EPJ Web of Conferences **364**, 02007 (2026) https://doi.org/10.1051/epjconf/202636402007
Quark Matter 2025

Quasinormal modes of nonthermal fixed points

Matisse De Lescluze[1,*]

[1]Department of Physics and Astronomy, Ghent University, 9000 Ghent, Belgium

Abstract. At the early stages of ultra-relativistic nuclear collisions, the system is expected to reach a Nonthermal Fixed Point. This makes them an important phenomenon to study for understanding thermalization in QCD matter. In this work we propose, analogously to how quasinormal modes play a prominent role in the relaxation of diverse physical systems to equilibria, ranging from astrophysical black holes to tiny droplets of quark-gluon plasma at the RHIC and LHC accelerators, that a novel kind of quasinormal mode governs the direct approach to self-similar time evolution of nonthermal fixed points. We compute the spectrum of these far-from-equilibrium quasinormal modes for a kinetic theory with a Fokker-Planck collision kernel in isotropic and homogeneous states. Our conclusion is that quasinormal modes of nonthermal fixed points give rise to a tower of progressively more decaying power-law contributions. This proceeding is based on the Flash Talk at Quark Matter 2025.

1 Introduction

Understanding thermalization of the quark-gluon plasma created in relativistic heavy-ion collisions is a central topic of contemporary research, both theoretically and experimentally. A key insight in this context is the notion of the hydrodynamic attractor [1], which explains why relativistic hydrodynamics can successfully describe the quark–gluon plasma even far from equilibrium: systems rapidly approach a universal evolution curve largely independent of their initial conditions. This universality arises from the decay of non-hydrodynamic quasinormal modes (QNMs), leaving only the slowest, hydrodynamic modes, to define the attractor and govern the subsequent evolution.

In our work [2] we recognize that QNMs should play another, so far unexplored role in thermalization dynamics. In the course of the past 20 years it has been predicted theoretically [3, 4] and then verified experimentally [5, 6] that overoccupied states are attracted to a transient albeit long-lived nonthermal fixed point (NTFP) regime characterized by a temporal self-similarity. In the simplest isotropic case, the weakly-coupled dynamics are described by a distribution function of particles $f(t, p = |\vec{p}|)$ depending on time t and the magnitude of spatial momentum p. The NTFP regime occurs in this setup when the temporal and momentum dependence effectively factorize

$$f(t, p) = A(t) f_s(B(t)p). \tag{1}$$

As a result, in the NTFP regime the form of a distribution function at a given time allows to predict its form to the future by a mere rescaling of the distribution function and momentum.

*e-mail: matisse.delescluze@ugent.be

The Boltzmann equation

$$\partial_t f(t,p) = C[f](t,p), \tag{2}$$

dictates then that

$$A(t) = B(t)^{\sigma} \quad \text{and} \quad B(t) = \left(\frac{t-t_*}{t_{ref}}\right)^{\beta}. \tag{3}$$

The number σ and the exponent β characterize the underlying dynamics, t_{ref} is a constant choosing normalization of f_s and t_* is an initial condition-dependent offset picking the origin of time, as 0 plays a special role for self-similarities such as (3) [7].

Our key insight on top of the aforementioned observed attractive nature of NTFPs is that $A(t)^{-1}f(t,p)$ becomes effectively time-independent when viewed as a function of $\bar{p} \equiv B(t)p$ and renders this configuration effectively static in a close similarity to the static nature of the thermal state or a final black hole in a merger or collapse. This allows for setting up an eigenvalue problem for perturbations around a NTFP regime

$$A(t)^{-1} f(t, \bar{p}/B(t)) \approx f_s(\bar{p}) + \delta f(t,\bar{p}). \tag{4}$$

2 Eigenvalue problem for QNMs

The emergence of exact NTFPs (1) is a consequence of the collision kernel C in Eq. (2) being a homogeneous functional of particle momenta [7]. This means that for a distribution function of the form in Eq. (4) the collision kernel behaves as

$$C[f](t,p) = A(t)^{\mu_\alpha} B(t)^{\mu_\beta} (\tilde{C}[f_s](\bar{p}) + \delta\tilde{C}[fs, \delta f](t,\bar{p})). \tag{5}$$

where we consider a small perturbation $\delta f \ll f_s$ to the scaling function f_s. μ_α is determined by the powers of f appearing in C, while μ_β is related to the powers of momentum p. In many cases, this property of the collision kernel emerges in the case of overoccupation $f \gg 1$.

To find the time dependence of the perturbations we plug the form of $f(t,p)$ in Eq. (4) into the Boltzmann equation (2) and consider only terms up to first order in δf. By making use of Eqs. (5) and (3), we obtain

$$B(t)\partial_{B(t)}\delta f\,(t,\bar{p})\,|_{\bar{p}=const} = \frac{1}{D_1}\delta\tilde{C}[f_s,\delta f](t,\bar{p}) - \sigma\delta f\,(t,\bar{p}) - \bar{p}\partial_{\bar{p}}\delta f\,(t,\bar{p})\,, \tag{6}$$

with D_1 a separation-of-variables constant. The above equation is solved by the Ansatz

$$\delta f(t,\bar{p}) = B(t)^{i\Omega}\delta f_\Omega(\bar{p}). \tag{7}$$

The sign convention for the imaginary unit was chosen so that in the following the decaying modes will be characterized by $\mathrm{Im}(\Omega) < 0$, as is the usual convention. Because Ω is constant we can pull the $B(t)^{i\Omega}$ factor through the operators on the right-hand side and eliminate it leading to an eigenvalue equation for the QNM frequencies Ω

$$i\Omega\delta f_\Omega(\bar{p}) = \frac{1}{D_1}\delta\tilde{C}[f_s,\delta f_\Omega](\bar{p}) - \sigma\delta f_\Omega\,(\bar{p}) - \bar{p}\partial_{\bar{p}}\delta f_\Omega\,(\bar{p})\,. \tag{8}$$

Eqs. (7) is the main results of our work. The QNM frequencies Ω induce a power law in time approach to (1) when $\mathrm{Re}(i\Omega)\beta < 0$, which is a self-consistency condition for the attractive nature of NTFPs observed in ab initio simulations and experiments. Furthermore, real parts of Ω's, if nonzero, would induce oscillatory behaviour in the logarithm of B, i.e. in $\log(t-t_*)$. This is reminiscent of the near equilibrium kinetic theory setup of [8].

3 Self-similarity and the role of conservation laws

To find a self-similar solution, $A(t)$ and $B(t)$ are related by a positive constant σ [7], see Eq. (3), through conservation laws. Consider for example conservation of energy density $\epsilon = \int d^d p \omega_{\vec{p}} f/(2\pi)^d$ with $\omega_{\vec{p}} \propto p$, for massless particles. At the NTFP this becomes

$$\epsilon = \int d^d p p^z f(t,p) = A(t)B(t)^{-d+z} \int d^d \bar{p} \bar{p}^z f_s(\bar{p}). \tag{9}$$

The energy density will be conserved when $A \propto B^{d+z}$. We define the conserved quantity to be completely contained in f_s, i.e. $A = B^{d+z}$ and $\sigma = d+z$. Now we can already derive some insights into perturbations of the form in Eq. (7). The energy density of the perturbation is

$$\delta\epsilon(t) = \int d^d \bar{p} \bar{p}^z \delta f(t,\bar{p}) = B(t)^{i\Omega} \int d^d \bar{p} \bar{p}^z \delta f_\Omega(\bar{p}). \tag{10}$$

It is clear that the above needs to be constant for the total energy density to be conserved. This means that either $\Omega = 0$ or $\delta\epsilon_\Omega \equiv \int d^d \bar{p} \bar{p}^z \delta f_\Omega(\bar{p}) = 0$. $\Omega = 0$ is a zero mode and corresponds to a NTFP with a shifted energy density. The non-zero modes come with the constraint $\delta\epsilon_\Omega \equiv \int d^d \bar{p} \bar{p}^z \delta f_\Omega(\bar{p}) = 0$, which will be used in the subsequent calculation of the QNM spectrum.

4 Calculation

We test the general lessons from the previous section in the explicit example of Fokker-Planck kinetic theory adopted from QCD applications, given by

$$C_{FP}[f](t,p) = K \left\{ \frac{I_a}{p^2} \partial_p \left(p^2 \partial_p f \right) + \frac{I_b}{p^2} \partial_p \left(p^2 f (f+1) \right) \right\} \tag{11}$$

where $K = \frac{\lambda^2}{4\pi}\mathcal{L} = \frac{\lambda^2}{4\pi}$, $I_a = \int \frac{d^3p}{(2\pi)^3} f(f+1)$ and $I_b = 2\int \frac{d^3p}{(2\pi)^3}\frac{f}{p}$, see e.g. [9]. λ is the ('t Hooft) coupling constant and the Coulomb logarithm $\mathcal{L}$ is given by $\log(\langle p \rangle / m_D)$, but we chose to set it equal to 1. Overoccupation and conservation of energy density leads to $\sigma = 4$ and the direct energy cascade NTFP $(\alpha, \beta) = (-4/7, -1/7)$ [10, 11].

To find the spectrum of QNMs we discretize Eq. (8) and solve it as a numerical eigenvalue problem. As input we need the scaling function, as determined in [2].

Fig. 1 shows the resulting QNM spectrum. There are seemingly unstable modes, $\mathrm{Im}(\Omega) > 0$, when no boundary conditions are imposed, the orange circles. By imposing that the modes carry no energy density, $\delta\epsilon = 0$, we eliminate the growing perturbations and the spectrum becomes that of an attractor, the blue asterisks. This means that, on the linear level, the NTFP is completely stable when the assumptions that allow for the direct energy cascade remain valid, i.e. overoccupation and conservation of energy density. The remaining modes are purely imaginary. They are very robust to changes in the UV cut-off and remain the same for the largest cut-off we consider, see [2] for more details. The QNM $\Omega = -7i$ can be understood by shifting δt_* in A and B in Eq. (1). This mode captures the prescaling phenomenon of [7] and shows our analysis is able to capture important physical effects. We expect infinitely many QNM frequencies further down in the lower half complex plane.

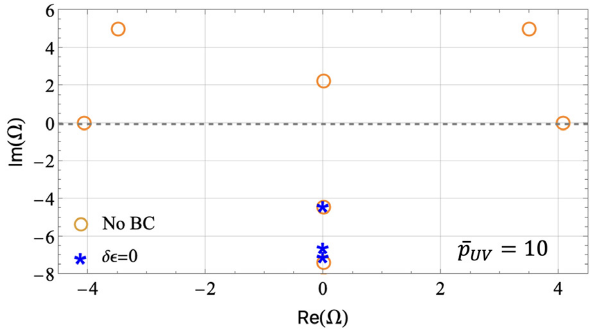

Figure 1. Results for the QNMs frequencies Ω calculated on a grid with cut-offs $\bar{p}_{IR} = 0$ and $\bar{p}_{UV} = 10$. The blue asterisks show the QNMs obtained from the eigenvalue equation (8) with the additional constraint that their energy density $\delta\epsilon$ is 0. See the discussion below Eq.(10). The orange circles is the result without imposing conservation of energy. This leads to unstable modes, $\mathrm{Im}(\Omega) > 0$, which is in disagreement with the attractive nature of NTFPs.

5 Conclusions and Outlook

Our paper aims to provide a bridge between far-from-equilibrium NTFPs and QNMs describing the transient approach to thermality. It is natural to envision a significant advancement of our understanding about the attractive nature of NTFP and phenomena in their vicinity coming from adopting the relevant knowledge and techniques from the fields of black hole perturbation theory and holography. There are many future directions that our work opens up. On the theoretical front, it is certainly understanding the QNM spectrum for other systems that model heavy-ion collisions more accurately, in particular systems undergoing longitudinal expansion. Finally, on the experimental front, it would be fascinating to directly observe QNMs of NTFP using their experimental realisations in cold atomic gases.

References

[1] M.P. Heller, M. Spaliński, Hydrodynamics beyond the gradient expansion: Resurgence and resummation, Phys. Rev. Lett. **115**, 072501 (2015). 10.1103/PhysRevLett.115.072501
[2] M. De Lescluze, M.P. Heller, Quasinormal modes of nonthermal fixed points, Phys. Rev. Lett. **135**, 091601 (2025). 10.1103/tz78-jfbw
[3] R. Micha, I.I. Tkachev, Relativistic turbulence: A Long way from preheating to equilibrium, Phys. Rev. Lett. **90**, 121301 (2003), hep-ph/0210202. 10.1103/PhysRevLett.90.121301
[4] J. Berges, A. Rothkopf, J. Schmidt, Non-thermal fixed points: Effective weak-coupling for strongly correlated systems far from equilibrium, Phys. Rev. Lett. **101**, 041603 (2008), 0803.0131. 10.1103/PhysRevLett.101.041603
[5] M. Prüfer, P. Kunkel, H. Strobel, S. Lannig, D. Linnemann, C.M. Schmied, J. Berges, T. Gasenzer, M.K. Oberthaler, Observation of universal dynamics in a spinor Bose gas far from equilibrium, Nature **563**, 217 (2018), 1805.11881. 10.1038/s41586-018-0659-0

[6] G. Martirosyan, C.J. Ho, J. Etrych, Y. Zhang, A. Cao, Z. Hadzibabic, C. Eigen, Observation of subdiffusive dynamic scaling in a driven and disordered box-trapped Bose gas (2023), 2304.06697.

[7] M.P. Heller, A. Mazeliauskas, T. Preis, Prescaling Relaxation to Nonthermal Attractors, Phys. Rev. Lett. **132**, 071602 (2024), 2307.07545. 10.1103/PhysRevLett.132.071602

[8] M.P. Heller, V. Svensson, How does relativistic kinetic theory remember about initial conditions?, Phys. Rev. D **98**, 054016 (2018), 1802.08225. 10.1103/PhysRevD.98.054016

[9] J. Berges, M.P. Heller, A. Mazeliauskas, R. Venugopalan, QCD thermalization: Ab initio approaches and interdisciplinary connections, Rev. Mod. Phys. **93**, 035003 (2021), 2005.12299. 10.1103/RevModPhys.93.035003

[10] M.C. Abraao York, A. Kurkela, E. Lu, G.D. Moore, UV cascade in classical Yang-Mills theory via kinetic theory, Phys. Rev. D **89**, 074036 (2014), 1401.3751. 10.1103/PhysRevD.89.074036

[11] J. Berges, K. Boguslavski, S. Schlichting, R. Venugopalan, Universal attractor in a highly occupied non-abelian plasma, Physical Review D **89** (2014), 1311.3005. 10.1103/physrevd.89.114007