

The electric conductivity of nuclear matter along the QCD phase transition line

Joseph Atchison^{1,*}, Yiding Han³, and Frank Geurts²

¹Abilene Christian University, 1600 Campus Ct, Abilene, 79601, Texas, USA

²Rice University, 6100 Main St, Houston, 77005, Texas, USA

³Baylor College of Medicine, 1 Baylor Plz, Houston, 77030, Texas, USA

Abstract. Transport coefficients play an important role in characterizing hot and dense nuclear matter, such as that created in ultra-relativistic heavy-ion collisions (URHIC). The electric conductivity can be accessed via the electromagnetic (EM) spectral function's low-energy transport peak, which can be measured via thermal dilepton emission. Several facilities including the Schwer-Ionen Synchrotron (SIS), the Relativistic Heavy-Ion Collider (RHIC), and the Large Hadron Collider (LHC) have potential to probe low energy dilepton signals in ongoing and future experiments.

We present our study on the electric conductivity of hot and dense nuclear matter. We implement the vector dominance model (VDM), in which the photon couples to hadronic currents predominantly through the ρ meson. Hadronic many-body theory is utilized to calculate the ρ -meson's self-energy, by dressing its pion cloud with π - ρ , π - σ , π - K , N-hole, and Δ -hole loops. Vertex corrections are introduced to maintain gauge invariance. We examine the transport peak and conductivity along a proposed phase transition line, and under conditions comparable to those expected in future experiments. We compare the transport properties for hadronic matter and a pion gas, to examine their individual contributions. Finally, we calculate the transport peak and find that the conductivity shows a decreasing tendency from high to low collision energies.

1 Introduction

Electromagnetic probes, including low-mass photons and dileptons, are produced throughout the nuclear fireball created in heavy-ion collisions (HICs), but do not interact through the strong force. They provide a probe of the medium's electromagnetic spectral function (ρ_{EM}) [1]. One can characterize the medium with transport coefficients such as the electric conductivity (σ_{el}). The conductivity is closely related to low-mass dilepton emission rates as both are proportional to ρ_{EM} . Ongoing and future measurements of very-low mass and momentum dileptons at the SIS, RHIC, and the LHC may measure the associated transport peak [2–4]. Current theoretical calculations of σ_{el} in hadronic matter implementing different methodologies vary significantly, motivating further study of σ_{el} .

In this proceeding we summarize the work of [5], which calculated σ_{el} for hot and dense nuclear matter. The pion plays a critical role in generating the transport peak, because higher

*e-mail: jia08a@acu.edu

mass particles are suppressed at low energies. Pions act as the primary charge carrier in nuclear matter. Pion scatterings off in-medium particles generate the pion’s width, and subsequently a finite electric conductivity. The pion’s medium interactions are quantified through the pion self-energy (Σ_π). Here we consider: S - and P - wave $\pi\pi$ -scattering [6], πN scattering [7], and P - wave scattering with thermal kaons [5]. Gauge invariance requires that ρ_{EM} be 4-dimensionally transverse, which is violated by the introduction of an energy dependent pion width. Transversality is restored by the introduction of vertex corrections that satisfy the Ward-Takahashi identities. We then evaluate ρ_{EM} and σ_{el} along a proposed QCD phase transition line, providing results with and without vertex corrections, and addressing the significance of each scattering process.

2 EM spectral function in hadronic matter

The electric conductivity is related to the the EM current-current correlator (Π_{EM}) by [8–10]: $\sigma_{el}(T) = (-e^2/3) \lim_{q_0 \rightarrow 0} [\text{Im}\Pi_{EM}^{ii}(q_0, \vec{q} = 0, T)/q_0]$. For small invariant mass ($\lesssim 1$ GeV), the EM-spectral function ($\rho_{EM} = -2 \text{Im}\Pi_{EM}$) is well described by the vector dominance model. The EM-current couples to the medium through the light vector mesons, with the dominate contribution coming from the ρ . For small invariant mass, ρ_{EM} may be approximated in terms of the ρ ’s spectral function ($\text{Im}D_\rho^{\mu\nu}$) as $\text{Im}\Pi_{EM}^{\mu\nu} \approx ((m_\rho^{(0)})^4/g_\rho^2)\text{Im}D_\rho^{\mu\nu}$, where $m_\rho^{(0)}$ is the ρ ’s bare mass and g_ρ is the $\rho\pi\pi$ coupling constant [1, 11]. Medium interactions are introduced through the ρ ’s self-energy ($\Sigma_\rho^{\mu\nu}$), as described in Refs. [7, 12]. Lorentz invariance is broken in medium, causing D_ρ to split into transverse and longitudinal modes for finite 3-momentum. At zero 3-momentum the two modes are equivalent, allowing one to express σ_{el} in terms of the transverse projection of $\Sigma_\rho^{\mu\nu}$ (Σ_ρ^T) [13]:

$$\sigma_{el} = \frac{e^2}{g_\rho^2} \lim_{q_0 \rightarrow 0} \text{Im} \left[\frac{-(m_\rho^0)^4}{q_0^2 - (m_\rho^0)^2 - \Sigma_\rho^T(q_0, \vec{q} = 0)} \right]. \quad (1)$$

In the low-energy regime $\Sigma_\rho^{\mu\nu}$, and in turn σ_{el} , are dominated by the Landau cut. This cut describes the absorption of a virtual ρ by a thermal pion [5, 6]. Resistance is generated through the pion’s self-energy (Σ_π), which can be resummed into the pion’s propagator, $D_\pi(k_0, \vec{k}) = 1/(k - m_\pi - \Sigma_\pi(k, T, \mu_B))$ [6, 12]. This process is depicted in figure 1.

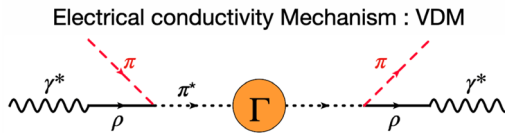


Figure 1. Virtual photon coupling to the medium through the ρ ’s Landau cut. Resistance is generated through $\pi\pi$, πN , and πK interactions.

3 Pion Interactions in medium

In medium the pion gains a finite width due to scatterings. We consider $\pi\pi$ S- and P-wave scattering through σ and ρ resonances, πK P-wave scattering through a K^* resonance, and πN scattering through nucleon and Δ resonances. The total pion self-energy is given by the

sum of the self-energies for each process as $\Sigma_\pi = \Sigma_\pi^\rho + \Sigma_\pi^\sigma + \Sigma_\pi^N + \Sigma_\pi^K$. The definition of the individual contributions can be found in Refs. [5, 7].

The ρ couples to a conserved current, thus $\Sigma_\rho^{\mu\nu}$ must be 4-dimensionally transverse. This is true if the Ward-Takahashi identities are satisfied:

$$q^\mu \Gamma_{\mu ab3}^{(3)} = g_\rho \epsilon_{3ab} (D_\pi^{-1}(k+q) - D_\pi^{-1}(k)), \quad (2)$$

$$q^\mu \Gamma_{\mu\nu ab33}^{(4)} = i g_\rho (\epsilon_{3ca} \Gamma_{\nu bc3}^{(3)}(k, -q) - \epsilon_{3bc} \Gamma_{\nu ca3}^{(3)}(k+q, -q)). \quad (3)$$

Where $\Gamma_{\mu ab3}^{(3)}$ is the $\rho\pi\pi$ vertex and $\Gamma_{\mu\nu ab33}^{(4)}$ is the $\rho\rho\pi\pi$ vertex. The identities are broken by the inclusion of Σ_π , however this can be remedied by introducing vertex corrections. For the πN corrections we adopt the corrections found in Ref. [7]. The corrections due to the $\pi\pi$ and πK interactions are numerically expensive, so we introduce effective corrections that are similar in form to those found in Ref. [7]. Although these corrections are not derived from Feynman diagrams, they exactly maintain the Ward identities and agree well with the corrections calculated in Ref. [6]. The effective corrections can be found in Ref. [5].

4 EM-spectral function at finite temperature and density

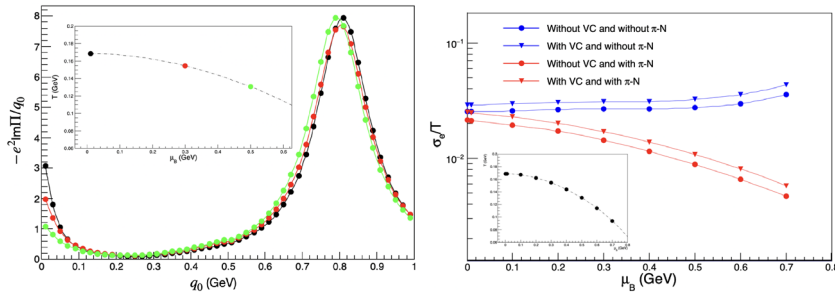


Figure 2. Left: The transverse projection of the EM-spectral function for three points along the transition line: $\{T = 157\text{MeV}, \mu_B = 1\text{MeV}\}$ (black), $\{T = 145\text{MeV}, \mu_B = 300\text{MeV}\}$ (red), and $\{T = 115\text{MeV}, \mu_B = 500\text{MeV}\}$ (green). Right: The electric conductivity along the transition line (insert). Red points include πN scattering, while blue points do not. Results including vertex corrections are plotted with circles, and results excluding vertex corrections are plotted with triangles.

We analyze ρ_{EM} at finite T and μ_B , normalized so that σ_{el} is extracted from the zero-energy limit. The left panel of figure 2 plots ρ_{EM} at three points on the proposed transition line, estimated using chemical freeze out data from published results [14, 15]. We focus on the low-energy transport peak, which reduces in height, but increases in width as one moves from left to right along the transition line. The reduction in height is due to the pion density falling with T , while the increased width is due to the rising nucleon density. We see little T or μ_B dependence at the ρ 's mass peak, due to our exclusion of direct ρN interactions.

We now examine σ_{el}/T as a function of T and μ_B , displayed in the right panel of figure 2. The vertex corrections provide approximately a 15% increase in σ_{el} . This occurs because the corrections introduce additional channels through which the ρ can couple to the medium. Next, we analyze the effect of the πN interaction. One sees that excluding the πN interaction results in σ_{el} rising along the transition line. This is due to the pion width decreasing with decreasing T . However, when the πN interaction is included σ_{el}/T decreases along the transition line. In this case, the pion width increases due to an increasing nucleon density. This

demonstrates the importance of both the pion and nucleon in determining σ_{el} . The pion act as the primary charge carrier, while nucleons provide the majority of the medium's resistance. Thus nuclear matter transitions from a more weakly interacting "pion gas" at high T and low μ_B to a more strongly interacting nucleon fill medium at low T and high μ_B .

5 Summary and future work

In this work, we used a quantum many-body approach to calculate ρ_{EM} in hadronic matter along a proposed phase transition line. We include vertex corrections to preserve gauge invariance. The EM-current is coupled to the medium through the ρ , and interactions are introduced through Σ_ρ . In particular, the Landau cut of Σ_ρ generates a transport peak. Medium interactions are introduced through Σ_π , so that a finite σ_{el} may be obtained. We consider pion interactions through P- and S- wave $\pi\pi$ -scattering, P-wave πK scattering, and πN scattering.

We find that the pion acts as the primary charge carrier in the medium, because transport through nucleons and kaons is heavily suppressed due to their large masses relative to the pion. In the absence of πN interactions the medium behaves as a more weakly interacting gas, with σ_{el} rising along the transition line. When πN interactions are included the medium becomes more strongly coupled, and σ_{el} falls along the transition line. We see that, while the pions act as a charge carrier, the nucleons provide a resistive medium.

The large broadening found at high μ_B indicates, that experiments at GSI, RHIC, and the LHC may be able to detect the transport peak by measuring very-low-mass (10 MeV) dileptons [2–4]. We intend to apply our model at finite momentum, to calculate thermal dilepton emission rates at low-invariant mass. This analysis will require further examination of the effective vertex corrections to ensure their reliability at finite momentum. Furthermore, at low T and high μ_B one may need to consider next to leading order effects, such as direct interactions of the ρ with nucleons and kaons. The importance of these effects under experimental conditions should be understood to enable appropriate comparisons to model calculations.

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