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Structure and evolution of shock waves in the presence of magnetic fields for heavy-ion collisions and astrophysics applications

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Abstract. This study presents a numerical and analytical model of hydrodynamic and magnetohydrodynamic piston-driven shock waves, developed to generate benchmark solutions for shock dynamics in the presence and absence of magnetic fields in core-collapse supernovae environments.

1 Introduction

Shock waves produced in matter under extreme conditions, such as in high-energy nuclear and astrophysical events, drive complex processes like energy transfer, particle acceleration, and plasma instability. Magnetic fields alter shock wave properties, influencing energy density, pressure, and turbulence in these extreme environments. In relativistic heavy-ion collisions (HIC), atomic nuclei collide at speeds approaching the speed of light, depositing enormous kinetic energy into a small volume and generating a quark-gluon plasma (QGP). This hot, dense medium rapidly expands hydrodynamically, forming structures akin to shock fronts and rarefactions, with velocities exceeding $0.6c$ and energy densities above $10\ \text{GeV/fm}^3$ [1]. In core-collapse supernovae (CCSNe), a similar hydrodynamic shock arises during the collapse of a massive star's iron core. As the core exceeds nuclear saturation density ($\sim 2.7 \times 10^{14}\ \text{g/cm}^3$), repulsive nuclear forces halt the collapse, triggering a rebound and launching a shock wave outward from the newly formed proto-neutron star (PNS). Matter at the bounce stage may be accelerated to mildly relativistic speeds, up to $0.3c$–$0.4c$, over distances of tens of kilometres [2].

In both HIC and CCSNe, magnetic fields strongly influence shock propagation. In non-central HIC, transverse nuclear motion generates extreme but short-lived fields ($\sim 10^{18}$ G, decaying within 10^{-23} s). These fields induce magnetohydrodynamic (MHD) effects in the QGP, such as transverse flow anisotropy and the chiral magnetic effect, where magnetic fields couple to axial quark currents [2]. Pre-collapse fields ($\sim 10^4$ G) can be amplified by flux freezing, differential rotation, and magnetorotational instabilities (MRI), reaching post-bounce strengths of 10^{15}–10^{16} G [2]. These fields exert significant pressure and tension, potentially collimating the bounce shock into bipolar outflows and contributing to explosion mechanisms in magnetorotational supernovae.

To model such extreme environments, relativistic magnetohydrodynamics (RMHD) provides a unified theoretical framework [3]. Despite scale differences, from femtometre-sized QGP fireballs to stellar explosions spanning thousands of kilometres, both systems are described by the RMHD equations. Muronga and Rischke (2004) extended causal dissipative hydrodynamics to describe the space-time evolution of quark-gluon plasma in heavy-ion collisions, demonstrating the critical role of viscosity and transport coefficients in quantitative modelling. This emphasis on causal formulations and dissipative effects motivates the present

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study's use of piston-driven shock waves as controlled benchmarks, ensuring that both analytical and numerical models can later be extended toward more realistic relativistic magnetohydrodynamic (RMHD) treatments relevant for core-collapse supernovae [4].

The piston-driven shock problem is particularly relevant to CCSNe physics [5]; in this setup, a boundary (the "piston") initiates a shock analogous to the outward-moving bounce shock in CCSNe, and, when extended to include magnetic fields and stratified density profiles , it encapsulates key features of magnetised stellar explosions, allowing semi-analytical solutions in idealised regimes. This study focuses on a preliminary MHD analysis of the piston-driven shock wave model within CCSNe applications, comparing numerical simulations to analytical descriptions of post-shock conditions in a stratified medium by varying state profiles such as pressure, density, and velocity to assess their influence on shock structure and stability, thus providing a foundation for understanding shock waves across diverse scales and environments.

2 One-Dimensional Piston-Driven Shock Wave Model

The piston-driven shock wave model provides a controlled framework for simulating shock formation via mechanical compression. A piston, located initially at $x = x_0$, begins moving with constant velocity U_p into a uniform medium at $t > 0$. This study is restricted to a one-dimensional planar geometry, with all variables depending only on x and t. The fluid obeys ideal MHD assumptions: infinite conductivity , frozen-in field lines, and flat spacetime metric $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. The four-velocity is $u^\mu = \gamma(1, v_x, 0, 0)$ with $\gamma = (1 - v_x^2)^{-1/2}$. The equation of state is $p = (\Gamma - 1)\rho\epsilon, \qquad h = 1 + \frac{\epsilon + p}{\rho}$, where ρ is proper density, p pressure, ϵ specific internal energy, and h specific enthalpy. Magnetic effects enter through the four-vector b^μ, contributing anisotropic stresses and wave modes such as Alfvén and magnetosonic modes.

Therefore, the governing equations; baryon number and energy-momentum conservation are given as follows:

$$\partial_\mu N^\mu = 0, \qquad N^\mu = \rho u^\mu, \tag{1}$$

$$\partial_\mu T^{\mu\nu} = 0, \tag{2}$$

with stress-energy tensor:

$$T^{\mu\nu} = (\rho h + p)u^\mu u^\nu + p g^{\mu\nu} - b^\mu b^\nu + \tfrac{1}{2} b^2 g^{\mu\nu}. \tag{3}$$

Across a discontinuity, the Rankine–Hugoniot relations apply. These relations guarantee conservation of mass, momentum, and energy flux across the s hock. In the HD limit ($b^\mu = 0$), these reduce to the Landau–Lifshitz piston relations [6]. In MHD, magnetic pressure and tension modify momentum conservation, leading to faster shocks and altered compression ratios. In the broader context of relativistic fluid dynamics, second-order dissipative frameworks have been developed to overcome limitations of the Eckart and Landau–Lifshitz first-order theories. Though this piston-driven setup is inviscid, this work forms the foundation for future extensions of incorporating viscosity and dissipation to build upon the realistic formulations employed by Muronga and Rischke in heavy-ion simulations [4].

2.1 Numerical Formulation

For computational use, Eqs. (1)–(2) are expressed in conservative form [7]:

$$\partial_t \mathbf{U} + \partial_x \mathbf{F}(\mathbf{U}) = 0, \tag{4}$$

with conserved vector and fluxes

$$\mathbf{U} = (D,\ S^x,\ \tau,\ \mathbf{b}_\perp)^T, \quad \mathbf{b}_\perp = (b^y, b^z), \qquad \mathbf{v}_\perp = (v_y, v_z),$$
$$\mathbf{F} = (Dv_x,\ S^x v_x + p^* - (b^x)^2,\ S^x v_x + p^* v_x - b^0 b^x - Dv_x,\ \mathbf{b}_\perp v_x - b^x \mathbf{v}_\perp)^T.$$

Here, $D = \rho\gamma$, is the conserved rest-mass density, $S^x = \rho h \gamma^2 v_x + b^2 v_x - b^0 b^x$ is the x-component of the total momentum density, $\tau = \rho h \gamma^2 - p + \frac{1}{2} b^2 - (b^0)^2$ is the total energy density minus rest-mass energy, v is the fluid three-velocity, and $p^* = p + \frac{1}{2} b^2$ is the total pressure. These expressions define the conserved quantities **U** and their fluxes **F** as functions of the primitive variables [3].

The governing equations, Eqs. (4), were solved numerically with the `PLUTO` framework [7], which uses the finite volume method and high-resolution shock-capturing schemes based on approximate Riemann solvers. For this study, the Harten-Lax-van Leer (HLL) solver. Time integration relied on a second-order Runge-Kutta method. This setup preserved conservation laws with numerical diffusion limited to a few cells around the shock, ensuring solutions accurately reproduced the Rankine–Hugoniot structure and maintained global conservation of mass, momentum, and energy.

3 Results and Discussion

To model the piston-driven shock wave models for both hydrodynamics (HD) and magneto-hydrodynamics (MHD) the following initial and boundary conditions shown in table 1 were used.

Table 1. Initial conditions and X1-boundary conditions for the MHD Numerical and Analytical piston-driven shock wave models.

Case	Region / Side	ρ	p	v_x	v_y	v_z	b_y	X1-Boundary
Pre-shock	Region 1	0.125	1.0	0.0	0.0	0.0	0.0	outflow
Post-shock	Region 2	ρ_2	p_2	0.5	0.0	0.0	0.0	moving piston
MHD	Region 1	0.125	1.0	0.0	0.0	0.0	1.0	outflow
Analytical	Region 2	ρ_2	p_2	0.5	0.0	0.0	1.0	moving piston
MHD	Region 1	0.125	1.0	0.0	0.0	0.0	1.0	outflow
Numerical	Region 2	ρ_2	p_2	0.5	0.0	0.0	1.0	moving piston
MHD	Region 1	0.125	1.0	0.0	0.0	0.0	0.1	outflow
Numerical	Region 2	ρ_2	p_2	0.5	0.0	0.0	0.1	moving piston
MHD	Region 1	0.125	1.0	0.0	0.0	0.0	0.5	outflow
Numerical	Region 2	ρ_2	p_2	0.5	0.0	0.0	0.5	moving piston

The model was validated for hydrodynamics case as shown in Fig 1 a-b). The HD and MHD piston-driven shock models reproduces analytical Rankine–Hugoniot solutions for density, pressure, and velocity. The post-shock plateau and piston and shock locations are accurate with $< 1.2\%$ and $< 1.5\%$ relative errors, respectively. A uniform transverse magnetic field is applied as indicated in Fig. 1 c). This addition modifies the downstream pressure:

$$p_2 = p_1 + \rho_1 V_s^2 \left(1 - \frac{1}{r}\right) - \frac{(r^2 - 1)}{2\mu_0} b_{y1}^2,$$

with compression ratio $r = \rho_2/\rho_1$ and upstream field b_{y1}. As the magnetic field increased b_y (Fig. 1 d)), there is acceleration of the shock, raising of the post-shock pressure, and slight reduction of the density. higher b_{y1} leads to shocks propagating faster, with higher downstream pressure and slightly lower density. Magnetic pressure diverts piston work, modifying energy distribution and compression. These trends agree with theory based on plasma beta and magnetosonic speeds.

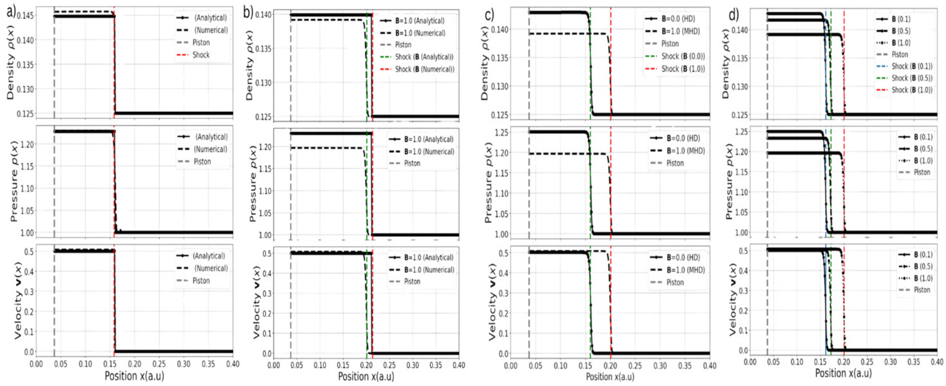

Figure 1. Profiles of density, pressure, and velocity a) HD Case: Comparison of analytical and numerical solutions for piston-driven shocks at $t = 0.08$ b) MHD case: Comparison of MHD and MHD analytical and numerical solutions for the piston-driven shock tube. Magnetic pressure raises post-shock pressure and lowers downstream density. d) Effects of the addition and increase of the magnetic field b_y investigation.

4 Conclusion

One-dimensional piston-driven shocks in HD and MHD were validated analytically and numerically. Magnetic fields accelerate shocks, elevate post-shock pressure, and reduce compression relative to HD. This work provides benchmarks for HIC and CCSNe shock studies and lays the foundation for future RMHD and dissipative extensions.

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