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NLO calculations at small x

Guillaume Beuf[1], *Tuomas* Lappi[2,3,*], *Heikki* Mäntysaari[2,3], *Risto* Paatelainen[3], and *Jani* Penttala[4,5]

[1]Theoretical Physics Division, National Centre for Nuclear Research, Pasteura 7, Warsaw 02-093, Poland

[2]Department of Physics, University of Jyväskylä, P.O. Box 35, 40014 University of Jyväskylä, Finland

[3]Helsinki Institute of Physics, P.O. Box 64, 00014 University of Helsinki, Finland

[4]Department of Physics and Astronomy, University of California, Los Angeles, CA 90095, USA

[5]Mani L. Bhaumik Institute for Theoretical Physics, University of California, Los Angeles, CA 90095, USA

Abstract. We discuss recent progress in next-to-leading order calculations for dilute-dense-processes in the small-x gluon saturation regime, especially for Deep Inelastic Scattering observables. A systematical formalism for such observables is provided by a combination of light cone perturbation theory for the dilute probe, combined with a Color Glass Condensate picture for eikonal scattering off the dense gluonic target. In particular we focus here on the recent calculation of the diffractive DIS structure function, which is expected to be more sensitive to gluon saturation than inclusive DIS observables.

1 Introduction

For understanding the initial stages of heavy ion collisions, we need to understand small-x gluons in a high energy nucleus. How many are there? How are they distributed in coordinate and momentum? What is the role of nonlinear interactions; is the infrared regulated by gluon saturation or by confinement? To understand these issues it is useful to use a picture where high energy collisions are understood as eikonal scattering. This picture can be used to relate high energy Deep Inelastic Scattering (DIS) to heavy ion collisions. After discussing this relation we will describe a recent calculation [1] (building on earlier partial results in [2]) at next-to-leading order (NLO) in the dipole picture of the diffractive structure function, which is a particularly powerful probe of saturation physics in DIS.

In order to measure the small-x glue, i.e. the color field, in a nucleus, one needs to send a dilute colored probe through it. At high energy the interaction is eikonal, meaning that the transverse coordinate of the probe is conserved in the scattering. The eikonal scattering amplitude for a quark is given by the *Wilson line*

$$\mathbb{P}\exp\left\{-ig\int^{x^+} \mathrm{d}y^+ A^-(y^+,x^-,\mathbf{x})\right\} \underset{x^+\to\infty}{\approx} V(\mathbf{x}) \in \mathrm{SU}(N_\mathrm{c}), \tag{1}$$

*e-mail: tuomas.v.v.lappi@jyu.fi

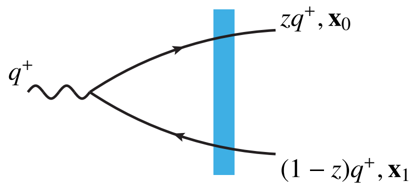

Figure 1. Diagram to be computed for leading order diffractive DIS, where the blue area denotes the shockwave of the target gluon field. The part of the calculation specific to diffractive DIS is relating the coordinates and longitudinal momenta of the final state $q\bar{q}$ system to the (measured) invariant mass M_X.

a path ordered exponential of the color field. In DIS, the color field is probed by a color-neutral quark-antiquark dipole, whose scattering amplitude is given by a *color dipole* operator

$$\mathcal{N}(r = |\mathbf{x} - \mathbf{y}|) = 1 - \left\langle \frac{1}{N_\mathrm{c}} \mathrm{Tr} V^\dagger(\mathbf{x}) V(\mathbf{y}) \right\rangle. \tag{2}$$

From basic group theory constraints it follows that such a dipole amplitude smoothly interpolates between perturbative small values as $r \to 0$ (color transparency) and saturates to one for large dipoles. The distance scale at which this happens $\sim 1/Q_\mathrm{s}$ defines a *saturation scale* Q_s, where even a weak coupling description is nonperturbative.

There is a 1-to-1 mapping from dipole amplitude to the glasma fields in the initial stage of a heavy ion collision. Starting from the Wilson lines $V_{(1,2)}(\mathbf{x})$ for the colliding nuclei (1) and (2) one calculates the Light Cone Gauge fields $A^i_{(1,2)} = \frac{i}{g} V_{(1,2)}(\mathbf{x}) \partial_i V^\dagger_{(1,2)}(\mathbf{x})$, which give the initial conditions for the glasma fields. Thus, high energy DIS directly probes the degrees of freedom which are needed for understanding the initial stage of a heavy ion collision, and the saturation scale Q_s becomes the dominant momentum scale for gluons that then equilibrate to become a quark-gluon plasma.

2 Dipole picture of Deep Inelastic Scattering and diffraction

For small-x_{Bj} DIS, the eikonal scattering limit corresponds to the dipole picture. Here, scattering amplitudes factorize into on one hand the development of a partonic component in the γ^*, to leading order a $q\bar{q}$ dipole, and on the other hand the eikonal scattering of this partonic state off the target. The leading order dipole picture has been used for many successful fits to HERA data. Adding the emission and interaction of a soft gluon one obtains a large logarithmic correction $\sim \alpha_\mathrm{s} \ln 1/x_{Bj}$, which can be resummed by absorbing it into a renormalization of the dipole amplitude by the Balitsky-Kovchegov [3–5] (BK) equation. Here next-to-leading order (NLO) means that this additional gluon is included not just in the soft limit, but with exact kinematics. Recently there have been many advances in performing such NLO calculations, and the results for the total DIS cross section have already been successfully confronted with HERA data for both massless [6] and massive quarks [7].

Diffractive DIS provides a subclass of DIS scattering that is particularly sensitive to gluon saturation, since in the perturbative dilute limit the cross section is quadratic in the gluon density of the target. Here the process is characterized by the target proton or nucleus staying intact (coherent) or disintegrating into a color neutral small invariant mass system (incoherent), separated from the γ^* remnants by a rapidity gap. The cross section is measured differentially in the invariant mass ${M_X}^2$ of the diffractive system, parametrized by $\beta = Q^2/(Q^2 + M_X^2)$. The rapidity gap size, and the interval of BK evolution of the target, is given by $\Delta y_\mathrm{gap} \sim \ln 1/x_\mathbb{P}$ with $x_{Bj} = x_\mathbb{P}\beta$. Here we are concerned with the limit, more relevant for EIC kinematics, where $x_\mathbb{P}$ is parametrically small but β not.

The amplitude for diffractive DIS at leading order in the dipole picture requires the computation of only one diagram, depicted in Fig. 1. This has been done in many references, but the result for general kinematics, with an eikonal interaction with the target, without assumptions about the impact parameter profile of the target, can be found in [2].

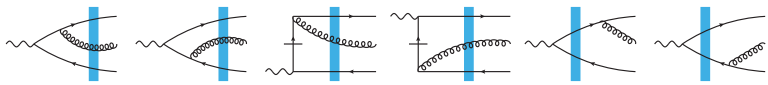

Figure 2. Radiative diagrams for the amplitude.

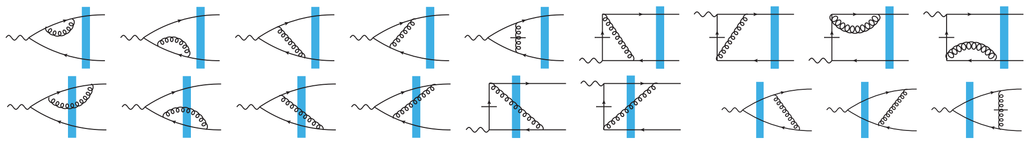

Figure 3. Virtual diagrams for the amplitude.

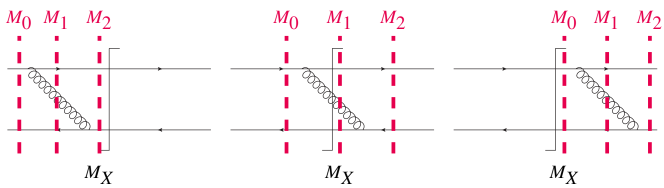

Figure 4. Diagrams with gluon exchange in the final state.

3 Diffractive DIS at NLO

The NLO diffractive DIS process, calculated in [1], involves a much larger set of corrections that need to be evaluated. In radiative contributions (Fig. 2) the gluon is part of the measured final state ${M_X}^2$. Virtual corrections are depicted in Fig. 3. From previous NLO computations one already knows the $\gamma^* \to q\bar{q}$ vertex corrections. Contributions where the gluon crosses the target but not the cut are loop corrections to amplitude, but "tree level diagrams" for the $\gamma^* \to$ partons wavefunction. They involve 3-point operators of Wilson lines because the gluon also interacts with the target and are responsible for the BK evolution of the LO amplitude. The most difficult part technically are the final state interactions, where a gluon is emitted and absorbed (or produced) after the target shockwave.

The main message of this paper is that we have calculated all these contributions in [1], and obtained a finite result for the diffractive structure function. This is a very appealing observable for saturation physics: it is a clean IR-safe observable, whose perturbative definition precisely matches the experimental one, without dependence on fragmentation functions or jet algorithms. It probes saturation dynamics in a kinematical regime that is much more accessible at the EIC than for example dijets.

We use a regularization with a cutoff α in k^+-integrals and $2-2\varepsilon$ transverse dimensions. Ultraviolet $\frac{1}{\varepsilon}$ and $\frac{1}{\varepsilon}\ln\alpha$ divergences cancel between the $\gamma^* \to q\bar{q}$ vertex, the loop in the quark propagator and the gluon crossing the shockwave. Collinear $\frac{1}{\varepsilon}$ divergences cancel between the quark propagator and final state emissions. A power law $1/\alpha$ cancels between normal and instantaneous exchanges in the final state, and a $\ln^2\alpha$ between the final state exchange and emission diagrams: here the pattern of cancellation is different for a fixed M_X final state than for dijets. A remaining single log $\ln\alpha$ is absorbed into BK evolution of the target.

In order to separate these different types of divergences we use a "reverse unitarity" trick. Consider for example the three contributions depicted in Fig. 4. They share the same vertices, but differ in which one of the states ($q\bar{q}$ before or after gluon exchange, or the $q\bar{q}g$ state) is the final one with invariant mass M_X. Thus the energy denominators (off-shell propagators) and final state invariant mass restrictions can be combined noticing that:

$$\frac{\delta(M_X{}^2 - M_2^2)}{(\Delta M_{21}^2 + i\delta)(\Delta M_{20}^2 + i\delta)} + \frac{\delta(M_X{}^2 - M_1^2)}{(\Delta M_{10}^2 + i\delta)(\Delta M_{12}^2 - i\delta)} + \frac{\delta(M_X{}^2 - M_0^2)}{(\Delta M_{01}^2 - i\delta)(\Delta M_{02}^2 - i\delta)}$$
$$= \frac{1}{2\pi i}\left[\frac{1}{(M_X{}^2 - M_0^2 - i\delta)(M_X{}^2 - M_1^2 - i\delta)(M_X{}^2 - M_2^2 - i\delta)} - \text{c.c.}\right], \quad (3)$$

with $\Delta M_{ij}^2 \equiv M_i^2 - M_j^2$. If one then expresses the transverse momentum dot products in the numerator in terms of the invariant masses M_0^2, M_1^2, M_2^2 of the three states, it becomes possible to combine the contributions before integration and separate the different divergence types into separate terms.

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