

Physics of soap films – inquiry problems for 12th grade of the Bulgarian school

Daniela Ivanova¹, Diana Jordanova²

^{1,2} Baba tonka High School of Mathematics, Ruse, Bulgaria, 18 Ivan Vazov str.

Abstract: For centuries, soap bubbles have captured people's imagination. Bubbles have been a source of entertainment, inspiration for paintings, an incentive for philosophers and poets to compare their short lives to human life, and, of course, a huge challenge for scientists. Newton studied iridescent colors, Joseph Plateau proved the existence of a minimal two-dimensional surface with a given closed boundary, and Lord Kelvin in 1887 searched for the arrangement of cells or bubbles with a minimal total surface area between them. According to him, one can dedicate whole life to discovering physical laws just by blowing a soap bubble. Studying soap bubbles at school is beneficial from both theoretical and practical perspectives. The report discusses the application of an inquiry approach to studying the topics of liquid properties and surface tension. The topic is studied in grade twelve of the profiled physics curriculum in Bulgarian School. Teaching takes place at several levels. Students are introduced to basic concepts, principles, and laws of fluids. A structured study is then conducted in which the surface tension coefficient is determined by several different methods. The properties of soap films are studied through guided investigation, in which students are presented with research questions, and they develop research procedures. The geometric and optical properties of two-dimensional membranes and the interaction between soap bubbles are studied. From the two-dimensional membrane between soap bubbles, we move on to three-dimensional bubbles and the study of their properties. Advantages of the approach under consideration are the creation of interest and engagement of students, deepening of their mathematical knowledge, and the opportunity for interdisciplinary lessons in chemistry and art.

Keywords: inquiry approach, surface tension, surface energy, wave interference.

1. Introduction

Physics in the 11th and 12th grades is an elective subject in the Bulgarian education system. Students who choose to study this subject are genuinely interested and actively engage in practical work. The purpose of this report is to present an idea for applying an inquiry-based approach to teaching the topic of the properties of liquids, which is covered at the end of the 12th grade. The method under consideration is suitable for older students, as they have already studied all the topics in the general physics course in previous years and have developed skills in using equipment, as well as in collecting and analyzing data. The topic on the properties of liquids is studied over the course of five class lessons. The first lesson begins with a historical overview of the study of soap bubbles and the development of interest in their experimental investigation and mathematical description.

The students are asked the question – *Is it possible to make a film out of pure water?* In attempting to answer this question, they become familiar with fundamental concepts, principles, and laws related to liquids.

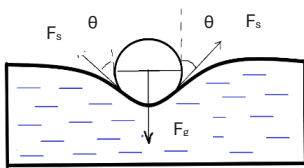
2. Structured inquiry

In seeking an answer to the main question, students become familiar with fundamental concepts, principles, and laws related to liquids. To understand the meaning of these concepts, their significance, and the reasons for their introduction, students carry out a series of inquiry-based tasks.

Students' research on topic begins with structured inquiry – the teacher asks essential questions and give students main procedures to complete the tasks.

2.1 Problem 1 - Construct a mosquito, made of copper wire, which can float on the water surface.

This is not a new experiment (Ref. 8,9,10), but it can be used for a more in-depth investigation of the properties of liquids. The teacher's question concerns the conditions under which a mosquito will float on the surface (Ref. 11). Students already know that cohesive forces act inward on the molecules at the surface of a liquid. The surface layer of the liquid behaves like an elastic membrane. If stretched, this force tends to contract it to a minimal area. Thus, the mosquito's ability to float on the surface (Fig.1) is possible due to the balance between gravitational force and surface tension (1).



$$F_g = F_s \cos \theta \tag{1}$$

$$Mg = \sigma \ell \cos \theta$$

Fig. 1 Cross section of wire on the water surface

The force of surface tension is proportional to the length of the stretched contour ℓ , through the surface tension coefficient σ . The floating model of the mosquito has six legs; therefore, the length of the contour can be obtained by (2):

$$\ell = 6[2(r + L)] = 12(r + L) \tag{2}$$

where radius of the wire is r and L is length of mosquito's leg floating on the water. If this length is small It can be assumed the legs are spherical (Fig.1) and then $\ell = 12\pi r$. If we assume that the mosquito's body is mainly composed of legs, the mass of the floating mosquito can be determined as (3):

$$M = \frac{12\pi r \sigma \cdot \cos \theta}{g} \tag{3}$$

The experimental tasks are related to determining the contact angle θ and the surface tension coefficient σ and obtain the mass of the mosquito so that it can float. The contact angle is measured based on image analysis. Students conduct research on different methods for determining surface tension. They chose, apply and compare two of them (Mulqueen, M., 2020). Two of the well-known methods that are easy to apply in school are drop weight method and Wilhelmy plate method (Mulqueen, M. Huibers, P. 2020).

2.1.1. Drop weight method

This is one of the simplest and most classical methods (Mulqueen, M. Huibers, P. 2020). A specific volume of liquid flows from a pipette in the form of drops, which are counted. (fig.2a). At the moment the drop detaches, its weight is equal to the surface tension force (4) that holds the drop together. (fig.2b).

$$Mg = 2\pi r \sigma$$

$$\frac{\rho V g}{N} = 2\pi r \sigma \tag{4}$$

It is assumed that the liquid does not wet the tube, and r is the inner diameter of the tube. To improve accuracy, the mass of N drops is measured.

It is assumed that the droplet is spherical, ρ is the liquid density, and the surface tension coefficient is obtained. (5):

$$\sigma = \frac{2\rho r^2 g}{3N} \tag{5}.$$



a) dropping droplets from a pipette

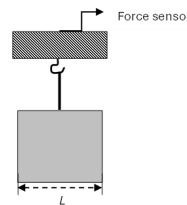
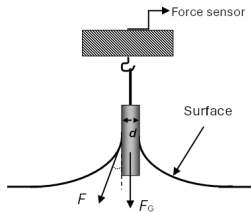
b) the droplet at the moment just before it falls from the pipette

Fig. 2 Implementation of the drop weight method

The advantages of this method are that it is fast, inexpensive, and requires only a small amount of water. A disadvantage is that the method is not static, and part of the drop remains on the tube when it detaches (Mulqueen, M. Huibers, P. 2020). A variation of this method is to drop the same number of pure water drops, whose surface tension coefficient is known.

2.1.2. Wilhelmy plate method

This method is based on immersing a porous plate between two fluids (water- air for example). This plate is linked to a very precise force sensor, which measures the force that has to be applied when pulling up the plate creating new surface (Fig.3a). The thickness of the plate is d and the width is L (fig. 3b). Using the Wilhelmy equation (6), the surface tension could be obtained.



a) side view of Wilhelmy plate

b) front view of Wilhelmy plate

Fig. 3 Implementation of the Wilhelmy plate method

$$\sigma = \frac{F}{l \cos \theta} \tag{6},$$

where l is the wetted perimeter of the plate, θ is the angle which is defined between the vertical line and the one tangential to the meniscus and F is the force measured by the sensor. This method is easy to use but a disadvantage is that it requires a large amount of liquid.

The students compare the methods – their advantages, disadvantages, and accuracy.

2.1.3. Using previous information make real model of mosquito (fig. 4)



Fig 4 Real model of mosquito

The students assemble a model of a mosquito (Ref. 8,9,10) and compare the experimental results for the mosquito's mass with the theoretically calculated one. The comparison of the results includes a discussion on how important it is which part of the mosquito's legs is in contact with the water.

2.2. Problem 2 - How would the mosquito's floating change in a soapy water?

Students are given reading materials, and with the guidance of the teacher, they understand what has changed in the properties of water when detergent is added to it. With the addition of soap to the water, surfactants appear, which adsorb onto the surface, reducing the surface tension. Soap molecules are long chains of fatty acid molecules that ionize in water. Thus, they have a hydrophilic head and a hydrophobic tail. In water, these molecules arrange themselves so that the polar part is in the water, while the nonpolar part is directed outward. (Fig. 5).

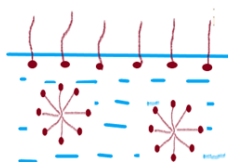


Fig. 5 Arrangement of soap molecules in water

When such a surface is stretched, the number of molecules per unit area decreases. As a result, molecules from the interior of the liquid migrate outward and restore the layer. The presence of surfactants on the surface disrupts the hydrogen bonding between water molecules at the surface, weakening the cohesive force and causing the surface tension to drop (Герузин, Я. Е. 1985). The students measure the surface tension at different concentrations of soap in water and find that in a soapy solution, the mosquito sinks.

2.3. Problem 3 - How big can a soap bubble get when one blows a bubble from a water droplet with radius r ?

The concept of such an evaluation was discussed by Geguzin (Гегузин, Я. Е. 1985) Soap solutions are capable of forming bubbles or thin films due to the presence of surfactant molecules that reduce surface tension and stabilize interfaces. Adding soap to the solution reduces the surface tension, allowing the free surface to stretch. A soap film has two adsorption surfaces (fig. 6).

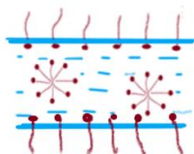


Fig. 6 Adsorption surfaces of soap film

The size of the bubble is determined by a combination of factors. As the bubble’s radius increases, the pressure difference between the inside and outside decreases due to the reduction in Laplace pressure. However, as the film stretches, it becomes thinner, and the liquid continuously evaporates. Despite the complex interplay of these factors, a simplified model can be constructed to estimate the limiting value of the bubble’s radius (Гегузин, Я. Е. 1985). When inflating a soap bubble from a droplet of soap solution with radius r , there are N_s soap molecules available in the solution, which can be distributed across the two surfaces of the film. During the formation of the soap film, these two free surfaces enclose a thin layer of soap water (fig.6). If „ a “ is the average intermolecular distance in the liquid and c is the concentration, then the number of soap molecules N_s can be estimated as (7):

$$c = \frac{N_s}{V} = \frac{N_s}{\frac{4}{3}\pi \left(\frac{r}{a}\right)^3}$$

$$N_s = \frac{4\pi r^3 c}{3a^3} \tag{7}$$

To cover both surfaces of the film with a single layer of molecules, the following number is needed (8) :

$$n = 2 \cdot 4\pi \left(\frac{R_b}{a}\right)^2 \tag{8}$$

The boundary radius R_b (9) is determined by the condition $N_s = n$

$$R_b = \sqrt{\frac{cr^3}{6a}}$$

$$R_b = const. \cdot c \cdot r^{\frac{3}{2}}$$

Since the bubble is inflated from a drop, the volume of the drop must be equal to the volume of the bubble film (10) with thickness h :

$$\frac{4}{3}\pi r^3 = 4\pi R^2 h \quad (10)$$

In this way, the limiting thickness (11) of the soap film can be determined:

$$h = \sqrt{\frac{2a}{c}} \quad (11)$$

- ✓ The limiting thickness of the film does not depend on the initial radius of the drop. The result is that the thinnest possible film consists of two layers of soap molecules stuck together."
- ✓ As the concentration increases, the film becomes thinner (12).

$$R_b = \frac{r^3}{3h} \quad (12)$$

$$h = \frac{r^3}{3R_b^2}$$

The dependence of the limiting radius on the concentration and the radius of the initial drop is investigated experimentally and compare experimental results with the theoretically derived values.

2.4. Problem 4 - Explore interactions between soap bubbles.

When two bubbles approach each other, they initially retain their spherical shape, then very briefly merge, being separated only by a membrane between them. The process can be recorded in slow motion and tracked with a video tracker. The assumption that the separating membrane is always flat is valid only when the sizes of the bubbles are equal (fig. 7).

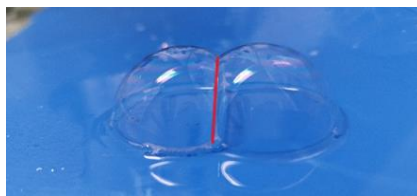


Fig. 7 Two bubbles of equal size in mutual contact

Students know that the pressure inside the bubble is greater than atmospheric pressure by $(4\sigma/R)$, which is the Laplace pressure. When there is a difference in the radii of the two bubbles, a pressure is exerted on the membrane (13).

$$\Delta p = p_1 - p_2 = 2\sigma \left(\frac{1}{R_1} - \frac{1}{R_2} \right) \quad (13)$$

The Laplace pressure for different curvatures is:

$$p_1 = \frac{2\sigma}{R_1}; \quad p_2 = \frac{2\sigma}{R_2}; \quad p_3 = \frac{2\sigma}{R_3}$$

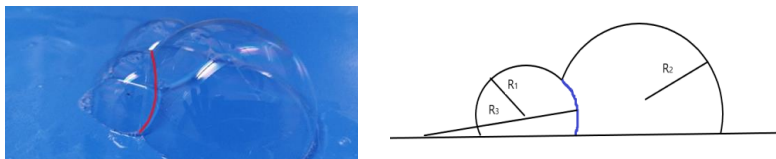


Fig. 8 Two contacting bubbles with different radii

In this case, the membrane bends because the pressure inside the smaller bubble is greater (fig.8).

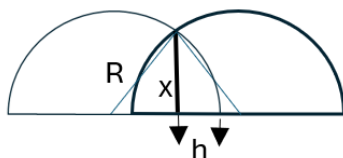
A functional dependence between the radii of curvature may be derived (14):

$$\frac{1}{R_3} = \frac{1}{R_1} - \frac{1}{R_2} \tag{14}$$

This relationship is investigated experimentally by measuring the radii of curvature from images and compare with with the theoretically derived value.

2.5 Problem 5 - What is the size of the separating membrane between the bubbles?

When two bubbles (in this case having equal radii) begin to make contact, a distinct interaction occurs at the point of contact - the water inside one bubble begins to flow into the water inside the other (Frederick, J et al, 1976). A portion of the two original surfaces vanishes, resulting in the formation of a single interfacial surface characterized by a radius of curvature x and an associated area (πx^2). The centers of the two balloons approach each other to a distance $2h \ll 2R$ (Fig. 9). At the onset of contact, the initial distance between the centers of the two balloons is $2R$, where R denotes the radius of each balloon.



Фиг. 9 Touching balloons sharing a common membrane.

Based on geometric considerations $R^2 = x^2 + (R - h)^2$ and due to the small value of h , the height of the membrane can be determined:

$$x^2 \approx 2hR. \tag{15}$$

Upon contact, the two boundary surfaces between the balloons disappear, resulting in a reduction of surface energy by:

$$\Delta E = 2(\pi R h)\sigma \approx \pi x^2 \sigma \tag{16}$$

This energy is released, enabling the formation of a membrane with height x . The value of

x is determined by the condition that the total energy of the system after contact — comprising the energy of the deformed balloons and the released surface energy — must not exceed the initial energy of the two separate balloons... The height of the membrane is always smaller than the radius; otherwise, this condition would be violated. The dimensions of the film are determined experimentally from images and compared with the radii of the bubbles.

3. Guided inquiry

After the students have become familiar with the properties of liquids and soap films and have conducted numerous experiments under the teacher's guidance, they can feel confident to carry out an independent investigation. The teacher poses the question – how can the thickness of a soap film be measured? Students are well acquainted with the phenomenon of light interference and can describe it both qualitatively and quantitatively. As a result, they can attempt to create a theoretical model of the interference pattern that could be produced when a soap film is illuminated with white light. A wire frame can be used to form a vertical planar film, or the boundary membrane between bubbles of equal size can be observed. The viscosity of the soap solution determines the drainage of liquid between the two layers of soap molecules. As a result, the draining film can be considered as a wedge-shaped plate with variable thickness (fig.10).

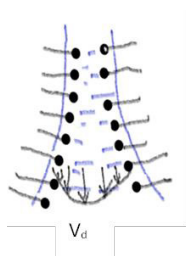


Fig. 10 A soap film flowing under the influence of gravity forms a vertical wedge

When illuminated with white light, interference fringes of equal thickness are produced. If the film thickness varies, there will be a difference in the optical path lengths of the waves, even if they are incident at the same angle. (fig.11)

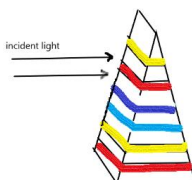


Fig. 11 Fringes of equal thickness

When light is incident at a certain angle onto a wedge-shaped film (Fig. 12) with a vertex angle ϕ , the rays reflected from the upper and lower surfaces of the wedge are not parallel but instead intersect at a point O' (ray 1')

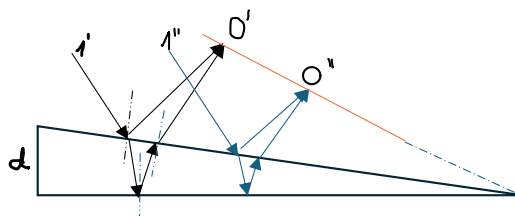


Fig 12 Light reflection from the wedge

Ray 1'', reflected from the upper surface of the wedge at a point with a different thickness, intersects with the ray reflected from the lower surface at point O''' (fig. 12). It can be shown that the points O' and O''' lie on a straight line passing through the apex of the wedge. The difference in the optical paths of the rays converging at points O' and O''' is determined by the thickness of the wedge at the respective points of reflection. Thus, each interference fringe arises as a result of reflection from the wedge at a specific thickness and is therefore referred to as a fringe of equal thickness *d*. Since the wedge thickness is small, the expression for the optical path difference under constructive interference condition in a plane-parallel plate can be applied as an approximation (17).

$$2d\sqrt{n^2 - \sin^2\alpha} - \frac{\lambda}{2} = m\lambda \tag{17}$$

where *n* is refractive index of soap solution, α is incidence angle, *m* is an integer.

As a result, the thickness could be obtained (18)

$$h = \frac{(2m+1)\lambda}{4\sqrt{n^2-1}} \tag{18}$$

When the film thickness is smaller than the wavelength of light, the intensity of the interference pattern approaches zero, and the film appears black. Over time, this effect becomes visible in the upper region of a vertically oriented film due to gravitational thinning. As the thickness increases, overlapping of interference maxima from different orders may occur.

The thickness can be determined in several ways. For example, by comparing the observed interference pattern with a known distribution of fringes as a function of wavelength. Isenberg (Isenberg, C.,1992) compares interference fringes of different orders with the corresponding film thickness. The students' task is to analyze the interference pattern of a vertical soap film (Fig. 13) using photographic images (Afanasyev, Y.D., 2011) or by comparing the obtained interference pattern with those obtained from other studies (by comparing the obtained interference pattern with those obtained from other studies (Lovett, D., 1994).

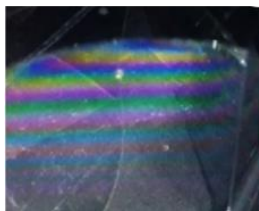


Fig. 13 Interference pattern of a vertical soap film

4. Conclusions

The use of an inquiry approach allows for a deeper understanding of the concepts of surface tension, surface energy, and Laplace pressure. Applying a research-based approach with older students who have more or less developed habits of collecting and processing data provides opportunities for more in-depth investigative work, with an understanding of the relationships being studied between the quantities. Students have the opportunity to work independently to compare the results of measuring the same quantity using different methods — in this case, the experimental determination of the surface tension coefficient. Creating a model of a "mosquito" allows for a comparison between the theoretically predicted results for its floating and the experimentally obtained values. Experiments with soap solutions of different concentrations make it possible to evaluate the sizes of the bubbles. The challenge here lies in creating identical initial conditions (initial drop size) and controlling the inflation of the bubbles as much as possible. Observing and experimenting with interacting bubbles leads to a deeper understanding of the concept of the minimum surface energy. When the bubbles come into contact, a boundary film is formed whose dimensions are smaller than the radii of the bubbles. By investigating the optical properties of soap films, in-depth knowledge is gained about the wave nature of light, as manifested in interference. By case study and proper experiments, general conclusions about the properties of liquids can be drawn. After experimenting with soap films, students will be able to answer the question of why a soap film cannot be made from pure water.

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