

# Higher order net-baryon number cumulants and baryon-strangeness correlations: Comparing QCD results on the pseudo-critical line with RHIC-BES II results on the freeze-out line

Jishnu Goswami<sup>1,\*</sup> and Frithjof Karsch<sup>1,\*\*</sup>

<sup>1</sup>Fakultät für Physik, Universität Bielefeld, Bielefeld, 33615, Germany

**Abstract.** We present lattice QCD results for ratios of net-baryon number cumulants along the pseudo-critical line and compare them with STAR measurements from the RHIC BES-II program. The ratio of first and second order cumulants,  $R_{12}^B$ , agrees well with corresponding net-proton number cumulants down to  $\sqrt{s_{NN}} = 11.5$  GeV or baryon chemical potentials  $\mu_B/T \leq 2$ . Likewise higher-order cumulant ratios,  $R_{31}^B$  and  $R_{42}^B$ , show no sign for the existence of a critical point in the parameter range explored with these cumulant ratios. A QCD critical point is unlikely to occur within the BES-II range in collider mode. Moreover, the results demonstrate that a non-interacting HRG description breaks down for  $\mu_B/T > 1$ . We further analyze baryon-strangeness correlations normalized by strangeness fluctuations, finding consistency with STAR data at large beam energies but deviations at lower energies. Comparisons of electric-charge and strangeness correlations with STAR and ALICE data also show agreement at high energies, while the deviations at lower energies emphasize the role of unobserved strange resonances and the need for controlled feed-down corrections in baryon-strangeness correlations.

## 1 Introduction

In this proceeding contribution we present updated results on net-baryon number cumulants and on correlations of baryon and strangeness number fluctuations, normalized by second-order strangeness fluctuations, along the pseudo-critical line of (2+1)-flavor QCD. The calculations are based on high-statistics data generated by the HotQCD collaboration, extending up to eighth order in the Taylor expansion of the pressure. We compare our results with point-like, non-interacting hadron resonance gas (HRG) model calculations as well as with Beam Energy Scan (BES-II) data obtained by the STAR collaboration at RHIC in the United States.

In our comparisons to HRG model calculations, we use two resonance lists: PDG-HRG, which includes only the 3- and 4-star states listed in the PDG booklet, and QMHRG2020 [1], which additionally incorporates 1- and 2-star resonances from the PDG booklet, together with further resonances predicted by quark model calculations. In the latter case, particular care has been taken to avoid double counting of states.

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\*e-mail: [jishnu@physik.uni-bielefeld.de](mailto:jishnu@physik.uni-bielefeld.de)

\*\*e-mail: [karsch@physik.uni-bielefeld.de](mailto:karsch@physik.uni-bielefeld.de)

Throughout this proceeding,  $T_{pc}(\mu_B)$  denotes the pseudo-critical line as a function of baryon chemical potential  $\mu_B$  obtained from determinations of the maximum of the chiral susceptibility;  $T_{pc}^0$  refers to the pseudo-critical temperature at vanishing chemical potentials.

## 2 QCD critical point constraints from the pseudo-critical line of (2+1)-flavor QCD

The pressure of (2+1)-flavor QCD at finite baryon, electric charge, and strangeness chemical potentials can be expressed in a Taylor expansion around  $\vec{\mu} = (\mu_B, \mu_Q, \mu_S) = \vec{0}$ ,

$$\frac{P}{T^4} = \sum_{i+j+k=\text{even}} \frac{\chi_{ijk}^{BQS}}{i! j! k!} \hat{\mu}_B^i \hat{\mu}_Q^j \hat{\mu}_S^k, \quad \chi_{ijk}^{BQS} = \frac{1}{VT^3} \left. \frac{\partial^{i+j+k} \ln Z}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu=0}, \quad (1)$$

with  $\hat{\mu}_X \equiv \mu_X/T$ , for  $X = B, Q, S$ . Here the generalized susceptibilities  $\chi_{ijk}^{BQS}$  encode fluctuations and correlations of conserved charges and are the basic observables that connect lattice QCD calculations with heavy-ion experiments.

In heavy-ion collisions, constraints on the net charge densities  $n_Q/n_B = 0.4$  and vanishing strangeness,  $n_S = 0$ , fix the values of  $\mu_Q$  and  $\mu_S$  as functions of  $\mu_B$  [2]. The expansion of cumulants and other thermodynamic observables thus effectively reduces to a one-parameter series in  $\hat{\mu}_B$ . The pseudo-critical line can be parameterized as

$$T_{pc}(\mu_B) = T_{pc}^0 \left[ 1 - \kappa_2 \hat{\mu}_B^2 + \kappa_4 \hat{\mu}_B^4 + O(\hat{\mu}_B^6) \right], \quad (2)$$

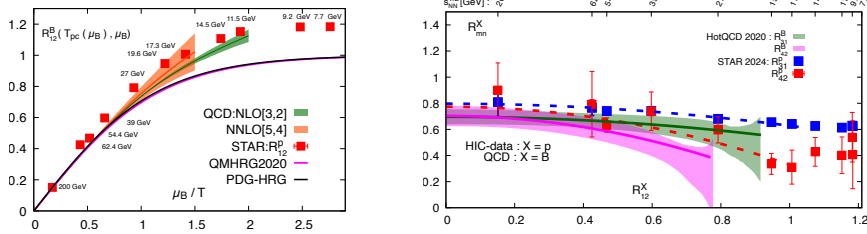
with  $T_{pc}^0 = (156.5 \pm 1.5)$  MeV,  $\kappa_2 = 0.012(4)$ , and  $\kappa_4 = 0.000(4)$  for  $n_S = 0$  [3]. The coefficient  $\kappa_2$  has been shown to be only weakly quark mass dependent, *i.e.*  $\kappa_2 = 0.015(1)$  for vanishing light quark masses [4], while the chiral phase transition temperature drops by about 25 MeV. It has been found to be  $T_c^0 = 132_{-6}^{+3}$  MeV [5]. If present at all, the QCD critical point (cep) is expected to show up at temperature  $T^{cep} < T_c^0$  [4, 6, 7]. The chiral phase transition temperature thus puts a bound on  $T^{cep}$  for the location of a possible CEP and as such also gives a lower bound on  $\mu_B^{cep}$ . Assuming that the curvature of  $T_{pc}(\mu_B)$  is well described by Eq. 2 up to  $\hat{\mu}_B \approx 3.6$ , where  $T_{pc}(\mu_B)$  would reach a temperature of about 132 MeV, one finds

$$T_c^{cep}(\mu_B) < 132_{-6}^{+3} \text{ MeV}, \quad \mu_B^{cep} > 477_{-20}^{+30} \text{ MeV}. \quad (3)$$

An even more stringent bound is obtained when assuming that a possibly existing tri-critical point in the chiral limit does not occur for  $\hat{\mu}_B < 2$ . In that case one concludes that no critical point can appear at physical values of the quark masses for  $T > 125$  MeV and  $\mu_B < 510$  MeV. This suggests that if a critical point exists at all, it must lie at values of  $\mu_B$  beyond those currently reached by BES-II in the collider mode. In the following sections we will show that fluctuation observables, especially skewness and kurtosis, are consistent with this conclusion.

## 3 Net-baryon number fluctuations on the pseudo-critical line

In Fig. 1, we present the ratio of mean and variance of the net-baryon number distributions,  $R_{12}^B = \chi_1^B/\chi_2^B$ , in (2+1)-flavor QCD as a function of  $\mu_B/T$  along the pseudo-critical line  $T_{pc}(\mu_B)$ . We find that  $R_{12}^B$  and the corresponding  $R_{12}^p = \chi_1^p/\chi_2^p$  of net-proton number fluctuations, taken from BES-II STAR data [8], are consistent with each other down to beam energies  $\sqrt{s_{NN}} \approx 11.5$  GeV. At larger baryon chemical potentials, both  $R_{12}^B$  and  $R_{12}^p$  exceed unity, in clear disagreement with point-like, non-interacting HRG-model predictions, which gives  $R_{12}^B \leq 1$  and  $R_{12}^p \leq 1$  for any value of  $\mu_B$ . This indicates the presence of more complicated baryonic interactions that are absent in a point-like, non-interacting HRG.



**Figure 1.** *Left* The ratio  $R_{12}^B$  obtained in QCD and HRG-model calculations as function  $\mu_B/T$  on the  $T_{pc}$ . *Right:* Skewness ( $R_{31}^B$ ) and kurtosis ( $R_{42}^B$ ) ratios as function of  $R_{12}^B$ . Also shown are the corresponding STAR results for net-proton number fluctuations.

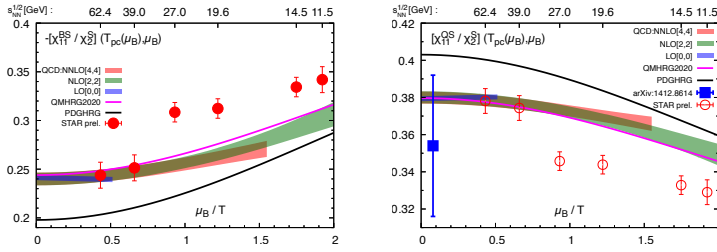
For higher-order cumulants, we find that QCD results for skewness ratio  $R_{31}^B \equiv \chi_3^B/\chi_1^B$ , and kurtosis ratio,  $R_{42}^B \equiv \chi_4^B/\chi_2^B$ , are consistent with STAR data down to  $\sqrt{s_{NN}} \simeq 19.6$  GeV and  $\sqrt{s_{NN}} \simeq 27$  GeV, respectively. In contrast, these ratios are unity in calculations with a point-like, non-interacting HRG-model. While these ratios approach unity also in QCD at low temperatures, they decrease by about 20–30% below unity already at  $T_{pc}^0$ . The point-like, non-interacting HRG-model thus fails to describe these observables even  $\mu_B/T = 0$ . The agreement of  $R_{12}^B$  with STAR net-proton results down to  $\sqrt{s_{NN}} \simeq 11.5$  GeV, together with the absence of any non-monotonicity in skewness and kurtosis ratios, provides further evidence against the existence of CEP in the  $(T, \mu_B)$  range probed in heavy ion collisions with beam energies  $\sqrt{s_{NN}} \geq 11.5$  GeV.

## 4 Baryon–strangeness number correlations on the pseudo-critical line

In Fig. 2 (left), we show the cumulant ratio  $\chi_{11}^{BS}/\chi_2^S$ , describing correlations of net-baryon and net-strangeness numbers,  $\chi_{11}^{BS}$ , normalized by the second-order strangeness fluctuations  $\chi_2^S$ . Results are shown along the pseudo-critical line  $T_{pc}(\mu_B)$ . In our previous publications, we studied truncation effects on Taylor series for second-order conserved-charge cumulants and concluded that our results are reliable for  $\mu_B/T \leq 1.5$  [9, 10]. In this range of  $\mu_B/T$ , QCD results agree well with STAR data only for  $\sqrt{s_{NN}} \geq 39$  GeV, while significant deviations appear for  $\sqrt{s_{NN}} \leq 27$  GeV. We also find consistency between QCD and QMHRG2020 up to  $\mu_B/T \simeq 2$ , while sizeable deviations from PDG-HRG results exist for all  $\mu_B/T \geq 0$ . Since QMHRG2020 includes many unobserved hadrons, this may point to the role of additional strange hadrons when constructing this observable from experimental data. In Fig. 2 (right), we present corresponding results for the ratio of electric charge–strangeness correlations,  $\chi_{11}^{QS}$ , and  $\chi_2^S$ ,

$$2 \frac{\chi_{11}^{QS}(T, \vec{\mu})}{\chi_2^S(T, \vec{\mu})} - \frac{\chi_{11}^{BS}(T, \vec{\mu})}{\chi_2^S(T, \vec{\mu})} = 1 + \frac{\Delta^{BQS}(T, \vec{\mu})}{\chi_2^S(T, \vec{\mu})}, \quad (4)$$

with  $\Delta^{BQS} = 0$  for strangeness neutral, isospin symmetric matter ( $n_S = 0, n_Q/n_B = 0.5$ ). For conditions relevant to heavy-ion collisions ( $n_S = 0, n_Q/n_B = 0.4$ ), the deviation from unity ( $\Delta^{BQS} > 0$ ) is less than 0.5%. Accordingly, the comparison of  $\chi_{11}^{QS}/\chi_2^S$  with STAR and QCD data shows the same trends observed for  $\chi_{11}^{BS}/\chi_2^S$ . Fig. 2 also includes the ratio  $\chi_{11}^{QS}/\chi_2^S$  obtained in [12] from strange particle yields measured by the ALICE Collaboration at the LHC [13, 14]. The second-order cumulants  $\chi_2^S$ ,  $\chi_{11}^{BS}$ , and  $\chi_{11}^{QS}$  were constructed from the measured yields, with feed-down corrections from  $\phi$ -mesons and neutral kaons [12]. These cumulants satisfy Eq. 4 up to uncertainties arising from feed-down contributions. As emphasized in [12], some decay channels contributing to  $\chi_{11}^{BS}$  are not known experimentally,



**Figure 2.** Baryon–strangeness (left) and electric charge–strangeness (right) correlations normalized by  $\chi^2_S$  along  $T_{pc}(\mu_B)$  [10]. Lattice QCD results are compared with preliminary heavy ion collision data from STAR presented at CPOD 2024 [11], and HRG model calculations using the QMHRG2020 list of hadrons. Also shown is a data point based on an analysis of data obtained by the ALICE collaboration [12].

and feed-down from higher kaon resonances as well as additional strange hadrons is difficult to control. Thus, a precise experimental determination of  $\chi_{11}^{BS}$  will remain challenging until these resonance contributions and their decay channels are better constrained.

## 5 Summary

We presented current constraints on the location of a possibly existing critical endpoint lattice QCD at non-vanishing chemical potential arising from studies of the QCD chiral phase transition temperature and discussed updated results for net-baryon number cumulants and baryon–strangeness correlations along the pseudo-critical  $(2+1)$ -flavor  $\bar{c}$ CD.

**Acknowledgements.** This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) Proj. No. 315477589-TRR 211 and the consortium PUNCH4NFDI supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) with project number 460248186 (PUNCH4NFDI).

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