

# Determination of space diffusion coefficient by heavy flavour $R_{AA}$ and $v_n$ in an event-by-event approach

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**Abstract.** The strong interaction in the non-perturbative regime typical of the system created in heavy-ion collisions has been studied in the past years through an effective Quasi-Particle Model (*QPM*). Recently, an extension to a more realistic model named *QPM<sub>p</sub>*, which incorporates momentum-dependent parton masses as entailed by QCD asymptotic free dynamics, leads to a good agreement with the IQCD quark susceptibilities previously underestimated in the simple *QPM*. A main general result is that for charm quarks the extracted space diffusion  $D_s(T)$  appears to be significantly larger (about 40 – 50 %) wrt the recent IQCD data. Within an event-by-event transport approach based on Langevin equations, we evaluate  $R_{AA}(p_T)$  and  $v_2(p_T)$  for D mesons in *PbPb* at 5 *TeV*. We propose a novel analysis that point out the crucial role of the momentum dependence of the interaction to draw solid conclusions about the agreement between the phenomenological approach and lattice QCD calculation as well as to determine the thermalization time of heavy quarks.

## 1 Introduction

Heavy quarks (HQs), such as charm and bottom, are powerful probes of the medium formed in ultra-relativistic heavy-ion collisions (uRHICs) [1, 2]. Produced out of equilibrium via perturbative QCD processes in the early stages and being their masses  $M_{HQ} \gg T$ , they are expected to thermalize more slowly than light quarks in the Quark-Gluon Plasma (QGP), giving information about the evolution of the system. The propagation of HQs has frequently been modeled in a Brownian motion since the large mass of HQs should generally lead to collisions with small momentum transferred. Under these conditions, the Boltzmann transport equation reduces to Fokker-Planck dynamics. This framework has been widely employed to compute key observables such as the nuclear suppression factor  $R_{AA}$  which describes the modification of heavy-quark spectra in nucleus–nucleus (AA) collisions with respect to proton–proton (pp) collisions, along with the elliptic flow  $v_2 = \langle \cos(2\phi_p) \rangle$ , which quantifies the azimuthal anisotropy in the angular distribution of heavy-flavour hadrons [1–6]. Among the effective models that have been developed to incorporate non-perturbative aspects of the interaction between HQs and plasma, there are Quasi-Particle Models (*QPM<sub>s</sub>*) [7, 8] which

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include temperature dependent masses and are able to reproduce the thermodynamic properties obtained in lattice QCD (IQCD). An alternative approach is provided by the TAMU framework, which models HQ–medium interactions via a potential kernel fitted to IQCD free energy—and more recently to the Wilson correlator—yielding a stronger momentum dependence of the drag coefficient [9, 10]. Recently, new IQCD data including dynamical fermions show a significant smaller spatial diffusion coefficient  $2\pi T D_s(T) \approx 1$  wrt the previous estimation in quenched approximation, suggesting a thermalization time  $\tau_{th} \approx 1.4 fm/c$  at  $T \approx 1.2 T_c$  at the charm mass scale compatible for  $p \rightarrow 0$  to the AdS/CFT predictions and substantially lower than phenomenological estimations. A central goal of this study is to assess whether such low values of  $D_s(T)$  can consistently reproduce the experimental trends observed in the main heavy flavour observables and the role which the momentum dependence plays in this context.

## 2 Initial conditions and transport approach for charm and bulk

In this work, the initial conditions of partons in coordinate space are described using a Monte Carlo Glauber model while in the momentum space, soft partons are initialized with a thermal equilibrium distribution and the charm quark with the Fixed Order + Next-to-Leading Log (FONLL) calculations. Further details are provided in Ref. [4, 5]. The bulk evolution of the QGP is described by the relativistic Boltzmann equation at constant  $\eta/s \approx 0.1$ , as extensively discussed in our earlier works on heavy-quark transport [11–14], while the propagation of HQs in the QGP is modeled within the Brownian motion approximation using an event-by-event Langevin approach [15] with the following equations:

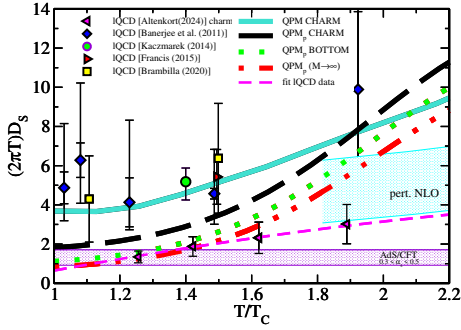
$$dx_j = \frac{p_j}{E} dt \quad (1)$$

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{j,k} \rho_k \quad (2)$$

where  $\Gamma$  and  $C_{j,k} = \sqrt{2D_p(E)}\delta_{jk}$  are directly connected to the *drag*  $A$  and *diffusion*  $D_p$  coefficients connected by fluctuation–dissipation relation  $D_p(p) = A(p)E(p)T$ . In the static limit  $p \rightarrow 0$ , the drag coefficient reduces to  $A(p \rightarrow 0) = \gamma$  and  $D_p$  can be related to  $D_s$  with  $D_s = \frac{T^2}{D_p} = \frac{T}{M_{HQ}\gamma} = \frac{T}{M_{HQ}}\tau_{th}$ . The drag coefficient is evaluated within different phenomenological approaches: among them, a recent extension of *QPM*, named *QPM<sub>p</sub>*, introduces momentum-dependent masses that asymptotically recover current quark masses, successfully reproducing both the IQCD Equation of State and light/strange quark susceptibilities [8]. In this approach, the drag coefficient at finite momenta  $A(p, T)$  is evaluated from the scattering matrices  $|\mathcal{M}_{HQ}|$  with effective coupling  $g(T)$  obtained from a fit to IQCD thermodynamics [16]. Finally, in our model, charm quarks hadronize through a hybrid mechanism combining coalescence and fragmentation [17–19].

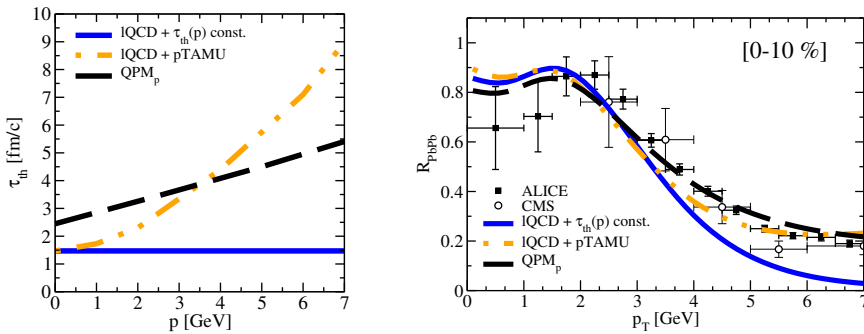
## 3 Results

In this section, we firstly discuss the  $D_s(T)$  obtained in *QPM<sub>p</sub>* in comparison to the recent IQCD data. Furthermore, we have also examined how different momentum dependence in the drag coefficient  $A$  and/or spatial diffusion coefficient  $D_s$ , such as the ones from the *QPM<sub>p</sub>* and *TAMU* approaches, can affect the description of the main heavy flavour observables. In Fig. 1, we show the  $D_s(p \rightarrow 0)$  for charm, bottom and infinite massive quark in the *QPM<sub>p</sub>* in comparison to the IQCD data. We find that the  $D_s(p \rightarrow 0)$  for charm in *QPM<sub>p</sub>* is smaller than the one in standard *QPM* and closer to the new IQCD data (pink left triangles) near  $T_c$ , but



**Figure 1.** Spatial diffusion coefficient  $2\pi T D_s$  in  $QPM$  and  $QPM_p$  approaches for charm quark compared to available IQCD data with dynamical fermions [20–22]. Further references to the IQCD data in quenched approximation, estimation in AdS/CFT and perturbative NLO are provided in Ref. [15].

still at least 40% larger. The  $D_s$  for  $M_{HQ} \rightarrow \infty$  and bottom quark is in very close agreement to the new IQCD data. It is worth noting that the  $D_s$  provides an estimate of the thermalization time  $\tau_{th}$  at vanishing momentum while, at finite momenta, the interaction strength may substantially decrease, resulting in a larger  $\tau_{th}$ . In this context, we have investigated how the momentum dependence of the drag coefficient, which is related to the thermalization time by  $\tau_{th}(p) = 1/A(p)$ , affects the description of the D-meson observables, also assessing the compatibility of the small value of  $D_s(T)$  of IQCD predictions to the experimental data. In this direction, we have studied different cases: the first one in which the  $D_s(T)$  is taken



**Figure 2.** Thermalization time  $\tau_{th}$  (left panel) and nuclear modification factor  $R_{AA}$  for the different cases discussed in the text. Experimental data from ALICE Coll.[23] and CMS Coll. [24]

from a fit to IQCD data shown Fig. 1 (pink dotted line) and the drag is obtained from FDT with  $A = \frac{T}{MD_s}$ . This case corresponds to an extreme scenario where the  $\tau_{th}$  is momentum independent as shown by the solid blue line in the left plot of Fig. 2. Another case in which the  $D_s(T)$  is fitted to IQCD data but imposing a  $p$ -dependence of drag coefficient  $A(p)$  (and  $\tau_{th}(p)$ ) as in the TAMU approach from Ref. [10]. We have also studied the case in which the  $D_s(T, p)$  comes totally from  $QPM_p$ . The corresponding  $\tau_{th}$  of the different cases (left plot of Fig. 2), show a strong momentum dependence in the TAMU approach (orange dot-dashed line), while the  $\tau_{th}$  in  $QPM_p$  (black dashed line) linearly increases starting from a value at  $p = 0$  which is about 50 % higher than IQCD predictions. In the right plot of Fig. 2, we show the  $R_{AA}$  in PbPb at  $5 TeV$  in 0 – 10 % for the different cases studied: we highlight that the first case (IQCD +  $\tau_{th}(p)$  const.), fails to reproduce the experimental data by the ALICE and CMS collaborations, yielding values that are excessively suppressed and tend toward zero. Apart from the previously discussed scenario, all other cases examined with different exhibit good agreement with the experimental data even if characterized by different  $D_s$  at  $p = 0$ . We

find similar results across different centralities classes and also for the D mesons anisotropic flows  $v_2$  and  $v_3$ . See Ref. [15] for more details.

## 4 Conclusions

In this work, we have investigated charm-quark propagation in the QGP using an event-by-event Langevin framework coupled to a hybrid hadronization by coalescence plus fragmentation to obtain the final hadron observables. We have focused on the key D mesons observables, such as the  $R_{AA}$  at top LHC energies, examining whether the small values of the charm quark  $D_s$  from recent IQCD data are compatible with experimental results. We find that, assuming an interaction strength in line with the latest IQCD results, only a strong momentum dependence, such as the one in the TAMU approach, can yields a  $R_{AA}$  consistent with experimental data. An extreme scenario with  $2\pi T D_s \simeq 1$ , as suggested by IQCD and AdS/CFT, but without momentum dependence, results in D mesons observables that deviate significantly from phenomenology. Moreover, a 50 % larger  $D_s(T)$ , combined with a milder momentum dependence as in the  $QPM_p$  scenario, still yields a good description of D-meson observables. Our analysis shows that, based on  $R_{AA}$ ,  $v_2$  and  $v_3$ , a clear distinction between the two scenarios is not achievable. A more comprehensive study, including additional observables or refined statistical methods, is needed to draw firmer conclusions.

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