, we want to mention an a priori criterion for the data quality. The extinction coefficient states how much (percentage) of the light is absorbed or scattered from particles at height *R*. Thus, this value has to be always positive. Looking into Eqn. 1 we see that also the derivative of Eqn. 2 has to be positive, even bigger than

$$\alpha_{\lambda_0}^{\mathrm{Ray}}(\mathrm{R}) + \alpha_{\mathrm{Ra}}^{\mathrm{Ray}}(\mathrm{R}) \quad (3)$$

Hence, a monotonically increasing signal *Y* is expected. For example, looking at Fig. 1(b), we deduce that for heights $R > 11000$ m, no more information is contained in the Raman signal.

2. The Basic Algorithms

We are using the well-known ordinary iterative Tikhonov-Phillips regularization (ITPR), the iterative Levenberg-Marquardt regularization (ILMR) and L-curve regularization parameter choice as fundamental methods, see e.g. [3-6].

The following four basic methods were used to compute an AOD using the analog signal P387A:

1. LMR over the interval [750 m, 10000 m].
2. ILMR
3. ITPR
4. LMR with SI determined a-priori with the cumulative sum from the Klett solution [7] $\beta_{\mathrm{Aer}}(R) * LR(R)$, see below.

Any signal *P* is given as a discrete point-set $P(R_i)$, $i=1,\dots,n$. In fact there is an equidistant grid $\{R_0,\dots,R_n\}$ with a grid length *h*. We need a discretization of *Y* as well. We approximate *Y* (*X* in the same way) as a piecewise constant

$$Y(R) \approx \sum_{i=1}^{n} y_i \phi_i(R) \quad (4)$$

with the base functions $\{\Phi_1, \dots, \Phi_n\}$, where Φ_i is 1 inside the interval $[R_{i-1}, R_i]$ and 0 outside.

But we need to consider smaller fractions of such intervals. The idea is to choose intervals with 'good' L-curves, i.e., a curve with L-shape and strong curvature. This should ensure that a good regularized solution could have been determined. We separate the interval of interest $[R_s, R_t]$ into smaller fractions

$$[R_{s_1}, R_{s_2}], [R_{s_2}, R_{s_3}], \dots, [R_{s_{N-1}}, R_{s_N}] \quad (5)$$

with $s = s_1 < s_2 < \dots < s_N = t$. Since the signal-to-noise ratio is non-constant, we cannot expect to find a regularization parameter that fits well to every part of the solution. Hence, by SI we may compute different regularization parameters for each interval. Sticking the solutions together might be an improved result.

Although this turns out to be very promising, especially for high altitudes, there are two issues. The first problem is to decide how to split the interval. At least two a-priori and one a-posteriori choices are possible.

1. Based on the noise of the data, choose an interval separation, s.t. every single interval has approximately the same noise level.
2. Based on the cumulative sum of the extinction solution $\alpha_{\mathrm{Aer}}=\beta_{\mathrm{Aer}} * LR$ from Klett's method (and the corresponding elastic lidar signal), determine an interval separation, such that

$$\sum_{j=s_i}^{s_{i+1}} \overline{\alpha}(R_j) \quad (6)$$

   is constant for every $i=1,\dots,N-1$. That is, in every interval, there may be the same amount of information (extinction). As we only use the elastic signal to choose the intervals, it is not necessary to use a precise Lidar Ratio (*LR*).
3. An a-posteriori interval choice method from Pornsawad [5] which introduces the interval length as an additional regularization parameter. We define

$$I_{\tilde{t},n} = [R_{\tilde{t}}, R_{\tilde{t}+n}] \quad (7)$$

   for $t \in \{s, \dots, t\}$ and $n < t - \tilde{t}$.

For more algorithm details, see [6]. ILMR and ITPR are based on the a-posteriori choice rule in point 3. Method 4 follows the idea from point 2.

3. Results

In this case study aerosols in heights above 5000 m were observed. Using the Klett method [7], the following AODs were derived with a constant and arbitrary Lidar Ratio of 50 sr, Fig. 2(a). As the AOD is the cumulative sum over the discrete solution, we expect its value to increase stronger the higher the extinction coefficient is. Hence up to 5 km, only low extinction occurs. Depending on the time step, beyond 5 to 7 km, the AOD increases significantly faster up to 15 km. Hence, in this interval, we expect aerosols. Now we will try to obtain similar solutions for the AOD from the Raman signal. To determine adequate results at these high altitudes, we consider SI. We will compare all three SI-methods from the previous section.

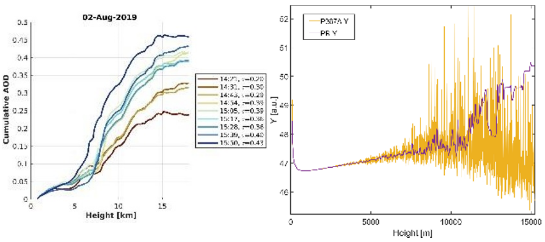

Figure 2. (a) Left: AOD for λ=355 nm. (b) Right: Term *Y* calculated with P387A and PR.

We decided on using the signal P387A as well as an arbitrarily smoothed signal PR. Up to about 11 km altitude this smoothed signal approximates P387A. Behind this altitude the signal is dominated by noise, see Fig. 3. Also, we see a clear influence of the overlap in the first heights. Hence, we set the minimum height to 750 m.

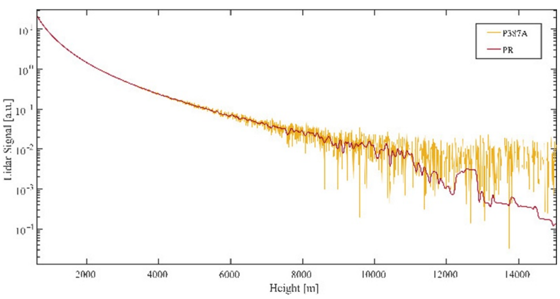

Figure 3. Lidar signal P_{387} analog and smoothed

The four basic methods were used to compute an AOD using the analog Signal P387A. Furthermore, one solution with the smoothed signal PR and the LMR was calculated over the whole interval [750 m, 10000 m]. Fig. 4 shows the AOD calculated with the different methods and SI for the time step 15:17 UT. The regularization parameters for the methods with multiple intervals can be read up in Tables 1-3.

Table 1. SI with ILMR for λ= 355 nm.

Interval [m]	Regularization parameter (number of iteration steps)	Curvature at L-Corner
[697, 2860]	19	8.93
[2860, 4329]	10	4.51
[4329, 6023]	8	0.95
[6023, 6817]	4	0.12
[6817, 8661]	6	0.052
[8661, 10220]	3	-0.0017

Table 2. SI with ITPR for λ = 355 nm.

Interval [m]	Regularization parameter	Curvature at L-Corner
[697, 2860]	6.43e+03	7.99
[2860, 5266]	8.70e+04	2.41
[5266, 6847]	1.83e+05	0.35
[6847, 9403]	7.39e+05	0.05
[9403, 8661]	1.00e+07	-0.0003

Table 3. SI derived from the cumulative sum of the solution from Klett method. The regularization parameters correspond to the solutions computed with the LMR.

Interval [m]	Regularization parameter (number of iteration steps)
[754, 6008]	12
[6008, 7792]	10
[7792, 8481]	15
[8481, 9995]	10

We observe that every solution produces a similar AOD at the end of the interval. However, after 10000 m, some solutions already produce negative results.

As expected, the solutions calculated for the whole interval tend to be over-smoothed and produce nearly constant extinction values. The iterated Tikhonov method does this as well. Every solution produces higher AOD values than the Klett method. This must not be an error but a sign that the Lidar ratio from Klett's method systematically underestimates the true extinction.

Comparing the quality of the solution, we observe that the iterated Levenberg-Marquardt method and the method that chooses the SI according to Klett's AOD fit best to the AOD from Klett's method. Both methods are able to determine a critical height between 7000 m and 8000 m, where the AOD starts to increase faster.

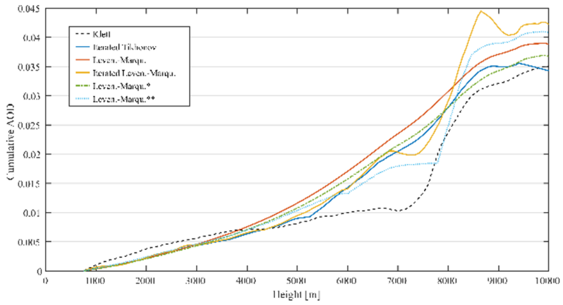

Figure 4. Different solution for the AOD for λ= 355 nm. For Leven.-Marqu.* the signal PR instead of P387A was used. For Leven.-Marqu.** the SI was chosen according to Table 3.

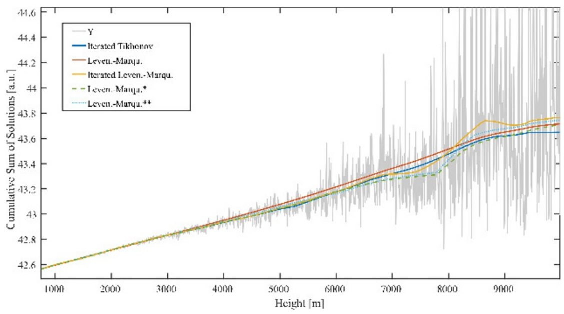

Figure 5. Cumulative sum of different solutions X for λ = 355 nm compared with Y. For Leven.-Marqu.*, the signal PR instead of P387A was used. For Leven.-Marqu.**, the SI was chosen according to Table 3.

4. Summary and Outlook

The studies showed that regularization of the Ansmann algorithm [1] is very promising. Regularization allows us to obtain adequate results in higher altitudes with weaker signals than classical methods do.

Since the signal-to-noise ratio of the signal decreases with height, we considered splitting the interval into smaller pieces and applying the regularization methods piece-wise.

We deduce that the regularization methods tend to produce over-smoothed solutions if the interval of interest is too large. Although SI is useful, solutions depend strongly on the particular SI. Strategies like the iterated Levenberg-Marquardt method [5] or the SI based on Klett's solutions [7] turn out to produce very promising results. Whether a solution is over-smoothed can also be checked by testing how well the cumulative sum of the derivative X approximates the given data Y. In Fig. 5, this is shown for the proposed methods. The different methods for SI should be applied and compared. Information on the behaviors of various solutions may help to determine the best SI.

Finally, we point out that due to the weak inelastic Lidar signal, Ansmann's method is restricted to certain heights. Regardless of which regularization method is used, if the signal does not provide information anymore, no adequate solution can be determined. We found adequate solutions for heights up to 10 km. However, choosing the correct algorithms and parameters for the problem is not yet easy and has to be further improved and automated.

5. References

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