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Bulk viscosity dependence on the symmetry energy

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Abstract. We show that for neutrino-transparent npe matter, the equation-of-state dependence of bulk-viscous transport coefficients can be completely determined by the nuclear symmetry energy. We also show that changes in the symmetry energy slope L can result in orders of magnitude changes in the bulk viscosity.

1 Introduction

Due to electric charge neutrality and β-equilibrium conditions, neutron stars exhibit isospin asymmetry that differs drastically from that of nuclei [1]. One key phenomenological tool that connects nuclear experiments to neutron star observations is the nuclear symmetry energy, E_{sym}, which quantifies the energy difference between pure neutron matter and symmetric nuclear matter. The symmetry energy plays a central role in constraining the equation of state (EoS) of neutron star matter [2].

In a previous work [3], we demonstrated that, within relativistic mean field models, the bulk viscosity arising from weak interactions is sensitive to variations in E_{sym}, in matter composed of neutrons, protons, and electrons (npe matter). In this proceeding, we show that for npe matter, the EoS dependence of the bulk-viscous transport coefficients can be fully computed from E_{sym}. Furthermore, we find that the pressure correction induced by deviations from β-equilibrium exhibits elastic behavior near nuclear saturation density—relevant to the early inspiral phase of binary neutron star mergers [4].

2 Bulk viscosity from chemical imbalance

We consider a binary neutron star merger at low enough temperatures and densities where the neutrino mean free path exceeds the star's radius [5] such that the system is then neutrino transparent and only contains protons (p), neutrons (n), and electrons (e). For such a system, β-equilibrium can be characterized by the difference in chemical potentials, $\delta\mu = \mu_n - \mu_p - \mu_e$, where $\delta\mu = 0$ represents β-equilibrium. The system can be described by a quasi-equilibrium

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thermodynamic state with the variables $\{s, n_B, Y\}$, where s is the entropy density, and $Y = n_e/n_B$ is the electron fraction. The system can be described by a set of dynamical equations

$$\nabla_\mu(su^\mu) = \frac{Q + \delta\mu\,\Gamma_e}{T}, \qquad \nabla_\mu(n_B u^\mu) = 0, \qquad u^\mu\nabla_\mu Y = \frac{\Gamma_e}{n_B}, \tag{1}$$

where Q is the radiative energy loss due to neutrino escape, Γ_e is the flavor equilibration rate of the direct and modified Urca processes, T is the temperature, and u^μ is the local 4-velocity.

Assuming a small and linear perturbation around β-equilibrium, one can approximate the flavor equilibration rate $\Gamma_e \approx \lambda\delta\mu$ [5–7]. One can then derive an equation of motion for the out-of-β-equilibrium correction to pressure, the bulk scalar $\Pi \equiv \mathcal{P} - \mathcal{P}|_{\delta\mu=0}$, [8, 9], which is the bulk-viscous Israel-Stewart equation [10],

$$u^\mu\nabla_\mu\Pi = -\frac{\Pi}{\tau_\Pi} - \frac{\zeta_0}{\tau_\Pi}\nabla_\mu u^\mu, \tag{2}$$

where the relaxation time τ_Π and the bulk viscosity ζ_0 are given by

$$\tau_\Pi = -\frac{n_B}{\lambda}\left(\frac{\partial\delta\mu}{\partial Y}\bigg|_{n_B}\right)^{-1}, \qquad \zeta_0 = -\frac{n_B^2}{\lambda}\left(\frac{\partial\mathcal{P}}{\partial\delta\mu}\right)_{n_B,\delta\mu=0}\left(\frac{\partial\delta\mu}{\partial Y}\bigg|_{n_B}\right)^{-1}\frac{\partial\delta\mu}{\partial n_B}\bigg|_Y. \tag{3}$$

If one assumes small amplitude baryon density oscillations $\sim e^{-i\omega t}$ around a spatially uniform $\beta-$equilibrated state, one can derive a frequency-dependent bulk viscosity [11],

$$\zeta(\omega) = \frac{\zeta_0}{1 + \tau_\Pi^2\omega^2}. \tag{4}$$

Different limits may lead to different phenomenologies [8]. When $\tau_\Pi\omega \ll 1$, Eq. (2) is reduced to $\Pi \sim -\zeta_0\nabla_\mu u^\mu$ and the system is in the Navier-Stokes viscous fluid regime. When $\tau_\Pi\omega \sim 1$, the system is in the resonant regime, leading to maximal dissipation. When $\tau_\Pi\omega \gg 1$, the system is in the *frozen* regime where the system is out-of-β-equilibrium, Y stays fixed, but the system does not dissipate. This *frozen* regime is relevant for early-to-late binary neutron star inspirals, where $T \sim 10^5$ K, tidal forces deform the stars with frequencies $\omega \sim$ kHz [12], and $\delta\mu \neq 0$.

3 Symmetry energy dependence of the transport coefficients

One can expand the energy per baryon, $E(n_B, \delta)$, in powers of $\delta \equiv 1 - 2\,Y$,

$$E(n_B, \delta) \approx E(n_B, \delta = 0) + E_{sym}(n_B)\,\delta^2 + O(\delta^4), \tag{5}$$

where $E_{sym}(n_B)$ is usually referred to as the nuclear symmetry energy [13]. One can further expand $E_{sym}(n_B)$ around the nuclear saturation density n_{sat},

$$E_{sym}(n_B) = S + L\left(\frac{n_B - n_{\text{sat}}}{3n_{\text{sat}}}\right) + \frac{K_{sym}}{2}\left(\frac{n_B - n_{\text{sat}}}{3n_{\text{sat}}}\right)^2 + \ldots, \tag{6}$$

where S is the symmetry energy at n_{sat}, L is the slope, and K_{sym} is the curvature. By noting that $\delta\mu = -2\frac{\partial E}{\partial\delta}$, we see that the transport coefficients given by Eq. (3) may only depend on the symmetry energy but not on the energy from the symmetric part, such that

$$\tau_\Pi \sim \frac{1}{E_{sym}(n_B)}, \quad \zeta_0 \sim \left(\frac{\partial E_{sym}(n_B)}{\partial n_B}\Big/E_{sym}(n_B)\right)^2. \tag{7}$$

At n_{sat}, these transport coefficients will depend only on S and L, as higher-order coefficients of the symmetry energy naturally drop out.

To test the validity of calculating the transport coefficients using the symmetry energy expansion, we employ a set of chiral EFT parameterizations that fit into the uncertainty band given by Ref. [14]. In Fig. 1, we plot the dimensionless bulk modulus $\zeta_0/(\tau_\Pi n_{\mathrm{sat}}^{4/3})$ as a function of n_B for all chiral EFT parameterizations. The blue curves are calculated directly from the chiral EFT parameterizations, while the red curves are calculated using the respective symmetry energy from each parameterization. As Fig. 1 shows, the calculations from the symmetry energy are in excellent agreement with the calculations without any approximations.

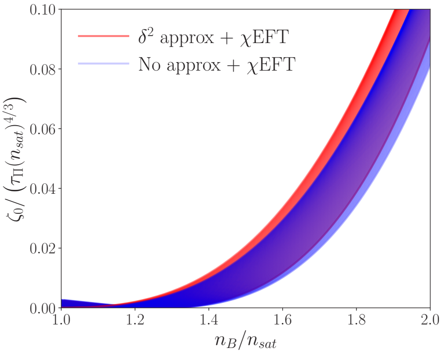

Figure 1. The dimensionless bulk modulus as a function of the baryon density. The blue curves are calculated directly from the chiral EFT parameterization. The red curves are calculated using the respective symmetry energy from each parameterization. The purple region denotes the overlap between blue and red curves, with darker colors reflecting a higher density of curves.

In Fig. 2, we adopt the ranges: $S \in [30, 40]$ MeV and $L \in [30, 150]$ MeV. We then plot ζ_0 vs. τ_Π using all combinations of S and L. As the figure shows, depending on which value of L we use, $\zeta_0(n_{\mathrm{sat}})$ can change by orders of magnitude.

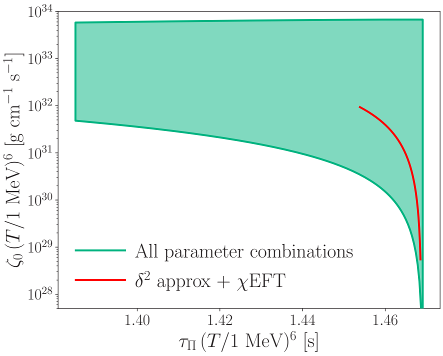

Figure 2. Bulk viscosity vs. relaxation time at n_{sat}. The green region is calculated using $S \in [30, 40]$ MeV and $L \in [30, 150]$ MeV. The red curve is calculated using the chiral EFT parameterizations.

4 Conclusion

In this paper, we showed that given the flavor equilibration rates, the relaxation time τ_Π and bulk viscosity ζ_0 can be completely written in terms of the symmetry energy, in the case of neutrino-transparent *npe* matter. We validate our calculations using a set of chiral EFT parameterizations. We also showed that changes in the symmetry energy slope L can induce orders-of-magnitude changes in ζ_0.

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