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Temperature dependence of heavy quark diffusion from 2+1 lattice QCD

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Abstract. We present new lattice results for the heavy quark diffusion coefficient in 2+1 flavor QCD in the temperature range from 163 MeV up to 10 GeV. Compared to previous lattice calculations with unphysical light quark masses, we consider near-physical values and a much wider temperature range. Our results for the spatial heavy diffusion coefficient near the crossover temperature are considerably smaller than the estimates obtained by comparing phenomenological models with experimental data and by T-matrix calculations, and are close to the AdS/CFT limit. At high temperatures, however, the spatial heavy diffusion coefficient increases and approaches the NLO weak coupling prediction within the estimated errors. We also find that the dependence of the spatial heavy quark diffusion coefficient on the heavy quark mass M_Q on the lattice is weaker compared to the model calculations.

1 Introduction

Heavy quarks are excellent probes of the quark-gluon plasma (QGP) produced in heavy-ion collisions (HIC). Because their masses $M \gg T$ are much larger than the typical thermal scale, they are produced in the early stages of the collisions and subsequently propagate through the entire plasma, providing unique insights into the non-equilibrium dynamics of the QGP.

In particular, the heavy-quark momentum diffusion coefficient κ (or its spatial counterpart D_s) characterizes the Brownian motion experienced by heavy quarks within the medium, and it is directly related to their thermalization rate. Physically, it describes how much momentum a heavy quark loses in the plasma and directly affects important experimental observables such as the nuclear modification factor R_{AA} and the elliptic flow parameter v_2 [1–3].

At weak coupling, κ can be computed perturbatively [4], but the convergence of the expansion is poor until very large temperatures. On the other hand, the AdS/CFT estimate [5, 6] depends on the coupling constant prescription and cannot be rigorously taken

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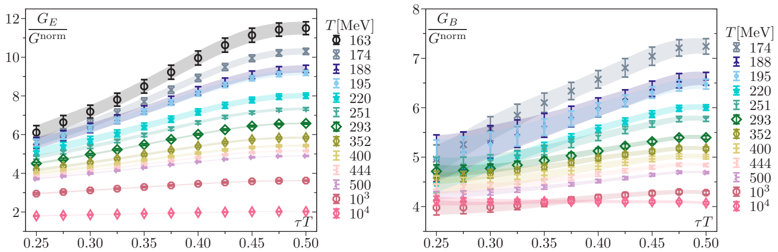

Figure 1. Continuum- and flow-extrapolated correlators as a function of τT for different temperatures [9].

as a lower bound. In contrast, Bayesian reconstructions and phenomenological analyses often yield values systematically larger than previous 2+1 lattice QCD results with unphysical pion masses [7, 8]. Therefore, a first-principles non-perturbative determination of D_s from lattice QCD with physical pion masses is essential.

In this work we present the latest lattice QCD determination of the heavy-quark diffusion coefficient at a physical pion mass, using heavy quark effective theory (HQET). Our study extends previous HotQCD calculations by reaching both lower temperatures close to the chiral crossover and higher temperatures up to 10 GeV, covering the full range where lattice QCD remains sensitive to κ. This proceeding summarizes the main strategy and results presented in detail in our recent publication [9].

2 Theoretical Framework

The heavy-quark momentum diffusion coefficient can be obtained from current–current correlations functions. It is defined in terms of the low-frequency limit of the spectral functions $\rho_{E,B}(\omega)$ as

$$\kappa_{E,B} = \lim_{\omega \to 0} \frac{T}{\omega} \rho_{E,B}(\omega)\,, \tag{1}$$

where the subscripts E and B denote chromo-electric and chromo-magnetic contributions, respectively. In HQET, the electric contribution corresponds to the infinitely heavy-quark limit, while the magnetic contribution represents the first mass correction. Thus, up to $O(T^2/M^2)$ corrections, the complete heavy-quark momentum diffusion coefficient can be written as [10]

$$\kappa = \kappa_E + \frac{2}{3}\langle \mathbf{v}^2 \rangle \kappa_B\,, \tag{2}$$

where $\langle \mathbf{v}^2 \rangle$ is the average thermal velocity of the heavy quark. Then, for a heavy quark with mas M, the diffusion coefficient κ is related to the spatial heavy-quark diffusion coefficient D_s via an Einstein relation:

$$D_s = \frac{2T^2}{\kappa} \frac{\langle \mathbf{p}^2 \rangle}{3MT}\,, \tag{3}$$

where $\langle \mathbf{p}^2 \rangle$ is the average thermal momentum of the heavy quark.

The spectral function is not directly accessible from the lattice. Instead, one computes Euclidean correlators:

$$G_{E,B}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{E,B}(\omega)\, K(\omega, \tau)\,, \tag{4}$$

with the integration kernel $K(\omega, \tau) = \cosh[\omega(\tau - 1/2T)]/\sinh(\omega/2T)$.

Reconstructing $\rho_{E,B}(\omega)$ from these correlators is a challenging problem, which requires noise reduction techniques and theoretical guidance on the infrared (IR) and ultraviolet (UV)

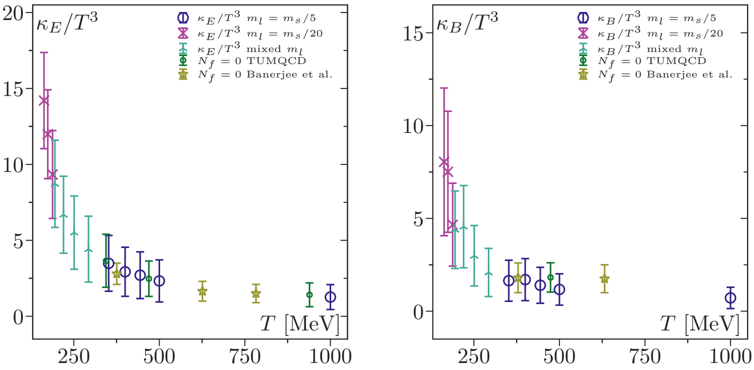

Figure 2. Temperature dependence of $\kappa_{E,B}/T^3$ [9]. Also shown are quenched QCD results for κ_B [14, 15].

behavior of the spectral function. At small frequencies the spectral function is determined uniquely by diffusion, while at large frequencies it can be computed perturbatively at LO and NLO. The spectral function is then modeled by smoothly interpolating between these two regimes using several Ansätze, which provides a controlled estimate of systematic uncertainties in $\kappa_{E,B}$.

We generated 2+1 flavor lattice QCD ensembles at several lattice spacings over a vast range of temperatures, from $T = 163$ MeV to 10 GeV, using the Highly Improved Staggered Quarks (HISQ) action [11] for the fermion sector and the tree-level improved Lüscher–Weisz action [12] for the gauge fields. For low temperatures the light-quark quark masses are tuned to its physical values. To reduce discretization effects, we apply Zeuthen flow [13] and perform a double extrapolation to zero flow time and zero lattice spacing, ensuring that the reconstructed correlators are under perturbative control at short distances while retaining sensitivity to transport coefficients in the infrared.

3 Results and Discussion

In Fig. 1 we show the correlators $G_E(\tau T)$ and $G_B(\tau T)$ at several temperatures. The correlators are normalized to remove the leading-order τ dependence. In both cases, the τT dependence decreases with temperature, becoming nearly τ-independent at the highest T. At small Euclidean times, the correlators are well described by perturbation theory, while at larger τT the clear temperature dependence reflects the infrared physics associated with heavy-quark diffusion. This provides the starting point for spectral reconstruction.

To reconstruct the spectral functions $\rho_{E,B}(\omega)$ we employ a set of Ansätze that interpolate smoothly between the infrared form $\rho^{\mathrm{IR}}_{E,B}(\omega) = \kappa_{E,B}\,\omega/(2T)$ and the ultraviolet behavior given by perturbative LO and NLO results. By fitting the Euclidean correlators with twelve different Ansätze and combining bootstrap resampling with model averaging, we obtain robust systematic estimates of $\kappa_{E,B}$.

The resulting values of κ_E/T^3 and κ_B/T^3 are shown in Fig. 2. Both coefficients decrease significantly with increasing temperature, consistent with expectations from weak coupling. The electric and magnetic contributions are comparable in magnitude.

Finally, in Fig. 3 we convert our results to the spatial diffusion coefficient $D_s = 2T^2/\kappa$ and compare them to other determinations. Close to T_c, our results are remarkably close to the AdS/CFT estimate $2\pi T D_s \simeq 1$ (for a fixed coupling), supporting the picture of a strongly coupled QGP. At higher temperatures, $2\pi T D_s$ grows, approaching the NLO perturbative QCD prediction. Across the full range $T = 163$ MeV–10 GeV, our results remain systematically

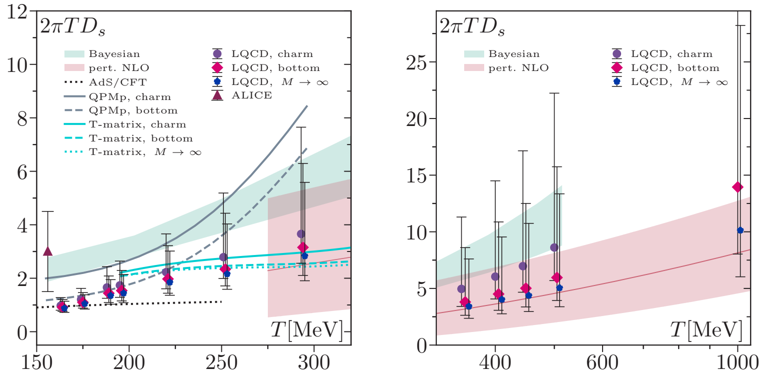

Figure 3. Final results for the spatial diffusion coefficients $2\pi T D_s$ for charm, bottom, and infinitely heavy quarks, compared with several estimates and determinations [9]. The figure is split into two parts: low-temperature behavior (right) and high-temperature behavior (left)

lower than Bayesian reconstructions and data-driven extractions from heavy-ion collisions, but are consistent with those obtained using the T-matrix formalism and with previous lattice QCD determinations.

4 Conclusions

We have presented a 2+1 lattice QCD determination of the heavy-quark diffusion coefficient at a physical pion mass. Our study covers a wide temperature range, from near the chiral crossover up to 10 GeV. Our results approach the strongly-coupled AdS/CFT estimate close to T_c, supporting the picture of the QGP as a strongly coupled and nearly perfect fluid. At higher temperatures, they gradually approach perturbative predictions, while remaining systematically lower than Bayesian reconstructions and data-driven extractions.

Overall, our results provide **the most comprehensive non-perturbative determination of heavy-quark diffusion to date**, bridging the low- and high-temperature regimes and offering valuable input for heavy-ion phenomenology.

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References

[1] A. Beraudo et al., Extraction of Heavy-Flavor Transport Coefficients in QCD Matter, Nucl. Phys. A **979**, 21 (2018), `1803.03824`. 10.1016/j.nuclphysa.2018.09.002

[2] X. Dong, Y.J. Lee, R. Rapp, Open Heavy-Flavor Production in Heavy-Ion Collisions, Ann. Rev. Nucl. Part. Sci. **69**, 417 (2019), `1903.07709`. 10.1146/annurev-nucl-101918-023806

[3] M. He, H. van Hees, R. Rapp, Heavy-quark diffusion in the quark–gluon plasma, Prog. Part. Nucl. Phys. **130**, 104020 (2023), `2204.09299`. 10.1016/j.ppnp.2023.104020

[4] S. Caron-Huot, G.D. Moore, Heavy quark diffusion in perturbative QCD at next-to-leading order, Phys. Rev. Lett. **100**, 052301 (2008), `0708.4232`. 10.1103/PhysRevLett.100.052301

[5] J. Casalderrey-Solana, D. Teaney, Heavy quark diffusion in strongly coupled N=4 Yang-Mills, Phys. Rev. D **74**, 085012 (2006), `hep-ph/0605199`. 10.1103/PhysRevD.74.085012

[6] C.P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, L.G. Yaffe, Energy loss of a heavy quark moving through N=4 supersymmetric Yang-Mills plasma, JHEP **07**, 013 (2006), `hep-th/0605158`. 10.1088/1126-6708/2006/07/013

[7] Y. Xu, J.E. Bernhard, S.A. Bass, M. Nahrgang, S. Cao, Data-driven analysis for the temperature and momentum dependence of the heavy-quark diffusion coefficient in relativistic heavy-ion collisions, Phys. Rev. C **97**, 014907 (2018), `1710.00807`. 10.1103/PhysRevC.97.014907

[8] B. Abelev et al. (ALICE), Suppression of high transverse momentum D mesons in central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, JHEP **09**, 112 (2012), `1203.2160`. 10.1007/JHEP09(2012)112

[9] D. Bollweg, J.L. Dasilva Golán, O. Kaczmarek, R.N. Larsen, G.D. Moore, S. Mukherjee, P. Petreczky, H.T. Shu, S. Stendebach, J.H. Weber (HotQCD), Temperature Dependence of Heavy Quark Diffusion from (2+1)-flavor Lattice QCD, JHEP (2025), `2506.11958`. 10.1007/JHEP09(2025)180

[10] A. Bouttefeux, M. Laine, Mass-suppressed effects in heavy quark diffusion, JHEP **12**, 150 (2020), `2010.07316`. 10.1007/JHEP12(2020)150

[11] E. Follana, Q. Mason, C. Davies, K. Hornbostel, G.P. Lepage, J. Shigemitsu, H. Trottier, K. Wong (HPQCD, UKQCD), Highly improved staggered quarks on the lattice, with applications to charm physics, Phys. Rev. D **75**, 054502 (2007), `hep-lat/0610092`. 10.1103/PhysRevD.75.054502

[12] M. Luscher, P. Weisz, On-shell improved lattice gauge theories, Commun. Math. Phys. **98**, 433 (1985), [Erratum: Commun.Math.Phys. 98, 433 (1985)]. 10.1007/BF01205792

[13] A. Ramos, S. Sint, Symanzik improvement of the gradient flow in lattice gauge theories, Eur. Phys. J. C **76**, 15 (2016), `1508.05552`. 10.1140/epjc/s10052-015-3831-9

[14] N. Brambilla, V. Leino, J. Mayer-Steudte, P. Petreczky (TUMQCD), Heavy quark diffusion coefficient with gradient flow, Phys. Rev. D **107**, 054508 (2023), `2206.02861`. 10.1103/PhysRevD.107.054508

[15] D. Banerjee, S. Datta, M. Laine, Lattice study of a magnetic contribution to heavy quark momentum diffusion, JHEP **08**, 128 (2022), `2204.14075`. 10.1007/JHEP08(2022)128