Hadron resonance gas is not a good model for hadronic matter in a strong magnetic field

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Abstract. We study the effect of magnetic field on particle yields and charge fluctuations in hadron resonance gas. We argue that the big changes in the proton yield and baryon number susceptibility are due to ill-defined description of higher-spin states, and that because of detailed balance, neutral resonances must be affected by the field too.

1 Introduction

The behaviour of QCD matter in strong magnetic field is of both practical and theoretical interest. Practical, since the magnetic fields generated in non-central heavy-ion collisions at ultrarelativistic energies are among the strongest in the universe [1], and theoretical, since it can be calculated from first principles using lattice QCD (LQCD) methods. Recently, the LQCD calculations of the fluctuations of conserved charges in finite magnetic field have been of particular interest (see e.g. Ref. [2]). It has been observed that a finite magnetic field has a nontrivial effect on those observables.

The hadron resonance gas (HRG) model has been successful in describing the LQCD results on the EoS and fluctuations of conserved charges at vanishing magnetic field [3]. In [4], the HRG model was generalised to include the effects of finite magnetic field. In this contribution we calculate the fluctuations of conserved charges and particle yields in a finite magnetic field using the HRG model [5]. We argue that the strong dependence on the field strength is an artefact of the model assumptions, and appears in a region where the model contradicts its own premises.

2 Hadron resonance gas in finite magnetic field

The HRG model is based on the approximation that in interacting hadron gas interactions mediated by resonances dominate. If these resonances are narrow, their contribution to the equation of state of the gas can be well approximated by describing them as noninteracting

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particles with the pole mass of the resonance. Thus the interacting system can be approximated as a gas of free hadrons and resonances. Furthermore, the hadrons and resonances are assumed to be pointlike and structureless, and the usual kinetic theory definitions of thermodynamic quantities apply.

In the presence of a constant external magnetic field eB pointing along the z direction, the system undergoes Landau quantisation in the xy plane [6]. Consequently, the dispersion relation for a charged particle ($Q \neq 0$) becomes

$$\epsilon = \sqrt{p_z^2 + m^2 + 2|Q|B\,(l + 1/2 - s_z)}, \tag{1}$$

where s_z is the z-component of the particle's spin and $l \in \{0, 1, 2, \ldots\}$ numbers the Landau levels. We note that the dispersion relation in Eq. (1) is exact for structureless spin-0 and spin-1/2 particles [6], but a similar relation can be derived for spin-1 and 3/2 particles. Likewise, the integration over transverse momenta in the thermodynamic integrals is replaced by a summation over Landau levels.

As seen in Eq.(1), even the lowest Landau level gives a positive contribution to the energy of pions (and other spin-0 particles). Thus the pion density in fixed temperature decreases with increasing magnetic field. The contribution of the lowest Landau level to the energy of spin-1/2 particles is zero, but we observe a small increase in their densities with increasing magnetic field due to quantisation of the energy levels (summation instead of integration), whereas the contribution to the energy of spin-1 and 3/2 particles is negative, and we observe a strong increase of the densities of ρ and Δ with increasing magnetic field [5].

Particle densities are not observables in heavy-ion collisions since final observed particle yields contain contributions from all decayed resonances. In the left panel of Fig. 1, we show the particle yields after decays, normalised to the yields at vanishing magnetic field. Even if the density of thermal pions decreases, they receive a large contribution from rhos, and thus the dependence of the pion yield on magnetic field is weak. The kaons behave similarly, but protons get a large contribution from Deltas, and therefore the proton yield strongly depends on the magnetic field. The other spin-1/2 baryons, Σ and Ξ, have way weaker dependence since there are way fewer known strange than non-strange resonances.

In a thermal medium, the second-order fluctuations and correlations of conserved charges are quantified by the generalised susceptibilities. In the right panel of Fig. 1 we show the HRG model net-baryon number susceptibility χ_{BB} as a function of the strength of the magnetic field, and compare it to the LQCD result [2] at T=145 and 155 MeV. We find that the

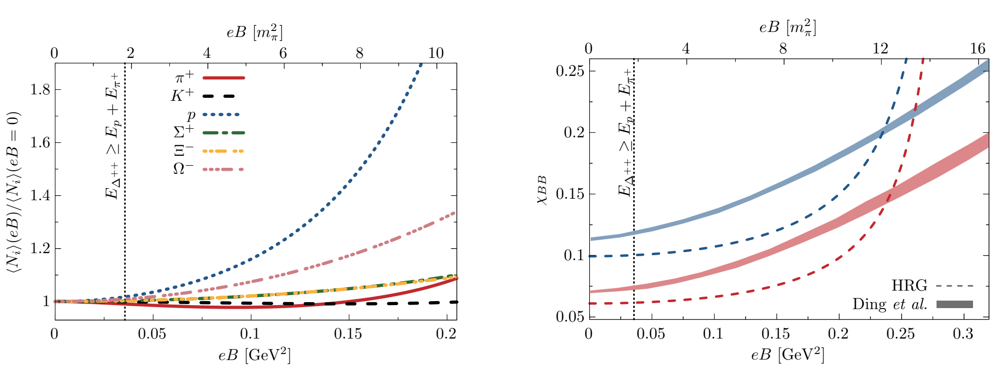

Figure 1. Particle yields normalised to the yields at vanishing magnetic field at $T = 155$ MeV (Left) and net-baryon number susceptibility at $T = 0.145$ (red) and 0.155 GeV (blue) (Right).

HRG model systematically underestimates the LQCD result for $eB \leq 0.22$ GeV^2 at both temperatures. If all the hadron states predicted by relativistic quark model are included in the HRG model, the lattice χ_{BB} is reproduced, but independent of the particle list, HRG model overshoots the lattice result at large values of eB. This behaviour is again due to the $s \geq 3/2$ resonances, which contribution ranges from 75% at vanishing eB to 95% at $eB = 0.3$ GeV^2.

At this point it is worth remembering that Eq. (1) neglects the compositeness of the states. Issues can arise when the scale of magnetic field resolves the structure of hadron states, i.e., $eB > m_\pi^2$. For example the contribution of the lowest Landau level to the energy of high-spin particles is negative, see Eq. (1). Therefore in strong enough magnetic field the dispersion relation of, say, Δ^{++}, becomes complex signalling instability. Likewise the concept of a resonance can become questionable: For the resonance to be able to decay, its energy must be larger than its daughter particles. E.g. for Δ^{++}, $E_{\Delta^{++}} \geq E_p + E_{\pi^+}$. But since the lowest energy level of Δ^{++} (π^+) decreases (increases) with increasing strength of the magnetic field, this requirement is no longer valid once $eB \gtrsim 0.0356$ GeV^2! Thus in stronger field it is not clear what is a resonance and what is a ground state hadron. In Fig. 1 this limit is shown as a vertical dotted line. Below this line, the effect of the magnetic field is negligible, but above it, our results are not to be trusted.

3 Detailed Balance

We assume neutral particles to have zero magnetic moments. Therefore, they are not affected by the magnetic field at all. However, in HRG in equilibrium detailed balance prevails, e.g., the decay rate of, say, rho mesons is equal to the scattering rate of pions forming them. The scattering rate depends on the densities and, as mentioned previously, pion density decreases with increasing magnetic field. On the other hand, decay rate depends on the resonance width, and it is known that a weak magnetic field causes a negligibly small change in the width of ρ^0 [7]. Thus in the presence of a magnetic field, the ρ^0 decay rate changes only if the ρ^0 density changes, and the change in rate is proportional to the change in density.

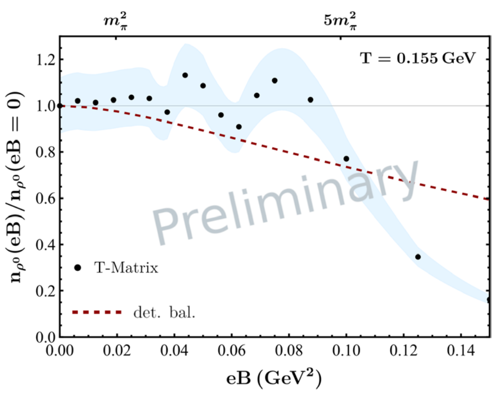

Figure 2. ρ^0 meson density at $T = 155$ MeV using the detailed balance and T-matrix approaches normalised to densities at $eB = 0$.

Unfortunately, we cannot evaluate the scattering rate of pions using the conventional kinetic theory calculation, since particles in magnetic field do not have well-defined momenta. Instead, we make a bold assumption that the magnetic field affects the pion scattering rate (and thus the ρ^0 production rate) only by changing their densities. We can evaluate an effective pion chemical potential such that $n_\pi(T, \mu_\pi, B = 0) = n_\pi(T, B)$. With the above-mentioned considerations, detailed balance requires that ρ^0 obtains a chemical potential, which is a sum of pion chemical potentials: $\mu_{\rho^0} = \mu_{\pi^+} + \mu_{\pi^-} = 2\mu_{\pi^+}$. Similar arguments apply to Δ^0 [5].

The ρ^0 density corresponding to this chemical potential, and scaled with its equilibrium density, is shown in in Fig. 2. Unlike the expectation that neutral particles should not be affected by the magnetic field, ρ^0 density decreases with increasing magnetic field, the effect being the stronger the colder the system. Since the densities of neutral resonances are affected via this mechanism, it is questionable what the proper treatment of charged resonances is.

Similar detailed balance approach leads to very different sensitivity to the magnetic field than the treatment of resonances as elementary particles according to Eq. (1).

4 Hadronic structure and thermal yields

The dispersion relation in Eq. (1) assumes structureless, i.e. pointlike, particles, and thus accounts only for very narrow resonances. To incorporate more general particle decay dynamics, which are essential for a reliable description of hadron yields at higher magnetic fields, one should employ a consistent model of hadronic interactions. As a first step towards such a model, we consider a $\pi - \rho$ gas where the $\rho_0\pi^+\pi^-$ interaction is described by the Lagrangian $\mathcal{L}_{int} = -g\,\partial_\mu\vec{\rho_\nu}\cdot\partial^\mu\vec{\pi}\times\partial^\nu\vec{\pi}$, with $g = 20.72$ GeV^{-2}, which is fixed from the vacuum decay width $\Gamma_{\rho\to\pi\pi}$= 150 MeV [9]. The one loop self energy ($\Pi^{\mu\nu}$) of the ρ_0 meson is evaluated in presence of an arbitrary magnetic field, employing the Schwinger propagator for the loop pions. The ρ_0 density in an external magnetic field is then obtained by constructing the in-medium spectral function. Re Π has not been taken into account in the spectral function. Therefore, there is no mass shift of ρ_0.

Fig. 2 shows the ρ_0 meson density in a magnetised medium, scaled with its density at zero field. The shaded band represents the uncertainty arising from neglecting Re Π. This preliminary result supports the underlying assumption of our detailed balance treatment: the magnetic field does affect the density of neutral resonances. It also shows the treatment's naivety, since the effect appears only at large field strengths and is non-monotonous. Further work is in progress to incorporate π–π scattering data for a unitary, model-independent treatment of hadron interactions [8].

Acknowledgments. Support by the program Excellence Initiative–Research University of the University of Wrocław of the Ministry of Education and Science (P.H, M.M. and B.K.), the Polish National Science Center (NCN) under the Preludium grant 2020/37/N/ST2/00367 (M.S.) and the OPUS Grant No. 2022/45/B/ST2/01527 (C.S., K.R. and P.M.L.), the Polish Ministry of Science and Higher Education (K.R.), and the World Premier International Research Center Initiative (WPI) under MEXT, Japan (C.S.) is gratefully acknowledged.

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