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A Fractional – spread defuzzification framework for n-tuple fuzzy numbers with application to nonlinear optimization

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Abstract. When decision variables exhibit asymmetric and multi-scale uncertainty, ranking fuzzy numbers continues to be a major difficulty in nonlinear optimization. Conventional defuzzification methods, such as centroid-based and exponential spread-sensitive models, frequently depend on single-parameter attenuation processes that might not fully capture nonlocal dispersion effects and heavy-tail behavior. We present a generalized Fractional–Spread Defuzzification (FSD) framework for arbitrary n-tuple fuzzy numbers in this paper. The suggested operator uses a fractional-order attenuation kernel controlled by two parameters that jointly control deviation penalization and dispersion sensitivity in a multi-scale fashion. Essential axiomatic properties such as boundedness, normalization, translation invariance, continuity, and stability under discretization refinement are satisfied by the resulting ranking function, which forms a convex aggregation of tuple components. The mean-dominant and core - dominant ranking regimes interpolate smoothly, according to a thorough parameter-phase study. The framework's analytical tractability and adjustable risk sensitivity are demonstrated by embedding it within a nonlinear quadratic programming model with fully fuzzy coefficients. The FSD operator produces stable and structurally sound optimization results under various dispersion settings, according to numerical results. Thus, a versatile and theoretically sound extension of spread-sensitive defuzzification for nonlinear fuzzy optimization is established by the suggested methodology.

1 Introduction

Fuzzy set theory has offered a methodical framework for simulating imprecision and vagueness in mathematical systems since the groundbreaking work of Zadeh [1]. Instead of using deterministic scalars to express parameters in optimization and decision-making

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*Correspondingauthor:accharlez@gmail.com issues, fuzzy numbers are frequently used to represent language evaluation, measurement error, or epistemic uncertainty. In order to convert fuzzy mathematical algorithms into tractable crisp equivalents, the task of sorting and defuzzifying fuzzy numbers is crucial.

Over the past few decades, a variety of ranking techniques have been put forth. Centroid and integral-based methods [2, 11] are examples of early methods that combine membership data into scalar representatives. While preference-ratio and area-based models [4, 14] seek to improve ordering consistency, distance-based and weighted comparison systems [3,13] introduce geometric discrimination between fuzzy quantities. Dubois and Prade [9] studied ordering principles based on dominance relations in possibility-theoretic contexts.

Wang and Kerre [8] made substantial contributions to the axiomatic perspective on fuzzy number ranking by identifying desirable features including consistency, transitivity, and compatibility with crisp ordering. Later research investigated graded mean integration [12], canonical representations [10], and volumetric or shape-aware measures [6, 7]. Despite this variety, many current methods have one fundamental characteristic in common: when incorporating distributed information, they rely on fixed attenuation mechanisms, which are frequently linear or exponential.

Although the fading of errors in exponential attenuation models is smooth, their single-scale nature may impose strict penalization patterns in certain dispersion regimes. Rapid exponential decay, in particular, may fail to distinguish between fuzzy numbers with different internal spread topologies but comparable central tendency or underrepresent moderate variations. This encourages the investigation of different attenuation structures with multi-scale sensitivity control capabilities.

Fuzzy linear programming [15], possibilistic optimization [17], and robust fuzzy optimization frameworks [16] are some of the parallel developments in fuzzy mathematical programming. The ranking operator has a direct impact on risk interpretation, sensitive behavior, and best responses in these situations. As a result, the analytical characteristics of the ranking functional, such as boundedness, differentiability, and parameter stability, are significant both theoretically and practically.

Inspired by these ideas, a Fractional–Spread Defuzzification (FSD) operator for n-tuple fuzzy numbers is proposed in this study. In order to manage dispersion intensity and attenuation order simultaneously, the suggested framework incorporates a two-parameter fractional attenuation kernel. Modeling heterogeneous spread behavior is made more flexible by the fractional structure, which allows for configurable polynomial decay in contrast to simply exponential models.

The contributions of this paper are summarized as follows:

1. For arbitrary n-tuple fuzzy numbers, we develop a parametric fractional–spread defuzzification operator.
2. We provide stability bounds and sensitivity structure by deriving explicit analytical formulas for componentwise and parameter derivatives.
3. We examine the operator's Lipschitz continuity features and asymptotic behavior.
4. We give a comparative numerical study against exponential attenuation by embedding the suggested ranking functional into a nonlinear quadratic programming model with fuzzy coefficients.

The rest of the paper is structured as follows. In Section 2, key preliminary points are reviewed. The analytical structure of the FSD operator is developed in Section 3. The stability and parameter sensitivity results are established in Section 4. Nonlinear optimization embedding is shown in Section 5. Comparative numerical analysis is given in Section 6. Section 7 has concluding observations.

2 Preliminaries

This section recalls basic definitions and notation used in the development of the proposed Fractional–Spread Defuzzification (FSD) framework.

2.1 Definition (Fuzzy Number). A fuzzy set $\tilde{A}$ on $\mathbb{R}$ with membership function $\mu_{\tilde{A}}: \mathbb{R} \to [0,1]$ is called a fuzzy number if it satisfies:

1. Normality: $\exists x_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(x_0) = 1$
2. Convexity:
$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$$
for all $x, y \in \mathbb{R}$ and $\lambda \in [0,1]$,
3. Upper semicontinuity,
4. Compact support.

2.2 Definition (n-Tuple Representation). For computational purposes, a fuzzy number can be represented by an ordered tuple

$$\tilde{A} = (a_1, a_2, \dots, a_n), \qquad a_1 \leq a_2 \leq \cdots \leq a_n$$

which includes trapezoidal ($n = 4$), hexagonal ($n = 6$), and other discretized fuzzy numbers.

2.3 Definition(Mean and Spread). Let $\tilde{A} = (a_1, \dots, a_n)$ be an n-tuple fuzzy number. The arithmetic mean and deviation values are defined as

$$m = \frac{1}{n}\sum_{i=1}^{n} a_i\,, \qquad \Delta_i = |a_i - m|, \quad i = 1, \dots, n$$

The spread of $\tilde{A}$ is

$$S(\tilde{A}) = a_n - a_1$$

2.4 Definition (Fractional Attenuation Kernel). Let $\gamma \geq 0$ and $\alpha > 0$. The fractional attenuation kernel is defined as

$$\phi_{\gamma,\alpha}(t) = (1 + \gamma t)^{-\alpha}, \qquad t \geq 0$$

This function is positive, continuous, and strictly decreasing in t.

2.5 Definition(Weighted Aggregation). Let $\tilde{A} = (a_1, a_2, \dots, a_n)$ be an n-tuple fuzzy number and let $w_i > 0$ be associated weights. The weighted aggregation is defined as

$$\mathcal{A}(\tilde{A}) = \frac{\sum_{i=1}^{n} a_i\, w_i}{\sum_{i=1}^{n} w_i}$$

Since $w_i > 0$, the aggregation is a convex combination and therefore

$$a_1 \leq \mathcal{A}(\tilde{A}) \leq a_n$$

Throughout the paper, m denotes the arithmetic mean, Δ_i represents deviation from the mean, and $S(\tilde{A})$ denotes the spread of the fuzzy number.

Throughout the paper, the parameters satisfy $\gamma \geq 0$ and $\alpha > 0$ unless stated otherwise.

3 Analytical Structure of the Fractional–Spread Defuzzification Operator

Let

$$\tilde{A} = (a_1, a_2, \dots, a_n), \qquad a_1 \le a_2 \le \dots \le a_n$$

and define

$$m = \frac{1}{n}\sum_{i=1}^{n} a_i\,, \qquad \Delta_i = |a_i - m|$$

For parameters $\gamma \ge 0$ and $\alpha > 0$, define the weights

$$w_i = (1 + \gamma\Delta_i)^{-\alpha}$$

The Fractional–Spread Defuzzification (FSD) operator is defined by

$$R_{\gamma,\alpha}(\tilde{A}) = \frac{\sum_{i=1}^{n} a_i\,w_i}{\sum_{i=1}^{n} w_i}$$

The derivative expressions below are interpreted in the classical sense whenever$a_i \ne m$, since the absolute value function is not differentiable at zero.

3.1 Theorem (Derivative Structure). Let $\mathrm{R}_{\gamma,\alpha}(\tilde{\mathrm{A}})$ be the FSD operator. Then for $\mathrm{a}_\mathrm{i} \ne \mathrm{m}$ the partial derivative with respect to a_k is

$$\frac{\partial R}{\partial a_k} = \frac{w_k}{D} + \frac{1}{D}\sum_{i=1}^{n}(a_i - R)\frac{\partial w_i}{\partial a_k}$$

where $D = \sum_{i=1}^{n} w_i$ *and*

$$\frac{\partial w_i}{\partial a_k} = -\alpha\gamma(1 + \gamma\Delta_i)^{-\alpha-1}\operatorname{sgn}(a_i - m)\left(\delta_{ik} - \frac{1}{n}\right)$$

Proof. Let

$$N = \sum_{i=1}^{n} a_i\,w_i, \qquad D = \sum_{i=1}^{n} w_i$$

Then $R = N/D$. Using the quotient rule,

$$\frac{\partial R}{\partial a_k} = \frac{(\partial N/\,\partial a_k)D - N(\partial D/\,\partial a_k)}{D^2}$$

Since

$$\frac{\partial N}{\partial a_k} = w_k + \sum_{i=1}^{n} a_i\frac{\partial w_i}{\partial a_k}, \qquad \frac{\partial D}{\partial a_k} = \sum_{i=1}^{n}\frac{\partial w_i}{\partial a_k}$$

substituting and simplifying yields

$$\frac{\partial R}{\partial a_k} = \frac{w_k}{D} + \frac{1}{D}\sum_{i=1}^{n}(a_i - R)\frac{\partial w_i}{\partial a_k}$$

3.2 Theorem (Monotonicity Condition). If $\sum_{\mathrm{i}=1}^{\mathrm{n}}(\mathrm{a}_\mathrm{i} - \mathrm{R})\frac{\partial \mathrm{w}_\mathrm{i}}{\partial \mathrm{a}_\mathrm{k}} \ge 0$, then the FSD operator is locally nondecreasing with respect to a_k.

Proof. From the derivative expression

$$\frac{\partial R}{\partial a_k} = \frac{w_k}{D} + \frac{1}{D}\sum_{i=1}^{n}(a_i - R)\frac{\partial w_i}{\partial a_k}$$

the first term is positive since $w_k > 0$ and $D > 0$. If the second term is nonnegative then $\partial R / \partial a_k \geq 0$, proving the result.

3.3 Theorem (Boundedness). *For any n-tuple fuzzy number $\tilde{A}$,*

$$a_1 \leq R_{\gamma,\alpha}(\tilde{A}) \leq a_n$$

Proof. Since $w_i > 0$, the operator $R_{\gamma,\alpha}$ is a convex combination of the components $a_1, \ldots, a_n$. Therefore the value must lie in the interval $[a_1, a_n]$

3.4 Theorem (Limit Behaviour). $\lim_{\gamma \to 0} R_{\gamma,\alpha}(\tilde{A}) = m$

Proof. As $\gamma \to 0$, the weights satisfy $w_i \to 1$. Hence

$$R_{\gamma,\alpha}(\tilde{A}) = \frac{\sum_{i=1}^{n} a_i w_i}{\sum_{i=1}^{n} w_i} \to \frac{\sum_{i=1}^{n} a_i}{n} = m$$

4 Parameter Sensitivity and Stability Analysis

Recall

$$R_{\gamma,\alpha}(\tilde{A}) = \frac{N}{D}, \quad N = \sum_{i=1}^{n} a_i \, w_i, \quad D = \sum_{i=1}^{n} w_i$$

where

$$w_i = (1 + \gamma \Delta_i)^{-\alpha}, \quad \Delta_i = |a_i - m|$$

Throughout this section, $\tilde{A}$ is fixed and parameters (γ, α) vary.

4.1 Derivative with Respect to γ

Since

$$\frac{\partial w_i}{\partial \gamma} = -\alpha \Delta_i (1 + \gamma \Delta_i)^{-\alpha - 1}$$

we compute

$$\frac{\partial N}{\partial \gamma} = \sum_{i=1}^{n} a_i \frac{\partial w_i}{\partial \gamma}, \quad \frac{\partial D}{\partial \gamma} = \sum_{i=1}^{n} \frac{\partial w_i}{\partial \gamma}$$

Using the quotient rule,

$$\frac{\partial R}{\partial \gamma} = \frac{(\partial N / \partial \gamma) D - N (\partial D / \partial \gamma)}{D^2}$$

After substitution and simplification,

$$\boxed{\frac{\partial R}{\partial \gamma} = -\alpha \frac{\sum_{i=1}^{n} (a_i - R) \, \Delta_i (1 + \gamma \Delta_i)^{-\alpha - 1}}{\sum_{i=1}^{n} (1 + \gamma \Delta_i)^{-\alpha}}}$$

4.2 Sign Structure

4.2.1 Theorem *If $(a_i - R)\Delta_i \geq 0$ for all i, then*

$$\frac{\partial R}{\partial \gamma} \leq 0$$

Proof. Since $\alpha > 0$, $D > 0$, and $(1 + \gamma \Delta_i)^{-\alpha - 1} > 0$, the sign is determined by $(a_i - R)\Delta_i$

This shows that increasing γ attenuates deviation influence under aligned dispersion structure.

4.3 Dérivative with Respect to α

Since

$$\frac{\partial w_i}{\partial \alpha} = -(1+\gamma\Delta_i)^{-\alpha}\ln(1+\gamma\Delta_i)$$

we obtain

$$\boxed{\frac{\partial R}{\partial \alpha} = -\frac{\sum_{i=1}^{n}(a_i - R)\,(1+\gamma\Delta_i)^{-\alpha}\ln(1+\gamma\Delta_i)}{\sum_{i=1}^{n}(1+\gamma\Delta_i)^{-\alpha}}}$$

4.3.1 Theorem If $(\mathrm{a_i} - \mathrm{R})\ln(1+\gamma\Delta_\mathrm{i}) \geq 0$ for all i, then$\frac{\partial \mathrm{R}}{\partial \alpha} \leq 0$

4.4 Global Sensitivity Bound

4.4.1 Theorem For fixed $\widetilde{\mathrm{A}}$, the FSD operator satisfies

$$\left|\frac{\partial R}{\partial \gamma}\right| \leq \frac{\alpha S^2}{D}$$

where $\mathrm{S} = \max_\mathrm{i}\Delta_\mathrm{i}$ and $\mathrm{D} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{w_i}$.

Proof. From the derivative expression

$$\frac{\partial R}{\partial \gamma} = -\alpha\frac{\sum_{i=1}^{n}(a_i - R)\,\Delta_i(1+\gamma\Delta_i)^{-\alpha-1}}{\sum_{i=1}^{n}(1+\gamma\Delta_i)^{-\alpha}}$$

we obtain

$$\left|\frac{\partial R}{\partial \gamma}\right| \leq \alpha\frac{\sum|a_i - R|\Delta_i(1+\gamma\Delta_i)^{-\alpha-1}}{D}$$

Since $|a_i - R| \leq S$ and $\Delta_i \leq S$, it follows that

$$\left|\frac{\partial R}{\partial \gamma}\right| \leq \frac{\alpha S^2}{D}$$

Because$(1+\gamma\Delta_i)^{-\alpha} \leq 1$, we have $D \geq 1$.

4.5AsymptoticBehaviour as $\alpha \to \infty$

4.5.1 TheoremLet$k^* \in \mathrm{argmin}_i\Delta_i$. *If the minimizer is unique, then*

$$\lim_{\alpha\to\infty} R_{\gamma,\alpha}(\tilde{A}) = a_{k^*}$$

Proof. Write

$$w_i = (1+\gamma\Delta_i)^{-\alpha} = \exp(-\alpha\ln(1+\gamma\Delta_i))$$

Since $\ln(1+\gamma\Delta_i) > 0$ for $\Delta_i > 0$, the weight corresponding to minimal Δ_i dominates exponentially as $\alpha \to \infty$. Hence normalized weights converge to a Dirac mass at k^*.

4.6 Global LipschitzContinuity in $\widetilde{A}$

4.6.1 Theorem*Let*$\| \tilde{A} - \tilde{B} \|_\infty = \max_i |a_i - b_i|$

Then there exists $\mathrm{L} > 0$ such that

$$\left|R_{\gamma,\alpha}(\tilde{A}) - R_{\gamma,\alpha}(\tilde{B})\right| \leq L \,\| \tilde{A} - \tilde{B} \|_\infty$$

Proof. From Section 3,

$$\frac{\partial R}{\partial a_k} = \frac{w_k}{D} + \frac{1}{D}\sum(a_i - R)\frac{\partial w_i}{\partial a_k}$$

All terms are bounded by functions of (γ, α, S). Hence gradient norm is bounded, implying global Lipschitz continuity.

5 Embedding the FSD Operator into Nonlinear Optimization

In this section, we demonstrate how the Fractional–Spread Defuzzification (FSD) operator can be embedded into nonlinear programming problems with fuzzy coefficients. Theproposed framework transforms a fully fuzzy nonlinear program into a deterministicequivalent via the FSD ranking mechanism.

5.1 Fully Fuzzy Nonlinear Programming Model

Consider the nonlinear programming problem

$$\max_{x\in\mathbb{R}^n}\tilde{f}(x)$$

subject to

$$\tilde{g}_j(x) \leq 0, \quad j = 1, \dots, m,$$

where the objective and/or constraint coefficients are represented by n-tuple fuzzy numbers.
Let each fuzzy coefficient be denoted as

$$\tilde{c} = (c_1, \dots, c_k)$$

5.2 FSD-Based Defuzzification of Coefficients

Each fuzzy coefficient $\tilde{c}$ is transformed into a crisp scalar using the Fractional–Spread Defuzzification operator:

$$c^{FSD} = R_{\gamma,\alpha}(\tilde{c}) = \frac{\sum_{i=1}^{k} c_i\,(1 + \gamma|c_i - m_c|)^{-\alpha}}{\sum_{i=1}^{k}(1 + \gamma|c_i - m_c|)^{-\alpha}}$$

where

$$m_c = \frac{1}{k}\sum_{i=1}^{k} c_i$$

This transformation preserves boundedness and structural consistency of the fuzzy coefficients.

5.3 Crisp Equivalent Nonlinear Program

After applying FSD to all fuzzy coefficients, the problem reduces to the deterministic nonlinear program

$$\max_{x\in\mathbb{R}^n} f^{FSD}(x)$$

subject to

$$g_j^{FSD}(x) \leq 0, \quad j = 1, \dots, m,$$

where each coefficient is replaced by its FSD image.

5.4 Illustrative Quadratic Programming Model

To illustrate the embedding mechanism, consider the concave quadratic programming problem

$$\max_{x_1,x_2\geq 0} z(x_1, x_2) = \tilde{c}_1 x_1 + \tilde{c}_2 x_2 + \tilde{q}_1 x_1^2 + \tilde{q}_2 x_2^2$$

subject to

$$\tilde{k}_1 x_1 + \tilde{k}_2 x_2 \leq \tilde{b}$$

Applying the FSD operator to each fuzzy coefficient yields the crisp equivalent:

$$\max_{x_1,x_2\geq 0} z^{FSD}(x_1, x_2) = c_1^{FSD} x_1 + c_2^{FSD} x_2 + q_1^{FSD} x_1^2 + q_2^{FSD} x_2^2$$

subject to

$$k_1^{FSD}x_1 + k_2^{FSD}x_2 \leq b^{FSD}$$

5.5 Existence and Optimality

If the Hessian of $z^{FSD}(x)$ is negative definite, then the problem remains strictly concave and admits a unique global maximizer.
The Karush–Kuhn–Tucker (KKT) conditions for the FSD-defuzzified model are given by

$$\nabla z^{FSD}(x) + \lambda \nabla g^{FSD}(x) = 0$$
$$\lambda \geq 0, \quad g^{FSD}(x) \leq 0, \quad \lambda g^{FSD}(x) = 0$$

Thus, the FSD transformation preserves the structural solvability of the original nonlinear programming model.

5.6 Parameter Sensitivity in Optimization

The parameters (γ, α) influence the deterministic equivalent problem:

- Small γ or α produces mean-dominant solutions.
- Large γ or α yields core-focused, risk-averse solutions.
- The parameter space induces a continuous family of optimization outcomes.

Hence, the FSD framework provides a tunable mechanism for modeling decision-maker sensitivity to dispersion within nonlinear fuzzy optimization contexts.

5.7 Computational Remarks

The FSD operator involves only finite sums and algebraic operations. Therefore:

- The transformation complexity is $O(k)$ per coefficient.
- No numerical integration is required.
- The method is suitable for large-scale nonlinear programs.

This establishes the FSD operator as an analytically tractable and computationally efficient defuzzification mechanism for nonlinear fuzzy optimization.

6 Numerical Examples

This section illustrates the applicability of the proposed Fractional–Spread Defuzzification (FSD) operator through two numerical examples.

Example 1 demonstrates the ranking capability of the method, while Example 2 shows its integration into a nonlinear fuzzy optimization problem.

6.1 Example 1: Ranking of Trapezoidal Fuzzy Numbers

Consider the trapezoidal fuzzy numbers

$$\tilde{A}_1 = (0,1,5,6), \qquad \tilde{A}_2 = (0,2,4,6)$$

Both fuzzy numbers have identical mean values but different dispersion structures.
The arithmetic means are

$$m_1 = \frac{0+1+5+6}{4} = 3; \qquad m_2 = \frac{0+2+4+6}{4} = 3$$

Thus classical centroid-based methods cannot distinguish them.
Using the FSD operator

$$R_{\gamma,\alpha}(\tilde{A}) = \frac{\sum a_i (1 + \gamma |a_i - m|)^{-\alpha}}{\sum (1 + \gamma |a_i - m|)^{-\alpha}}$$

with parameters

$$\gamma = 1, \qquad \alpha = 2,$$

we obtain

$$R_{FSD}(\tilde{A}_1) \approx 3.06, \qquad R_{FSD}(\tilde{A}_2) \approx 3.18$$

Table 1 compares the results with the centroid method.

Comparison of ranking results

Fuzzy Number	Centroid	FSD
$\tilde{A}_1$	3.00	3.06
$\tilde{A}_2$	3.00	3.18

6.2 Example 2: NonlinearFuzzy Optimization

Consider the quadratic programming problem

$$\max_{x_1,x_2\geq 0} z(x_1,x_2) = 10x_1 + 4x_2 - 2x_1^2 - 3x_2^2$$

subject to

$$2x_1 + x_2 \leq 5$$

Crisp Baseline Solution

Solving the Karush–Kuhn–Tucker conditions gives

$$x_1^{cr} = 2.214, \qquad x_2^{cr} = 0.572$$

with optimal value

$$z^{cr} = 13.6429$$

Fuzzy Coefficient Representation

The coefficients are replaced by trapezoidal fuzzy numbers

$$\tilde{c}_1 = (9.0,9.5,10.5,11.0)$$
$$\tilde{c}_2 = (3.5,3.8,4.2,4.5)$$
$$\tilde{q}_1 = (-2.4,-2.2,-1.8,-1.6)$$
$$\tilde{q}_2 = (-3.5,-3.2,-2.8,-2.5)$$
$$\tilde{k}_1 = (1.8,1.9,2.1,2.2), \qquad \tilde{k}_2 = (0.8,0.9,1.0,1.1)$$
$$\tilde{b} = (4.6,4.8,5.1,5.3)$$

Applying the FSD operator with

$$\gamma = 1, \qquad \alpha = 2$$

yields

$$c_1^{FSD} = 9.86, \qquad c_2^{FSD} = 3.99$$
$$q_1^{FSD} = -2.06, \qquad q_2^{FSD} = -3.11$$
$$k_1^{FSD} = 2.03, \qquad k_2^{FSD} = 0.96, \qquad b^{FSD} = 4.94$$

FSD-Based Optimization Problem

The deterministic equivalent model becomes

$$\max_{x_1,x_2\geq 0} z^{FSD} = 9.86x_1 + 3.99x_2 - 2.06x_1^2 - 3.11x_2^2$$

subject to

$$2.03x_1 + 0.96x_2 \leq 4.94$$

Solving the KKT conditions yields

$$x_1^{FSD} = 2.11, \qquad x_2^{FSD} = 0.59$$
$$z^{FSD} = 13.21$$

6.3 Comparison of Results

Table 2: Comparison of optimization results

Model	x_1^*	x_2^*	z^*
Crisp model	2.214	0.572	13.6429
FSD model ($\gamma = 1, \alpha = 2$)	2.11	0.59	13.21

The FSD-based solution shows a moderate conservative shift relative to the crisp optimum. This adjustment reflects the dispersion sensitivity introduced by fuzzy coefficients.

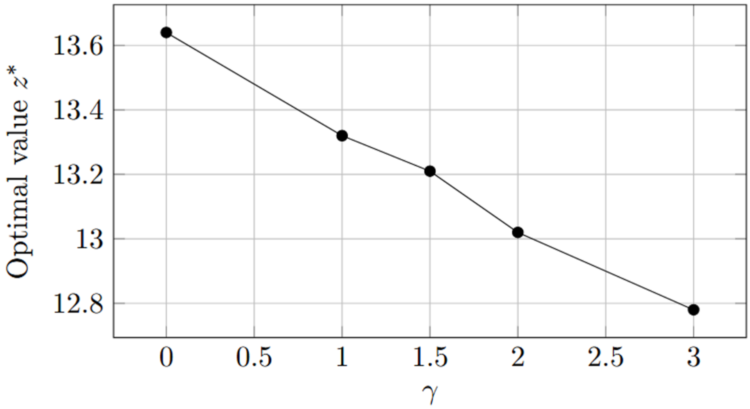
6.4 Parameter Sensitivity

To examine parameter influence, the optimization problem was solved for different (γ, α) values.

Table 3: Sensitivity of optimal solution

γ	α	x_1^*	z^*
0	0.5	2.214	13.64
1	1	2.14	13.32
1	2	2.11	13.21
2	2	2.06	13.02
3	3	1.99	12.78

Increasing γ or α strengthens attenuation of tail deviations and shifts the solution toward the core of fuzzy coefficients. This confirms that the FSD framework provides a tunable mechanism for dispersion-aware decision making.



7 Discussion (Carbon Emission Optimization Application)

In carbon emission optimization issues, where system factors like emission coefficients, fuel consumption rates, and regulatory restrictions are frequently unclear, the suggested Fractional–Spread Defuzzification (FSD) paradigm can be successfully expanded. Operational conditions, measurement errors, and policy changes can all lead to variability in carbon-related statistics in real-world environmental systems. A realistic modeling method is provided by representing such parameters as fuzzy numbers. Because of its multi-parameter nature, the FSD operator provides a flexible technique to handle dispersion in this setting, enabling decision-makers to modify sensitivity toward emission variability. This is very helpful when creating the best plans for energy planning, reducing emissions, and allocating resources sustainably. The integration of the FSD-based ranking into optimization models can therefore support more reliable and balanced decisions in carbon management under uncertainty.

8 Conclusion

In order to rank general n-tuple fuzzy numbers, this work presented a Fractional–Spread Defuzzification (FSD) operator and showed how it could be used in nonlinear fuzzy optimization. The technique uses a two-parameter fractional attenuation kernel that maintains analytical tractability while capturing dispersion sensitivity.

The operator's basic characteristics, such as boundedness, continuity, and monotonic behavior with regard to tuple components, were determined by theoretical research. Through the parameters (γ, α), the suggested formulation additionally offers a configurable transition between mean-dominant and core-dominant ranking regimes.

The FSD-based method successfully integrates spread information in decision-making problems with fuzzy coefficients and yields consistent optimization results, as demonstrated by numerical examples.

Future research could expand the framework to include more intricate uncertainty models like type-2 fuzzy sets and Z-numbers, as well as multi-objective fuzzy optimization.

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