A Graph-Theoretic Model for Battery Monitoring Systems Using Weighted Friendship Graphs and Harary Index Evaluation

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Abstract. This study presents a graph-theoretic approach to model the communication architecture of a Battery Monitoring System (BMS) using an undirected, edge-weighted Friendship Graph. In this representation, each vertex corresponds to a battery module or sensor, while a central vertex symbolizes the controller system that coordinates overall communication. The edges denote communication links, and their associated weights capture two critical parameters, they are the physical distance between nodes and the effective distance determined by the infrared (IR) sensor's radiation range. To assess the efficiency of communication within this architecture, the Harary Index is adopted and extended to incorporate these edge weights. The Harary Index, a well-known topological invariant in graph theory, reflects the closeness of vertices and thus serves as a measure of communication robustness and fault tolerance. By calculating the Harary Index for both physical distances and radiation distances, the study derives an average index that quantifies the monitoring efficiency of the system. This combined measure aids in predicting faults, improving communication reliability, and minimizing the risk of damage in the BMS. The findings emphasize the determination of an optimal placement strategy for sensors and the controller unit, thereby offering practical guidelines for designing efficient BMS communication networks.

1 Introduction

Battery Monitoring Systems (BMS) are essential components in modern energy applications, playing a decisive role in ensuring the safety, efficiency, and prolonged life of batteries. They are widely deployed in electric vehicles (EVs), renewable energy storage systems, and other battery-powered technologies where reliability is paramount. As the complexity and scale of battery arrays continue to grow, the demand for advanced monitoring frameworks becomes even more critical. Conventional BMS designs focus on monitoring parameters such as voltage, current, and temperature, yet with increasing energy densities and modular configurations, there arises a pressing need for more robust, fault-tolerant, and computationally efficient communication structures.In this context, the present study introduces a graph-theoretic modelling approach to capture the communication architecture of a BMS. Specifically, a Friendship Graph structure is employed to represent the interaction between battery modules, sensors, and the central control unit. In this representation, each vertex denotes a sensor or battery module, while a central vertex signifies the controller that governs the overall communication. Edges are used to indicate the communication links, and they are assigned weights corresponding to physical distances between modules as well as distances based on infrared (IR) sensor radiation ranges. This dual-weight modelling allows for a more accurate reflection of real-world sensor placement and communication efficiency in BMS networks.To evaluate the effectiveness of such a model, the study employs the Harary Index, a topological index traditionally used to measure graph connectivity and closeness. The Harary Index is extended in this study to incorporate edge weights, enabling it to capture not only the logical connectivity but also the physical and radiation-based constraints inherent to BMS communication systems. By calculating the Harary Index for both distance measures and averaging the results, the model provides a quantitative measure of monitoring efficiency, fault prediction capability, and potential damage prevention within the system. This average Harary Index serves as a decision-making tool for identifying optimal sensor and controller placements, thereby offering guidelines for designing cost-effective and reliable BMS architectures.

2 Literature Review

Graph theory has long been a powerful mathematical tool for modelling and analysing the structural properties of networked systems. It has been widely applied in computer communication networks[14],transportation systems[13], and wireless sensor networks[1], where connectivity, fault tolerance, and efficiency are of primary concern.

In the field of Battery Monitoring Systems (BMS), researchers have proposed multiple architectural models to address the challenges of safety, reliability, and scalability. Conventional frameworks include centralized architectures, where all data is processed at a central controller [12],distributed architectures that assign monitoring responsibilities across multiple controllers[5], and modular architectures that strike a balance between scalability and efficiency [6], Comparative surveys highlight that modular and distributed approaches offer higher fault tolerance but increase communication and synchronization costs, whereas centralized approaches, though simple, become bottlenecks as system size increases [11].Recent surveys on BMS communication topologies [7],[8] emphasize the need for network-aware design strategies that optimize both monitoring efficiency and fault resilience. For instance, [7]analysed wired and wireless BMS architectures, showing that network topology directly impacts latency and energy efficiency. Similarly, [8]surveyed fault-diagnosis methods for lithium-ion batteries, identifying gaps in communication-efficient and sensor-based monitoring strategies.The Harary Index, introduced by [15], measures the closeness of a graph by summing the reciprocals of shortest-path distances between vertex pairs. Initially employed in chemical graph theory[9], it has since been adapted to evaluate communication efficiency[4],[2]. Studies show that networks with higher Harary Index values demonstrate stronger connectivity and robustness, making it suitable for assessing fault-tolerant systems. However, its application to weighted graphs, particularly in sensor-based BMS networks, has received little attention. This represents a significant research gap that the present study aims to address.The Friendship Graph, also called the Dutch Windmill Graph, introduced by Erdős, Rényi, and Sósin [3], is another relevant structure. Its central vertex connected to multiple triangular subgraphs naturally represents centralized control systems where a master controller interacts with modular sensor-battery units. Although it has been studied in theoretical graph research[10], its adoption for modelling BMS communication remains unexplored.While surveys exist on BMS architectures [7], [7], communication protocols[11]and fault-detection strategies [8], there is a lack of studies that integrate graph-theoretic measures such as the Harary Index with practical BMS network design. This work bridges that gap by proposing a Friendship Graph-based model extended with weighted Harary Index analysis to quantify monitoring efficiency and fault tolerance in sensor-based BMS architectures.

3 Methodology

3.1 Graph Construction

In this study, the Battery Monitoring System (BMS) is represented as an undirected, edge-weighted Friendship Graph. A Friendship Graph is formed by connecting several triangular subgraphs to a single central vertex, creating a windmill-like structure. This property makes it an excellent choice for modelling BMS architectures, where the controller unit acts as the central coordinating node and each battery–sensor pair forms a triangular wing with the controller.Each vertex in the constructed graph represents a component of the system: either the central controller, a battery module, or a sensor. Every wing of the graph consists of three nodes—one controller, one battery, and its corresponding sensor. The edges represent the communication links among these components. Specifically, the controller communicates with both the battery and the sensor, while the battery and sensor are also directly connected to exchange data. This triangular structure not only captures the monitoring relationship but also ensures redundancy in communication, as alternative paths exist between nodes if one link fails.The edges in the graph are assigned weights that correspond to two practical measures: (i) the physical distance between components, such as the wiring length or placement in the system, and (ii) the infrared (IR) radiation distance, which reflects the sensing or communication range of IR-based sensors. Considering both physical and radiation distances allows the model to capture real-world constraints in BMS communication.The graph is constructed step by step by first defining the central controller node, then pairing each battery with its sensor, and finally forming triangular connections with the controller. As more battery–sensor modules are added, the Friendship Graph expands with additional wings, while the controller remains the single point of coordination. This results in a scalable and hierarchical representation of the BMS network.This modelling approach has several advantages. It reflects the centralized nature of most BMS designs, provides a clear mathematical structure for analysing communication efficiency, and introduces a systematic way to evaluate fault tolerance. Since each triangular wing has multiple communication paths, the network can withstand individual link failures while maintaining overall monitoring functionality. Moreover, by incorporating weighted distances, the model can be used to optimize the placement of sensors and controllers, ensuring efficient communication with minimal delays and reduced chances of data loss.

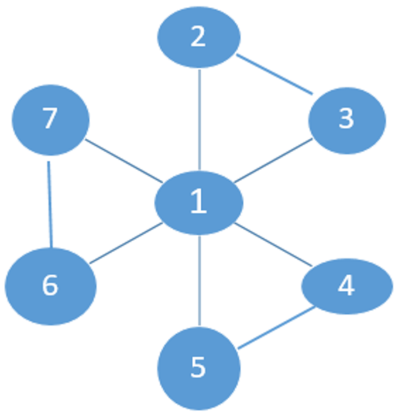

Fig. 1. Friendship graph with 3 wings

Fig. 2. Friendship graph modelled BMS

Overall, constructing the BMS as an undirected, edge-weighted Friendship Graph provides both a realistic and mathematically tractable framework for analysing and improving the design of monitoring systems in large-scale battery applications.

3.2 Edge Weights

In the proposed BMS graph model, each edge is assigned a weight to represent the cost of communication between nodes. Two types of edge weights are considered.

1. Physical Distance: This weight corresponds to the actual spatial separation between components, such as wiring length between a battery and the controller. It reflects installation complexity, energy loss due to longer connections, and potential delays in communication. Shorter physical distances generally indicate more efficient and reliable communication.
2. Radiation Distance: This weight captures the effective communication cost when infrared (IR) sensors are used. Unlike physical distance, radiation distance depends on factors such as line-of-sight, orientation, signal attenuation, and sensor strength. Even nearby components may face higher communication costs if the IR path is obstructed or weak.

Both weights can be calculated from placement geometry or estimated from sensor specifications and environmental conditions. Together, they provide a realistic measure of communication efficiency in the BMS network. Physical distance highlights wiring and structural constraints, while radiation distance reflects sensing and signal-related limitations.By incorporating both measures, the model ensures a balanced evaluation of system performance. This dual-weight approach enables designers to optimize the placement of sensors and controllers for minimal communication cost, higher fault tolerance, and improved monitoring efficiency.

3.3 Weighted Harary Index

The Harary Index H(G) for a graph G is traditionally defined as:

$$H(G) = \sum_{u,v \in V} \frac{1}{d(u,v)} \tag{1}$$

where d(u, v) is the shortest path distance between the vertex u and v.

In this part Harary index$H_{\{physical\}}(G)$ is calculated using physical distance as edge weights and $H_{\{radiation\}}(G)$ is calculated using radiation distance as edge weights

graph-theoretic modelling of the Battery Monitoring System (BMS), a Python-based tool has been developed. This tool performs four key tasks:

1. Constructs a Friendship Graph with n specified wings, where each wing corresponds to a battery–sensor–controller triad.
2. Accepts edge weights for both physical distances and radiation-based communication distances.
3. Calculates weighted Harary Indices for efficiency analysis.
4. Provides graph visualization to illustrate component connectivity and system topology.

The Harary Index of a weighted graph is given by:

$$H_w(G) = \sum_{u,v\in V} d_w(u,v) \qquad (2)$$

where $d_w(u,v)$ denotes the shortest weighted distance between nodes u and v.

To utilize the full efficiency of the dual influence of wiring layout and communication quality, this study introduces a weighted average Harary Index, denoted as:

$$H_w(G) = \alpha\, H_{physical}(G) + (1-\alpha)H_{radiation}(G) \qquad (3)$$

Here:

- $H_{physical}(G)$ is the Harary Index computed using edge weights based on physical distances between BMS components.
- $H_{radiati\ on}(G)$ is the Harary Index computed using radiation distances, which reflect infrared signal quality, line-of-sight, and robustness of wireless communication.
- The weighting parameter is defined as: α ∈ [0,1]
- The parameter α provides flexibility for system design priorities. For example:
    - A higher α emphasizes minimizing wiring length and installation cost.
    - A lower α prioritizes communication clarity and reliability in wireless or IR-based setups.

(i.e) The choice of the weighting factor α ∈ [0,1] in $H_w(G)$ depends on how important the physical or radiation distance in BMS design. In this study, an equal weighting of α=0.5 is initially adopted to balance both perspectives.

This implementation effectively bridges theoretical graph modelling with practical BMS design, offering both quantitative performance metrics and visual insights into system efficiency, reliability, and fault tolerance.

Algorithm-Weighted Friendship graph and Battery Monitoring System

1. Input
    a. Number of wings – n
       (i.e) Number of batteries and sensors
    b. Physical distance between vertices
       (i.e) Distance between
          control unit and sensor
          Sensors and corresponding batteries
          Control unit and batteries
    c. Radiation distance
       The distance between battery and sensor
2. Weighted Friendship graph is constructed based on input distance
3. Shortest paths between all pairs of vertices is calculated.
4. Harary index based on physical distance and radiation distance is calculated using parameter
5. Weighted Harary index is calculated using parameter α.
6. Output is interpreted and graph visualization is done.

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4 Results and Discussion

The proposed method is validated through several sample configuration of battery – sensor- controller network. Each configuration modelled as a friendship graph with edge weight representing either physical distance (wire length) or radiation distance based on communication. The weighted Harary Index is computed using

$$H_w(G) = \alpha\, H_{physical}\,(G) + (1-\alpha)H_{radiation}\,(G) \qquad (4)$$

And $\alpha = 0.5$

Table 1 Average Harary Index Value calculation

Distance between Sensor and Battery	Distance between Sensor and central unit	Distance between central unit and Battery	RadiationDistance between Sensor and Battery	AverageHarary index value
1	2	3	1	7.95
2	3	4	2	4.9821
1	1	4	1	11.75
2	1	4	1	10.375
3	4	2	1	6.6625
3	4	4	3	4
2	4	4	3	4.125
3	2	3	1	7.45

4.1 Effect of Sensor and Battery Placement

The results confirm that the placement of batteries and sensors creates a significant impact on monitoring efficiency. (ie) Higher value of $H_w(G)$ corresponds to better communication flow, reduced path redundancy, and greater fault tolerance. Conversely, lower value of $H_w(G)$ indicates poorly distributed sensor placements indicating vulnerabilities in both wiring efficiency and communication quality.

4.2 Relationship Between Edge Weights and Harary Index

The inference of the proposed method witnesses an inverse relationship between the edge weights (distances) and the Harary index:

- If the physical distance between the nodes, ie the wiring length increases then the $H_{physical}\,(G)$ decreases. It is trivial that the long wiring can initiate latency and increase in cost. This reduces the connectivity and hence lowers the efficiency of the system.
- Similarly, in case of wireless communication, the $H_{radiation}\,(G)$ depends strongly on the strength of links. When communication is clear line-of-sight, strong infrared or RF links, distances are effectively small, and $H_{radiation}\,(G)$ remains high. For instance, when line-of-sight is obstructed or infrared communication is degraded, the calculated Harary Index drops sharply. Precisely, the Harary index decreases whenever edge weights increase, whether due to longer wiring or weaker wireless links.

- This highlights the Harary index as a sensitive global indicator of network efficiency, directly reflecting physical constraints like cost, latency, and signal degradation.

4.3 Fault Simulation and Sensitivity Analysis

The tool was also used to model fault conditions such as:

- Removal or failure of battery, sensor considered as deficiency in node which decreases $H_w(G)$ due to connectivity disruption.
- In case of signal degradation either due to environmental attenuation or due to the fault leads to a noticeable drop in $H_{radiation}\ (G)$
- The weighting parameter α acts a tuning knobwhich shift system emphasis between wiring cost minimization (α→1) and communication robustness (α→0).

These simulations demonstrate the model's ability to predict performance under both design choices and unexpected operational disturbances.The values of the weighted Harary Index with respect to α ∈ [0,1] is determined. This demonstrate how the distance between the sensor and battery decides the setup of this network along with αvalue. Given below is the result obtained when considering α =0.25, Sensor – battery distance -3, Sensor- Central unit distance – 4, Central unit – Battery distance – 4, Sensor- battery radiation distance – 3.(Values in sixth row of above table).

```
--- Fault Simulation ---
After removing B1 → Harary Index: 2.9167
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--- Signal Degradation ---
Original Radiation H : 4.0
Degraded Radiation H: 2.0
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--- Alpha Sensitivity Analysis ---
Alpha = 0.00 → H = 4.0
Alpha = 0.25 → H = 4.0
Alpha = 0.50 → H = 4.0
Alpha = 0.75 → H = 4.0
Alpha = 1.00 → H = 4.0
```

4.4 Applications and Implications

1. Sensor Placement Optimization
   - By comparing Harary Index values across different layouts, the tool provides clear guidance on where to position sensors and controllers for maximal efficiency.
   - This is particularly important in large-scale BMS installations where poor sensor placement could cause monitoring bottlenecks.
2. Fault Prediction and Early Warning
   - A sudden drop in $H_w(G)$serves as an early indicator of faults, such as damaged sensors, degraded IR communication, or disconnected battery modules.
   - This makes the method highly applicable in predictive maintenance frameworks for EVs, smart grids, and renewable energy storage systems.
3. System Design and Cost Efficiency

- o The framework provides engineers with a computational decision-making tool to balance wiring cost and wireless reliability.
- o Instead of trial-and-error physical setups, designers can use simulations to identify cost-effective yet robust architectures before hardware implementation.

Conclusion

This work presents a graph-theoretic model for Battery Management Systems by representing components as vertices of friendship graph with edge weights based on physical and radiation distances. The approach highlights how sensor placement and network configuration influence efficiency, robustness, and fault tolerance. The model also simulates failures and signal degradation, offering practical insights for system optimization. It enables cost-effective, reliable BMS design and can be extended with dynamic topologies, real-time feedback, and hardware validation in future studies.

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