

Uncovering Nuclear Structure Patterns through AIML: A Case Study of β^- Decay Rates

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Abstract. Artificial intelligence and machine learning (AIML) techniques are widely used for recognising patterns in data and building models to describe complex relationships. Unlike traditional analytical models, machine learning (ML) models do not require an explicit theoretical formula in advance; instead, they learn the underlying trend from the data itself. This makes ML effective for problems involving nonlinear behaviour and multiple parameters, particularly in nuclear physics, where experimental information is often unavailable and theoretical models involve complex many body interactions. This work presents a preliminary exploration of ML based regression methods applied to β^- decay systematics. In stellar environments or storage ring experiments in terrestrial laboratories, atoms may become fully ionised, allowing β^- decay to unoccupied atomic orbitals (bound-state β^- decay) as an additional channel alongside decay to the continuum. The relative importance of these channels depends sensitively on nuclear properties such as the decay Q value, mass number, and proton number of the daughter nucleus, making reliable prediction challenging. In this paper, ML regression models are trained on theoretically calculated decay rates for some nuclei to understand these dependencies and assess the potential of ML in uncovering systematic trends in weak interaction processes.

1 Introduction

β^- decay is a weak interaction process through which a neutron inside an atomic nucleus transforms into a proton accompanied with the emission of an electron and an antineutrino. Atomic effects, such as ionization, can significantly modify of β^- decay. Ionization creates vacancies in atomic orbitals. The emitted β^- electron may be captured into one of these vacant bound states. This gives rise to an additional decay channel known as bound state β^- decay, alongside the conventional decay to the continuum. The opening of this channel alters the total decay rate and, consequently, the β^- decay half-life of the parent nucleus [1, 2].

In stellar environments, the high temperatures and densities can result in atoms being partially or fully ionized. Under such conditions, bound-state β^- decay can become comparable to, or even dominate over the continuum decay channel [3, 4]. Therefore, understanding β^- decay in highly ionized atoms is essential for accurately modelling stellar

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nucleosynthesis and elemental abundance evolution. Terrestrially, highly ionised atoms can be produced in storage ring experiments, enabling direct investigation of β^- decay in highly charged ions. In this context, Gupta *et al* [5] have recently calculated the bound state and continuum β^- decay rates separately for several nuclei of astrophysical importance under storage ring like conditions. These calculations establish upper limits for bound state β^- decay rates. Furthermore, the ionization induced reduction in half-lives of long-lived radionuclides has potential implications for nuclear waste management, particularly for mitigating the persistence of long-lived fission products [6].

For highly or fully ionized atoms, the competition between continuum and bound state β^- decay depends on several interrelated nuclear and atomic parameters, including the Q value of the transition, the degree of ionization, ionization potential depression, and the atomic and mass numbers of the daughter nucleus. The strong coupling between these parameters created considerable complexity which makes reliable predictions challenging through purely analytical or semi-analytical frameworks. Thus, robust theoretical calculations are required to determine which decay channel dominates for a given nucleus under specific ionization conditions [3, 4].

In this context, artificial intelligence and machine learning (AIML) techniques offer a different and promising approach. Machine learning (ML) models are particularly well suited for identifying patterns and correlations in high dimensional, nonlinear data sets without requiring explicit functional forms [7]. This capability is especially valuable in nuclear physics, where experimental data are often limited and theoretical models must account for complex many body interactions and competing physical effects. In recent years, ML methods have been successfully applied to a wide range of nuclear physics problems, including nuclear mass and decay property predictions, uncertainty quantification, parameter inference, and the discovery of interpretable physical models [8, 9, 10].

In this work, we present a preliminary exploration of ML based regression techniques applied to β^- -decay systematics in highly ionized atoms. Such conditions are relevant to stellar environments and terrestrial storage ring experiments, we train ML models using theoretically calculated decay rates for a selected nuclei. Through this approach, we investigate how β^- -decay behaviour varies with key nuclear and atomic parameters and assess the potential of ML methods to uncover systematic trends in weak interaction processes that may not be readily apparent within conventional theoretical treatments.

2 Methodology

The analysis was carried out for the observable - the bound to continuum decay rate ratio. For each β^- active radionuclide, several transitions may occur from a given parent level to different daughter levels. Each of these transitions is characterised by a unique Q-value, Q_n . On the other hand, both the bound and continuum decay rates depend strongly on the Q_n , along with other nuclear parameters such as the mass number A and atomic number Z. Consequently, for transitions originating from a fixed parent nucleus, the bound to continuum decay rate ratio is not a constant; rather, it increases as the transition Q_n decreases [5].

This observable was treated as the dependent variable and analysed as a function of the neutral atom Q_n , for fixed values of the mass number A and atomic number Z. However, for each A, Z combination, only a limited number of discrete data points are available. To overcome this issue, we have performed a two-step regression strategy based on artificial intelligence and machine learning (AIML).

Gaussian Process Regression was used as the first step of the analysis to reconstruct a smooth underlying dependence on Q_n from the sparse data. GPR is a non-parametric regression technique that models the data as a realization of a Gaussian process defined by a covariance kernel, which encodes smoothness of the curve [11]. GP defines a probability

distribution over possible functions rather than fitting deterministic curve, and mathematically represented as

$$f(Q_n) \sim GP(m(Q_n), k(Q_n, Q'_n)). \quad (1)$$

Where, mean function $m(Q_n)$ is assumed to be as zero, as a neutral prior. $k(Q_n, Q'_n)$ is the Kernel function which encodes physical smoothness of the observable. It captures the correlations between nearby Q_n values and provides uncertainty estimation for predictions. A radial Basis Function (RBF) kernel multiplied by a constant kernel was used to enforce smooth variation with Q_n

$$k(Q_n, Q'_n) = \sigma_f^2 \exp\left(-\frac{(Q_n - Q'_n)^2}{2l^2}\right). \quad (2)$$

The kernel hyperparameters were optimized through marginal likelihood maximization. Once trained on the original data points, the GPR model was evaluated on a dense grid of Q_n values within the data range, generating a smooth set of interpolated points. The process helped to create more data points for subsequent fitting by providing an idea of the global trend.

Symbolic regression was then used to extract an explicit mathematical expression describing the Q_n dependence of each observable [12]. Symbolic regression searches directly over mathematical expressions rather than fixed parameterizations by constructing a function

$$y(Q_n) \approx F(Q_n; \theta). \quad (3)$$

Where, the expression F is built from a predefined (based on physical significance) operator set.

However, in low data regimes the SR method prone to overfitting [13]. To mitigate this problem, symbolic regression was applied to a combined dataset consisting of the original data points and the GPR-generated points, with the original measurements assigned higher weights. This ensured that the resulting expressions remained anchored to the true data while using the GPR output only to constrain the overall functional shape. The expression complexity was restricted to obtain physically meaningful forms. This procedure yielded stable analytical representations for the observable across different A and Z values.

3 Data Source

Producing fully ionized atoms in a terrestrial laboratory is experimentally challenging. Consequently, only a very few experiments have been performed to date [14, 15], and among them, bound and continuum state β^- decay rates have been measured separately for only one nucleus, ^{207}Tl [16]. Therefore, reliable estimates of λ_b and λ_c must predominantly rely on theoretical calculations. Takahashi *et al.* [3, 4] provided theoretical decay rates for bare atoms of a few nuclei of astrophysical relevance. Nearly four decades later, Gupta *et al.* [5] carried out comprehensive calculations using updated nuclear and atomic parameters for a much larger set of nuclei. In this present work, we adopt the decay rates of ^{95}Nb , ^{123}Sn , ^{136}Cs , ^{152}Eu , and ^{155}Eu from Gupta *et al.* [5], in which

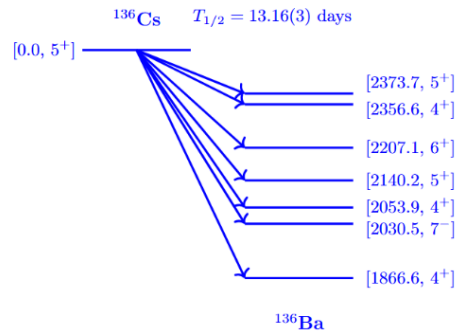


Figure 1. β^- transitions from the ground state of ^{136}Cs [17].

λ_b and λ_c were evaluated for fully ionized atoms under terrestrial conditions. These calculations yield the maximum possible λ_b and, consequently, the maximum λ_b to λ_c ratio.

4 Results and Discussions

To clearly demonstrate the analysis strategy adopted in this work, we begin with the nucleus ^{136}Cs and present the methodology in a step-by-step manner. This nucleus undergoes β^- decay from its ground state to several excited states of the daughter nucleus ^{136}Ba (Fig. 1). Among the energetically allowed transitions, six correspond to allowed or first non-unique forbidden β^- transitions, in accordance with the β decay selection rules. Each transition has a distinct decay energy Q_n and nuclear matrix element, resulting in different contributions to the total decay rate. The corresponding ratios of the bound to continuum state β^- decay rates for these allowed and non-unique forbidden transitions are displayed in Fig. 2.

To systematically analyse the behaviour of this ratio, we first employ GPR method to generate a dense and smooth representation of the available discrete data. The GPR framework not only interpolates between the calculated data points but also provides an estimate of the associated uncertainty, thereby allowing us to assess the reliability of the interpolation. From this procedure, a mean interpolated dataset is obtained, as illustrated in Fig. 2a. These interpolated data points are then used as input for SR, which aims to identify an explicit analytical expression describing the underlying trend. The SR predicted trend is shown in Fig. 2b and reveals that the bound to continuum decay rate ratio follows a power law dependence. This result suggests a simple and systematic functional relationship governing the competition between the two decay β^- channels across different transitions

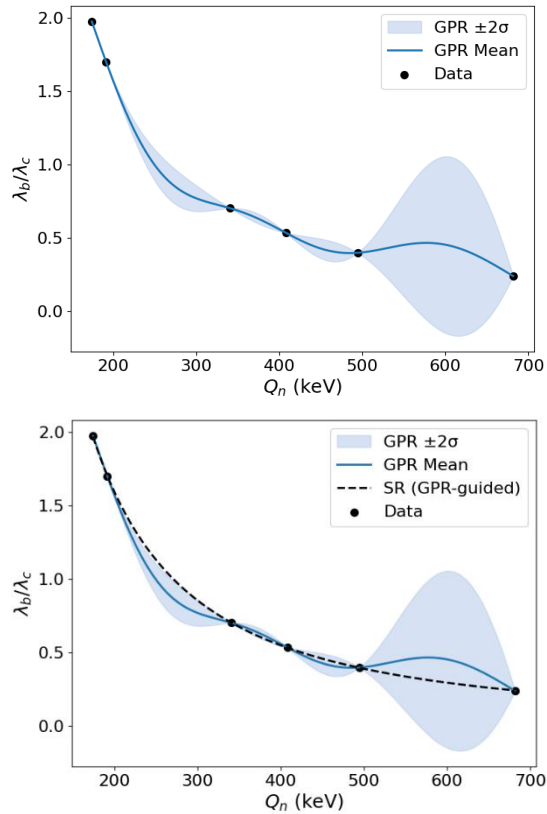


Figure 2. Top (a) : Application of only GPR on the data of ^{136}Cs ; Bottom (b) : Application of SR on interpolated data by GPR

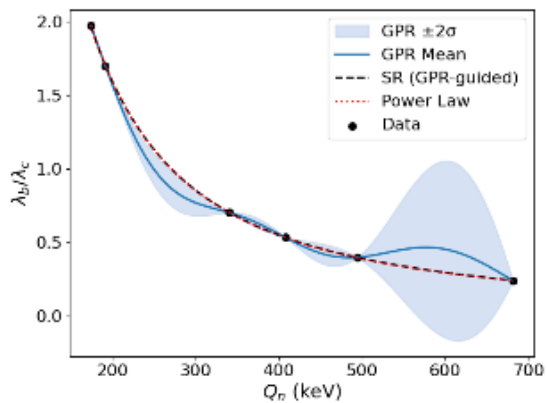


Figure 3 : Comparison of SR predicted function with Power Law fitted curve in case of ^{136}Cs .

$$\frac{\lambda_b}{\lambda_c} = (a / Q_n)^b. \quad (4)$$

Fig. 3 presents a direct comparison between the decay rate ratios reproduced by the fitted power law function and the original calculated data points. The close agreement observed across the entire range demonstrates that the GPR and SR method jointly successfully captures the underlying functional dependence of the λ_b and λ_c ratio. In particular, the SR-derived expression reproduces both the overall trend and the quantitative behaviour of the data with high fidelity, thereby validating the robustness of the combined GPR–SR approach adopted in this work. This agreement provides confidence that the extracted functional form is not an artefact of interpolation, but rather reflects a physically meaningful relationship inherent in the decay process.

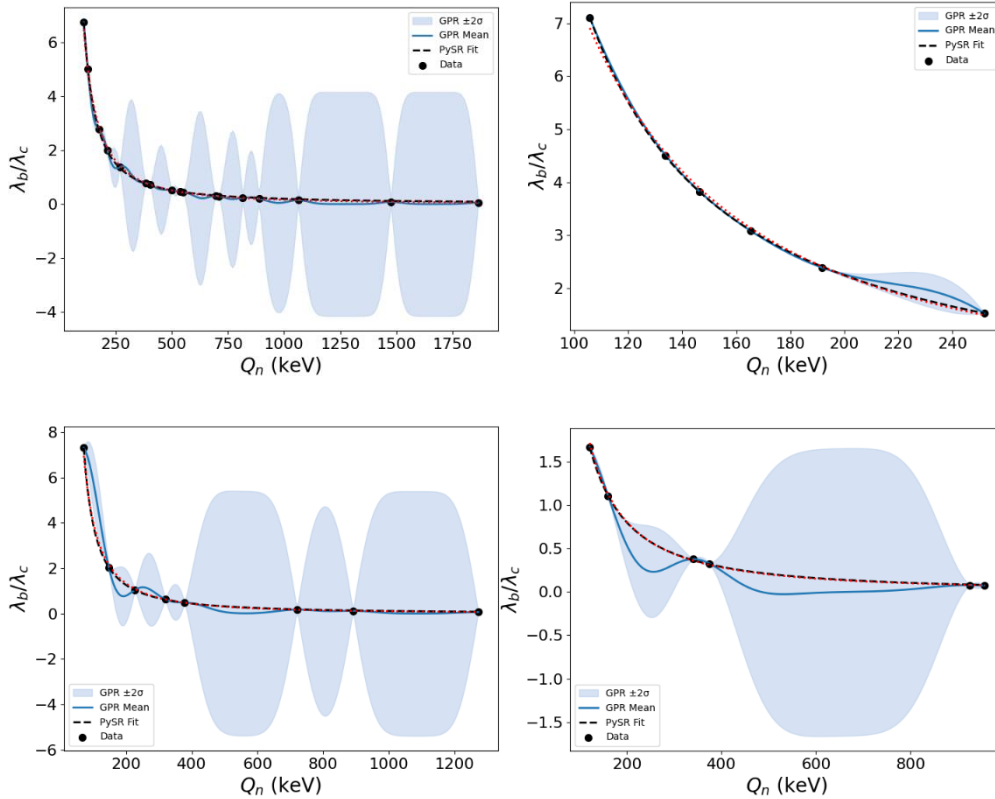


Figure 4. Stepwise determination of the functional relationship between Q_n and λ_b/λ_c using GPR and SR. The shaded GPR band represents the statistical $\pm 2\sigma$ uncertainty of an unconstrained Gaussian process. The negative part of the band has no physical meaning, as the decay-rate ratio is strictly positive, and merely reflects the lack of data constraints in this region. Top left: β^- decay of ^{152}Eu ; Top right: β^- decay of ^{155}Eu ; Bottom left: β^- decay of ^{123}Sn ; Bottom right: β^- decay of ^{95}Nb .

Encouraged by this level of consistency, we extend the same methodology to four additional nuclei, as shown in Fig. 4, in order to investigate whether a similar systematic behaviour persists. For each nucleus, the λ_b and λ_c ratios corresponding to different transitions are analysed following the same GPR interpolation and SR fitting procedure. In all cases, the analysis shows a clear power law dependence, indicating a universal trend across different

nuclei. The extracted fitting parameters for each nucleus are presented in Table 1 which provides a quantitative representation of the observed systematics and enables direct comparison among the studied cases.

The behaviour of the pattern is likely governed by the interplay between the available lepton phase space volume of the bound and continuum decay. The consistency of the extracted functional form across multiple nuclei highlights the robustness of this methodology and its suitability for studying bound and continuum state β^- decay rates in case of fully ionized atoms, in terrestrial conditions.

Parent	Parameter a	Parameter b
^{95}Nb	1.7279×10^2	1.5115
^{123}Sn	2.3886×10^2	1.5957
^{136}Cs	2.7077×10^2	1.5439
^{152}Eu	3.3314×10^2	1.6657
^{155}Eu	3.1529×10^2	1.7692

Table 1. Fitting parameters as per Eq. 4.

Based on these findings, the present framework can be systematically extended to a large range of nuclei of astrophysical relevance, as well as for possible candidates for storage ring experiments. In particular, applying the same GPR–SR methodology to additional isotopic chains will allow us to investigate how the fitted power law parameters depend on fundamental nuclear structural properties such as the decay Q-value, nuclear charge Z and mass number A. Such an analysis may thereby enable predictive estimation of bound state β^- decay contributions for nuclei for which detailed calculations are not yet available. Moreover, this approach has the potential to provide compact, physics informed parametrisations of stellar β^- decay rates that can be readily incorporated into large scale nucleosynthesis and stellar evolution models.

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