NON-LINEAR ANALYSIS OF THE DYNAMIC VERTICAL TRACK-BRIDGE INTERACTION OF A SIMPLY SUPPORTED SKEWED RAILWAY BRIDGE

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Abstract. The objective of the present study is to investigate numerically the contribution of the vertical track to the vibratory response of a skewed girder bridge, incorporating single ballasted track, under the excitation of trains circulating at constant speed. Compared to a straight beam-type structure, the present model considers two types of ballasted track behavior: one nonlinear elastic, stiffening with the applied force, and the other linear elastic. This comparison also accounts for the structural skew of the structural deck, a geometric detail that is often overlooked. This approach aims to assess the extent to which incorporating such nonlinear effects of the interaction mechanism can alter the bridge's behavior, particularly at resonance. The partial differential equations governing the vertical motion of the mechanical system are solved using two numerical methods to approximate the solution. The numerical results indicate that this type of nonlinearity in comparison to linear model cannot be neglected and its contribution is still moderate to the response of the mechanical system.

Keywords: Resonance, Numerical simulation, Runge-Kutta method, Finite Difference Method, Dynamic Response.

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1 Introduction

Over the past decades, the development of high-speed rail lines, both in developed and developing countries, and the increase in train operating speeds on certain existing lines (exceeding 200 km/h), has generated growing interest among engineers and researchers in railway bridge dynamics.

In structures, depending on the nature of the excitation, three vibration regimes can generally be distinguished: ambient, free, and forced. Ambient vibration, characterized by a low amplitude, is induced by environmental loads. Free vibration occurs primarily when the excitation ceases, for example, when the train leaves the bridge. This type of vibration is commonly used to extract the modal parameters of structures, such as their natural frequencies and mode shapes. Finally, forced vibrations occur when the structure is subjected to an external force, that, in the present study, is due to a high-speed train. In this case, the behavior of the bridge structure is characterized by higher vibration amplitudes. From a mechanical standpoint, the analysis of single-degree-of-freedom systems shows that, under resonance conditions, the vibration amplitude is primarily governed by the system's damping. This constatation also is still verified for multi-degree-of-freedom systems as well as for continuous systems. Physically, a resonance condition is known as the state in which the excitation frequency - i.e., the ratio between the train speed and a characteristic length D, such as the distance between bogies or the length of a carriage - coincides with one of the natural frequencies of the bridge. The occurrence of such a phenomenon, particularly in ballasted-track structures, may compromise structural stability by inducing cracking of load-bearing elements, ballast instability, or rail displacement/deformation [1,2]. For this reason, actually Eurocode 1 (EC1) [3], European standard, requires that, in any dynamic analysis, the vertical acceleration at the deck level of railway bridges - whether existing or newly constructed - must remain strictly below the prescribed limit value of 3,5 m/s^2 for structural deck incorporating ballasted tracks and 5 m/s^2 for slab tracks.

Globally, the vibratory response of a railway structure is complex, as several interaction mechanisms may contribute to it, such as track–bridge interaction (TBI), soil–structure interaction (SSI), and train–bridge interaction (VBI). Focusing on the first mechanism, a considerable number of numerical models, ranging from two-dimensional (2D) descriptions to three-dimensional (3D) representations, can be found in the literature [4–8]. The most common approach consists in modeling the bridge as a coupled mechanical system in which the rails and the deck are represented by uniform beams - upper and lower - connected by a vertical viscoelastic foundation (Winkler model), which accounts for the coupling effect between the ballasted track and the bridge deck. The consideration of nonlinearity in this type of interaction is often neglected, and studies addressing this aspect remain relatively limited [9,10].

With the objective to capture the effect of the nonlinear behavior of the ballasted track on the dynamic response of a skewed girder bridge, particularly at resonance, the present numerical study provides a comparison between two types of track behavior: linear and nonlinear. This analysis also takes into account a geometric feature that is often neglected, namely the structural skew exhibited by some bridges in plan. From a mechanical standpoint, the presence of such an angle in the plan configuration of single- or doubletrack bridges tends to increase their first fundamental frequencies, as reported in the literature [11,12], thereby producing a stiffening effect at low frequencies.

The present paper is structured into three sections as follows: Section 2 gives the main theoretical basis used to model the problem under investigation; Section 3 is devoted to the analysis of the applicability of this theory to a real bridge; and Section 4 summarizes the main conclusions of the study, along with some perspectives for future work.

2 Modelling theory

Consider the coupled system given in Fig. 1, consisting of two uniform Bernoulli-Euler (B–E) beams of identical length L , simply supported (S-S) and vertically connected by a viscoelastic foundation modeled by vertical spring–damper elements. The system is subjected to the motion of a succession of moving loads representing the action of a train traveling at constant speed V over a single-track ballasted railway bridge. Neglecting the effects of rotational inertia as well as the horizontal displacement of the two beams, the partial differential equations (PDEs) governing the behavior in plane description of each beam in the mechanical system can be written as follows:

$$\begin{aligned} &EI_r \frac{\partial^4 w_r}{\partial x^4} + \rho A_r \frac{\partial^2 w_r}{\partial t^2} + c_r \frac{\partial w_r}{\partial t} + k_f\left(w_r - w_b\right) + c_f\left(\dot{w}_r - \dot{w}_b\right) = \sum_{k=1}^{N_v} F_k \delta\left(x - \left(Vt - x_k\right)\right)\left[H\left(t - t_k\right) - H\left(t - t_k - L/V\right)\right] \\ &EI_b \frac{\partial^4 w_b}{\partial x^4} + \rho A_b \frac{\partial^2 w_b}{\partial t^2} + c_b \frac{\partial w_b}{\partial t} + k_f\left(w_b - w_r\right) + c_f\left(\dot{w}_b - \dot{w}_r\right) = 0 \end{aligned} \quad (1)$$

With $w_i(x,t)$ EI_i, ρA_i, c_i are respectively the flexural stiffness, mass per unit length, and the damping coefficient of the i-th beam, with $i = r\ or\ b$, r and b designate respectively rail and bridge. k_f et c_f are respectively the stiffness and damping of the viscoelastic layer (Winkler foundation). $\delta(.)$ et $H(.)$ denote the Dirac and Heaviside functions, respectively, N_v is the number of axles, F_k is the k-th moving load, and x_k is the position of the k-th axle on the beam measured from the left support. The computation time adopted to evaluate the forced response of the system is equal to the time interval during which the train is on the bridge.
The effects of wheel–rail interaction and track irregularities are neglected.

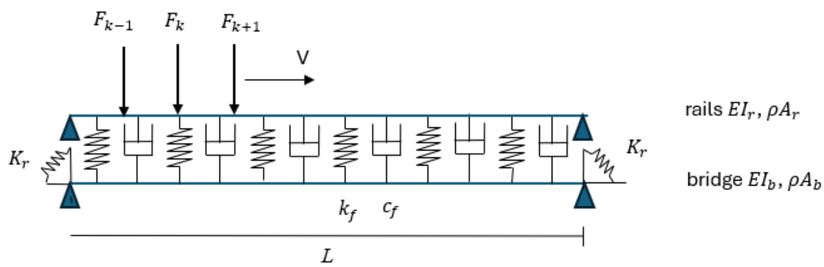

Fig. 1. Simplified model for TBI in a skewed bridge

To account for the mechanical skew of the structure, two rotational springs were installed at the ends of the lower beam representing the structural deck, based on the simplified model proposed by Nguyen and Goicolea [11,12]. This is a two-dimensional model in which the response of the beam-type structure is assumed to be governed by bending modes. The stiffness of the rotational springs is also provided in [11]:

$$K_r = \frac{2GJ}{L\cot^2(\beta)} \tag{2}$$

where GJ is the torsional stiffness of the deck (lower beam), β is the skew angle of the bridge.

Still referring to Fig. 1, the upper beam representing the rails is assumed to be isostatic. However, this simplifying assumption does not fully reflect reality, since the rails are infinite, and in the transition zone, a bending moment occurs due to the variation in stiffness. Although its influence remains moderate (on the frequency particularly), it is possible to extend the length of the rails [10] in the simulation. As for the lower beam, it is simply supported with partial fixity, in accordance with the boundary conditions given by Eq. (3):

$$\begin{cases} w_b(0,t) = w_b(L,t) = 0 \\ EI_b \dfrac{\partial^2 w(0,t)}{\partial x^2} = K_r \dfrac{\partial w_b(0,t)}{\partial x} \\ EI_b \dfrac{\partial^2 w(L,t)}{\partial x^2} = -K_r \dfrac{\partial w_b(L,t)}{\partial x} \end{cases} \tag{3}$$

This thus allows the extraction of the mode shapes, whose expression is given by: Eq. (4 , 5):

$$\phi_i(x) = \gamma_{1i}\left(\cosh\left(\frac{\lambda_i x}{L}\right) - \cos\left(\frac{\lambda_i x}{L}\right)\right) + \gamma_{2i}\sinh\left(\frac{\lambda_i x}{L}\right) + \sin\left(\frac{\lambda_i x}{L}\right) \tag{4}$$

$$\gamma_{1i} = \eta\frac{\sinh(\lambda_i) - \sin(\lambda_i)}{2\sinh(\lambda_i) + \eta(\cosh(\lambda_i) - \cos(\lambda_i))}; \quad \gamma_{2i} = -\frac{2\sin(\lambda_i) + \eta(\cosh(\lambda_i) - \cos(\lambda_i))}{2\sinh(\lambda_i) + \eta(\cosh(\lambda_i) - \cos(\lambda_i))};$$

$$\eta = \frac{4k}{\lambda_i}; \quad k = \frac{K_r L}{4EI} \tag{5}$$

and λ_i are the roots of the frequency equation, which is given as follows:

$$2k^2\cosh(\lambda)\cos(\lambda) - 2k^2 + k\lambda\sinh(\lambda)\cos(\lambda) - k\lambda\cosh(\lambda)\sin(\lambda) - \frac{(\lambda)^2}{4}\sinh(\lambda)\sin(\lambda) = 0 \tag{6}$$

It is important to note that the effect of SSI can be incorporated through a model in which viscoelastic supports are added, but this is beyond the scope of the present investigation, which aims to isolate the effect of TBI from other potential factors, as highlighted previously.

In this model, the aim is to assess the extent to which the dependency of the ballast stiffness to the vibration amplitude affects the vibratory response of the skewed bridge crossed by moving trains at high speeds, the following non-linear approximation of the vertical stiffness of the single layer of Fig.1 is adopted: $k_f = k_0 \left| \frac{w}{w_0} \right|^{\alpha-1}$,with $\alpha \geq 1, k_0$ and w_0 are respectively the linear stiffness, and initial displacement. w represents the relative displacement between the rail and the structural deck. The damping of the ballast layer is modeled using a linear viscous damping coefficient. Nonlinearity in damping is outside of the scope of the actual study.

In order to solve Eq. (1), as the studied coupled is becomes non-linear when the vertical foundation stiffness is introduced, the mode superposition method cannot be used to projet Eq. (1) into an ordinary differential equation (ODEs) whose solution can be computed in time domain numerically. Therefore, the solution of Eq. (1), obtained using the Galerkin method [14], can take the following expression:

$$w_r(x,t) = \sum_{i=1}^{\infty} \varphi_i(x) q_i(t) = \sum_{i=1}^{\infty} \sin\frac{i\pi x}{L} q_i(t)$$
$$w_b(x,t) = \sum_{j=1}^{\infty} \phi_j(x) g_j(t) \tag{7}$$

where $q_i(t)$, $g_j(t)$ are, respectively, the generalized vertical displacements of the rail beam and the bridge, and $\phi_i(x)$ are the trial functions (mode shapes of the lower beam given by Eq. (4) verifying the defined boundary conditions. By substituting Eq. (2) into Eq. (1), multiplying each term by its associated trial functions $\varphi_j(x)$ and $\phi_n(x)$, and invoking the orthogonality conditions, the resulting ODEs can be written as:

$$\ddot{q}_i + 2\xi_r\omega_{ri}\dot{q}_i + \omega_{ri}^2 q_i + \frac{2k_0}{\rho A_r L} I_1 + \frac{2c_f}{\rho A_r L}\int_0^L \sin\frac{i\pi x}{L}\left(\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L}\dot{q}_i(t) - \sum_{j=1}^{\infty}\phi_j(x)\dot{g}_j(t)\right)dx = -\sum_{k=1}^{N_v}\frac{2F_k}{\rho A_r L}\sin\frac{i\pi(Vt-x_k)}{L}\left[H(t-t_k) - H(t-t_k-L/V)\right]$$
$$\ddot{g}_j + 2\xi_b\omega_{bi}\dot{g}_i + \omega_{bj}^2 g_i + \frac{2k_0}{\rho A_b L} I_3 + \frac{2c_f}{\rho A_b L}\int_0^L \phi_j(x)\left(\sum_{j=1}^{\infty}\phi_j(x)\dot{g}_j(t) - \sum_{i=1}^{\infty}\sin\frac{i\pi x}{L}\dot{q}_i(t)\right)dx = 0$$

(8)

with

$$I_1 = \int_0^L \sin\frac{i\pi x}{L}\left|\frac{\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L} q_i(t) - \sum_{j=1}^{\infty}\phi_j(x) g_j(t)}{w_0}\right|^{\alpha-1}\left(\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L} q_i(t) - \sum_{j=1}^{\infty}\phi_j(x) g_j(t)\right)dx \tag{8a}$$

$$I_3 = \int_0^L \phi_j(x)\left|\frac{\sum_{i=1}^{\infty}\sin\frac{i\pi x}{L} q_i(t) - \sum_{j=1}^{\infty}\phi_j(x) g_j(t)}{w_0}\right|^{\alpha-1}\left(\sum_{j=1}^{\infty}\phi_j(x) g_j(t) - \sum_{i=1}^{\infty}\sin\frac{i\pi x}{L} q_i(t)\right)dx \tag{8b}$$

$$\omega_{ri} = \left(\frac{i\pi}{L}\right)^2\sqrt{\frac{EI_r}{\rho A_r}},\quad \omega_{bi} = \left(\frac{\lambda_i}{L}\right)^2\sqrt{\frac{EI_b}{\rho A_b}},\quad i = 1,2,...$$

Since Eq. (8) is nonlinear, obtaining an analytical solution is not straightforward. In the present study, by considering only the first longitudinal bending mode, Eq. (8) can be solved directly using the ODE45 command in MATLAB in conjunction with the central difference scheme (FDM), and the time step used to be $10^{-3} s$. All calculations are performed at the bridge mid-span. The maximum response is presumed to occur at this section.

3 Numerical results

In this section, the objective is to examine whether the viscoelastic foundation, whose behavior is non-linear, and which models the vertical interaction between the rails and the structural deck, influences the dynamic behavior during the passage of trains at constant speed. To this end, the bridge under study, whose geometric characteristics - assumed to be time-invariant - are presented in Table 1, is the same as that previously analyzed by Nguyen and Goicolea [11].

Table 1. Physical characteristics of the coupled mechanical system [5, 11]

Parameter	Value
Length $L(m)$	24
Flexural stiffness of the rail beam $EI_r\left(Nm^2\right)$	1.2831×10^{7}
Flexural stiffness of the bridge beam $EI_b\left(Nm^2\right)$	4.4547×10^{10}
Torsional stiffness of the bridge beam $GJ\left(Nm^2\right)$	3.4228×10^{10}
Skew angle	10°
Mass per unit length of the rail beam ρA_r	120
Mass per unit length of the bridge beam ρA_b	9654
Poisson ratio υ	0.25

For this bridge, using Eq. (6), the following first natural frequencies are obtained: $f_1 = 5.878 Hz$, $f_2 = 23.344 Hz$ .These identified frequencies are in good agreement with those issues from Finite Element simulation (FE) in [11]. Regarding the external excitation, since the forced-response behavior of the structure is predominant, it is induced by the excitation of the standard HSLM A1 train (from left to right) [3] in the interval [50-300] km/h. Geometrically, 18 intermediate coaches, each 18 m long (characteristic distance D), constitutes this train. Its modeling is performed according to the moving load concept, which represents the train as a succession of vertical loads with the same axle weight and arrangement, while neglecting any wheel–rail interaction (which is generally a source of additional damping). To develop a model that sufficiently accounts for this interaction, the train can be represented as a system of rigid bodies. It should be noted that, as preconized by the European design code, a dynamic analysis may be required if the train's maximum allowable speed exceeds 200 km/h.

3.1 Validation of results

As stated earlier, two numerical methods were used to solve Eq. (8): the RKM method and the FDM method. Fig. 2 presents a comparison between the two methods in the maximum response–speed plot. Overall, the results obtained from both methods show good agreement. In terms of maximum absolute response, the curves also highlight the presence of three resonance peaks associated to multiples resonances of the first bending mode, which coincide with the following theoretical critical speeds:

$$V_1^{(2)} = \frac{f_1 D}{2} = \frac{5.878 \times 18 \times 3.6}{2} = 190.45\, km/h \;,\; V_1^{(3)} = 127\, km/h \;,\; V_1^{(4)} = 95\, km/h \quad (9)$$

It can therefore be observed that, even in the presence of the track, whose behavior is nonlinear, Eq. (9) remains fundamentally valid, and speeds at which the vertical acceleration could exceed $3.5\, m/s^2$ can be determined easily.

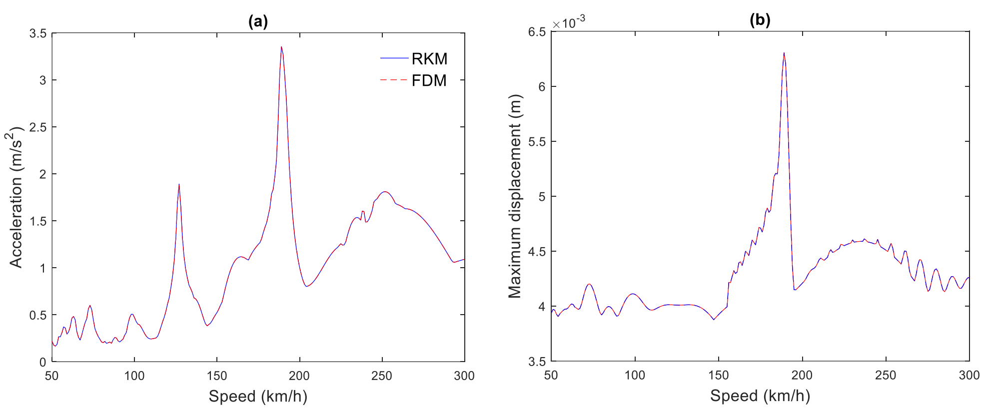

Fig. 2. Vertical response calculated at mid-point section of the beam under the HSLM A1: a) acceleration, b) displacement. $\alpha = 1.1$

It is well established that the time step significantly affects the convergence of numerical results. Fig. 3 illustrates the influence of the time-step size on the mid-span vertical acceleration and displacement when train A1 crosses the bridge with the first critical speed (Eq. (9)), corresponding to the second resonance of the fundamental mode. The obtained results indicate that increasing the time-step size apparently leads to a deterioration in the convergence of the numerical solution. Therefore, it can be reasonably concluded that larger time steps reduce the accuracy and convergence of the computed response, as expected.

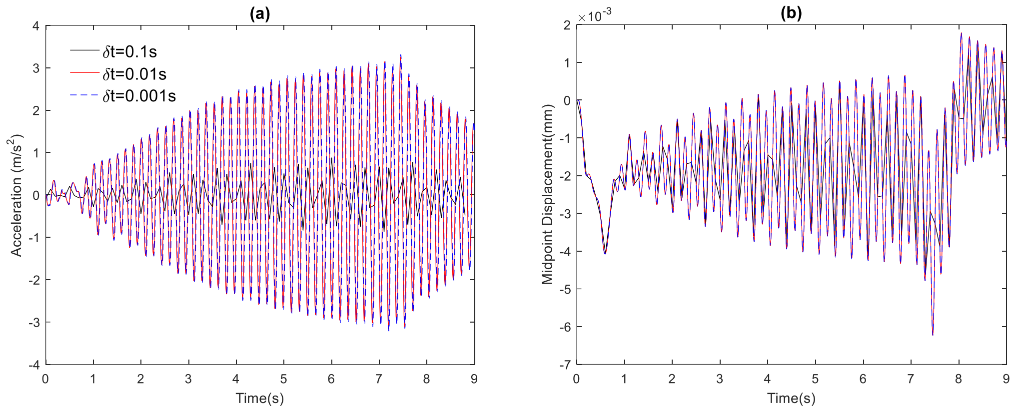

Fig. 3. Effect of time step on the computed mid-span bridge response under HSLM A1 at 190km/h : a) displacement, b) acceleration. $\alpha = 1.1$

3.2 Mechanical parameters effect

In this subsection, the effect of the mechanical parameters of the nonlinear viscoelastic foundation is studied. The results are presented through maximum response–speed plots, as shown in Figs. 4, 5, and 6. As a preliminary analysis of these figures, it can be clearly observed that, regardless of the mechanical skew of the structure β - which is assumed to affect particularly the first natural frequency compared with a non-skewed (straight) structure [11,14] - the resonance peaks previously identified remain unchanged. This behavior is observed irrespective of the values of stiffness k_f, damping c_f, and the power-law exponent α of the viscoelastic foundation. The linear case, represented for $\alpha = 1$, has been treated and the obtained results are in good match with those of Fig. 6. Furthermore, the curves indicate that the effect of the track is relatively moderate and can generally be neglected. This behavior can be properly explained by the fact that the flexural stiffness of the structure is higher than that of the rails (UIC 60 used here).

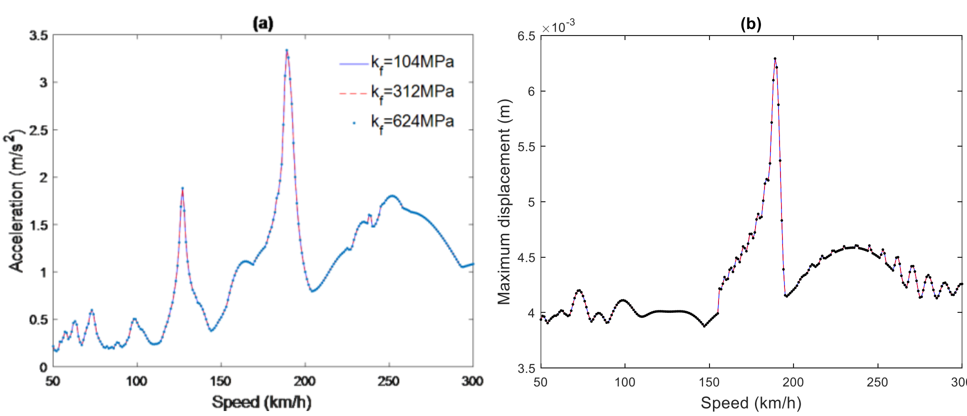

Fig. 4. Stiffness of the foundation effect on the vertical dynamic response of the bridge under the HSLM A1: a) acceleration, b) displacement. $\alpha = 1.1$

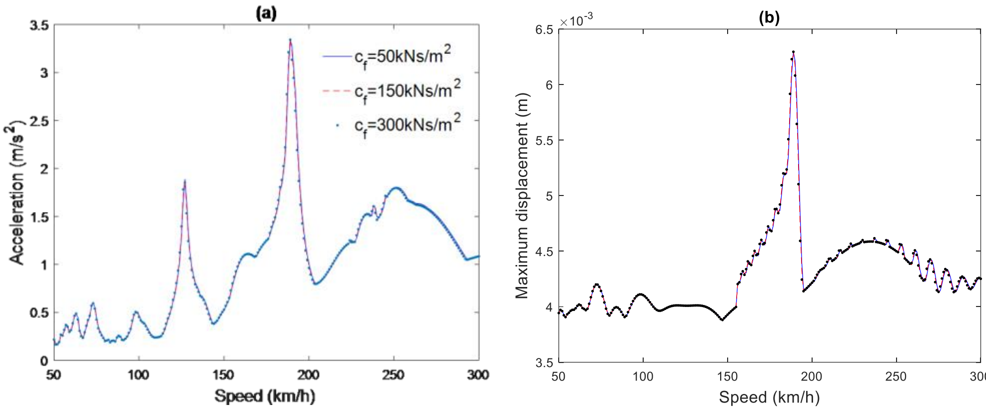

Fig. 5. Damping of the foundation effect on the vertical response of the bridge crossed by HSLM A1: a) acceleration, b) displacement. $\alpha = 1.1$

Referring again to Fig. 6, to summarize the content of the curves in more detail: although the behavior is generally nonlinear and for different values of α, $\alpha = 1.1$, 1.3 and $\alpha = 1.5$, the maximum response is not significantly affected by this nonlinear behavior. This indicates, overall, that the coupled effect between the rail, rail pads, sleepers and ballast can be relatively neglected, since, as noted above, the flexural stiffness of the structural deck is higher. In addition, it can be noted that values of the power-law exponent α greater than 2 may also be considered in the numerical simulations, even though the structural response is predominantly governed by the first flexural mode (simplification made in Fig. 1).

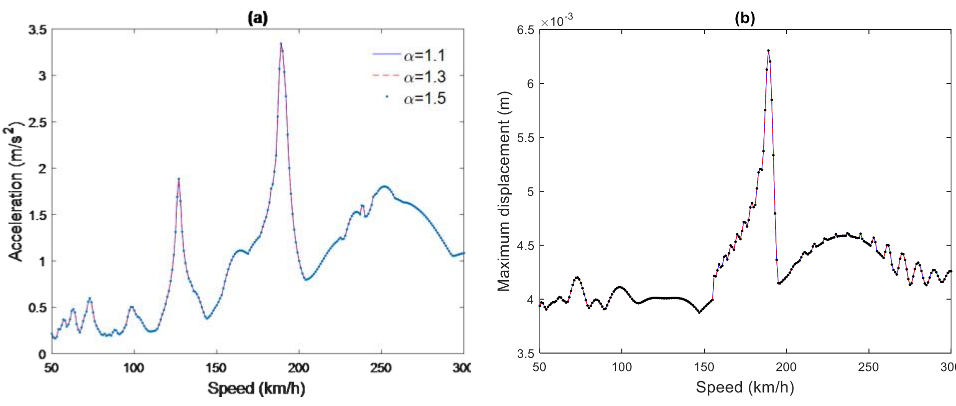

Fig. 6. Effect of the power order α on the vertical dynamic response of the bridge crossed by HSLM A1: a) acceleration, b) displacement.

4 Conclusion

The dynamic response of a skewed girder, single-ballasted-track railway bridge subjected to moving trains traveling at constant speed is investigated in the present study, considering both linear and nonlinear track behavior. The coupled system is modeled using a double-beam representation in which the ballasted track and the structural deck are idealized as two Bernoulli–Euler (B–E) beams connected vertically through a single layer modeled as a continuous distribution of spring–damper elements. The solutions of the equations of motion of the system are found using the forth-order Runge–Kutta method (RKM) and the central finite difference method (FDM). The obtained results show that, even when the nonlinear behavior of the track is taken into account, the estimation of the resonant speed remains reasonably accurate. Furthermore, the results indicate that, in the absence of track irregularities, the influence of the track can be neglected, and modeling the composite structure as a uniform beam provides an acceptable approximation of its dynamic behavior.

Data Availability Statement:

Data are available from the corresponding author upon reasonable request.

Conflicts of Interest:

The authors declare that they have no competing interests.

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