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Mesh Convergence and Error Analysis in Finite Element Modeling of Beam Structures

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Abstract. Checking for mesh convergence is important as it will give you an idea of the quality of your finite element solutions. A cantilever beam with a uniform loading is used as a test case in this study to examine the effect of mesh refinement on the numerical accuracy. Analytical results obtained from Euler–Bernoulli beam theory were used as reference values for the finite element predictions. Meshes were created and gradually refined along a series and comparisons between the displacement and the stress response were made. Relative error, root mean square error (RMSE), Richardson extrapolation and Grid convergence index (GCI) were used to investigate the discretisation error and convergence properties. The numerical results approached the analytical solution as the mesh density was refined, with the displacement approaching the solution faster than the stress. The convergence order was approximately second order for the stress quantity of interest observed at the midspan of the beam. The discretization uncertainty for the fine mesh was estimated to be about 1.3%. The study also seeks to show the impact of stress concentrations around constrained areas and to discuss practical considerations for the interpretation of convergence based on stresses. The outlined procedure here may be used as a reference procedure for verification studies using beam-type finite element models and other structural analysis techniques.

Keywords. Finite element analysis, mesh convergence, discretization error, Grid Convergence Index, Euler–Bernoulli beam, stress assessment, verification.

1. Introduction

Finite Element Analysis (FEA) is a fundamental tool in modern engineering that has come to be relied upon as the basic method for predicting structural response to complex systems. FEA is a popular method, but is always affected by the discretisation error, which results from approximating a continuous domain by a finite mesh [1]. It cannot be avoided; it is only

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possible to reduce it by systematically refining the mesh, and verification is an integral part of reliable simulation practice [2].

A lot of attention has been paid recently to verification procedures, which aim to measure the discretization error in numerical simulations. In computational fluid dynamics (CFD), Richardson extrapolation and Standardized and reproducible metrics of numerical correctness are well established in the field of CFD using GCI based approach [3,4]. Their application in structural and solid mechanics is, however, not as systematic, although the discretization error can have a significant impact on the response quantities of interest at the global or local level [5]. There are indeed means of verification, but their use is still not systematic enough for the use of the structural finite element analysis in real engineering application.

However, recent works have highlighted the need for considering the mesh resolution as one of the critical factors leading to epistemic uncertainties in finite element modelling (FEM), and that these can have significant effects on the prediction of stress, deformation fields and failure, when not properly managed [6,7,15]. Further, the convergence behavior is highly dependent on the chosen Quantity of Interest (QoI) with the convergence being smoother for the displacement than the stress that is more sensitive to gradients and numerical approximations [8].

Beam structures are useful for verification studies as there are analytical solutions available due to classical beam theory. The most common theory used for thin construction is the Euler-Bernoulli beam theory which assumes small shear deformations. This can be used as a trustworthy analytical reference for validation [9,10]. These benchmark problems allow a systematic study of the properties of discretization error and convergence in a controlled environment.

The primary goal of this work is to examine the convergence of a mesh of a controlled beam problem and to demonstrate a practical verification procedure, which can easily be replicated in finite element studies. The finite element method is not discussed here; rather, the effect of mesh refinement on the numerical accuracy and on the uncertainty of the discretization. For this purpose, analytical beam solutions are used as reference values and compared to the numerically generated solutions using increasingly fine meshes.

The investigation is a combination of multiple verification tools in one workflow. The solution accuracy and convergence properties are evaluated using solution Richardson extrapolation, the Grid Convergence Index, and relative error and root mean square error (RMSE). Particular attention is given to the behavior of displacement and stress quantities because these responses often exhibit different convergence patterns [11-16].

The present work also aims to provide quantitative information that can help engineers to pick the optimal mesh density for structural evaluations. Apart from convergence tendencies, the interpretation of numerical data is affected by the stress concentrations and mesh dependent effects.

Unlike many studies that report mesh sensitivity only through graphical comparisons, the present work includes a structured error evaluation procedure together with uncertainty estimation. The complete workflow was implemented using ANSYS and may be readily applied to similar beam-type finite element problems.

2. Methods

2.1. Analytical reference model

Consider a prismatic, homogeneous, linearly elastic beam of length L , rectangular cross-section $b \times h$, second moment of area $I = \frac{bh^3}{12}$, and Young's modulus E, as shown in Figure 1.

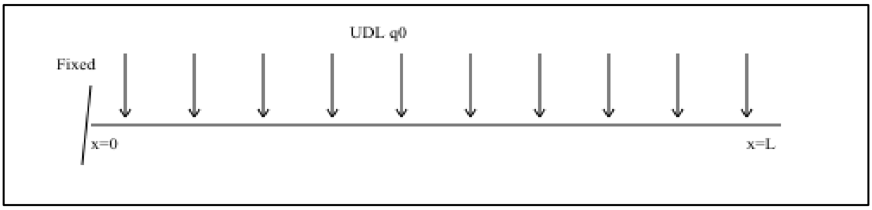

Fig. 1. Cantilever beam geometry, coordinate system, boundary conditions, and uniformly distributed load (UDL).

For small deflections under static transverse loading $q(x)$ (force per unit length), the Euler–Bernoulli beam equation is:

$$EI\frac{\mathrm{d}^4 w}{\mathrm{d}x^4} = q(x),$$

where $w(x)$ is the transverse deflection.

For a cantilever beam fixed at $x = 0$ and free at $x = L$, the boundary conditions are:

$$w(0) = 0, \quad w'(0) = 0, \quad M(L) = EIw''(L) = 0, \quad V(L) = EIw'''(L) = 0.$$

For the benchmark case of uniform loading $q(x) = q_0$, the closed-form deflection is:

$$w(x) = \frac{q_0 x^2(6L^2 - 4Lx + x^2)}{24EI},$$

and key derived quantities are:

$$\theta(x) = w'(x), \quad M(x) = \frac{q_0(L-x)^2}{2}, \quad V(x) = q_0(L-x).$$

The normal bending stress at distance y from the neutral axis is:

$$\sigma_{xx}(x,y) = -\frac{M(x)}{I}y = -E\,y\,w''(x),$$

so the maximum tensile/compressive stress for a rectangular section occurs at $y = \pm c$, $c = h/2$:

$$\sigma_{\max}(x) = \frac{|M(x)|c}{I}.$$

The Euler-Bernoulli theory ignores shear deformation, so the applicability is restricted to beam slenderness. For small slenderness Timoshenko-theory effects cannot be neglected. This motivates (i) sufficiently slender geometries for the benchmark and (ii) the explicit labeling of the analytical reference as an Euler–Bernoulli solution.

2.2. Validation Strategy

The validation approach is based on comparison between analytical and numerical solutions. Finite elements findings are compared with analytical solution of the Euler-Bernoulli beam theory.

The validation is based on the comparison of the following: displacement at selected interior points, bending stress at midspan and global error trends with mesh refinement. This ensures computational reliability of the numerical model and the physical consistency.

2.3. Finite element discretization concept and implementation

The beam is discretized in N elements in FE form (h-refinement) and an estimated displacement field w h (x) is created by use of element shape functions and nodal DOFs. For The element stiffness is obtained based on the weak version of the governing equation, and the nodal DOFs are usually transverse displacement w and rotation θ for Euler–Bernoulli beam elements. This can be done using dedicated beam elements, or using 3D solid elements, and the convergence principles and discretization-error reporting are both the same.

2.4. Quantities of interest and mesh convergence design

The mesh convergence study must include a list of QoIs which are (a) physically meaningful and (b) not dominated by singular behavior. When assessing convergence in structural analysis, the stress at geometric discontinuities or idealized boundary/load application points can have non-convergent (diverging) trends, because the actual continuum solution may contain stress singularities, so stresses away from the singular points or averaged/linearized stresses are suggested.

This study employs two Quality of Interest (QoIs): - Displacement at a point inside the tube (x=0.30L) away from the clamp where the bending moment field is smooth. - Bending stress at a point at the outer fiber of midspan (x=0.50L) calculated from the curvature/bending moment recovery. The GCI estimation is recommended to be carried out in three stages: coarse, medium and fine, and the ratio of the refinement between the two levels is recommended to be constant, r=h_2/h_1. The refinement here is using r=2 (element length is halved each refinement).

2.5. Error metrics

Suppose that Q_"ref" is an analytical reference for a QoI, and Q_h the FE value associated with a mesh size h. Relative error is:

$$e_{\%}(h) = \left|\frac{Q_h - Q_{\text{ref}}}{Q_{\text{ref}}}\right| \times 100\%.$$

The RMSE of the deflection curve is computed to evaluate field-level agreement for sampled points x_i, when evaluating deflection:

$$\text{RMSE}_w = \sqrt{\frac{1}{N_s}\sum_{i=1}^{N_s}\big(w_h(x_i) - w(x_i)\big)^2}.$$

For mesh-triplet solutions (Q_1, Q_2, Q_3) on fine/medium/coarse meshes $(h_1 < h_2 < h_3)$, the observed order of convergence p (for constant $r = h_2/h_1 = h_3/h_2$) is estimated via Richardson extrapolation:

$$p = \frac{\ln\left|\frac{Q_3 - Q_2}{Q_2 - Q_1}\right|}{\ln(r)}.$$

The Richardson extrapolated value is:

$$Q_{\text{ext}} = Q_1 + \frac{Q_1 - Q_2}{r^p - 1}.$$

A conservative discretization-uncertainty estimate is reported for the fine/medium pair, namely the Grid Convergence Index (GCI):

$$\text{GCI}_{21} = F_s \frac{|Q_1 - Q_2|}{|Q_1|(r^p - 1)} \times 100\%,$$

where F_s is a safety factor commonly used for three-grid studies and is three grid width thick [4].

2.6. ANSYS-oriented workflow for replication

The numerical workflow is easily repeatable in an ANSYS Static Structural analysis, as was done in published beam studies, which model cantilevers with ANSYS Mechanical / Design Modeler and utilize quadratic elements for increased accuracy. One set-up is practical, and it is: Geometry: make a prismatic beam of length L, width b, height h. Material: specify linear elastic isotropic properties (e.g., steel-like E,ν); retain small-strain, small-deflection assumption for equivalence to Euler–Bernoulli assumption. Boundary conditions: Fixed Support on x=0 face (cantilever clamp). Loading: Apply UDL on top face, so that q_0 is matched.

2.6.1 Element option:

Effective for skinny beams, suitable for comparison with beam theory, beam-element technique (1D line body). Solid element approach (3D): Gives stress contours and 3D effects, but needs stress extraction and needs to be aware of singularities. [12] Outline of the mesh strategy: mesh is generated with systematic refinement, the global size is halved for each level of meshing (coarse, medium, fine). Don't use different level meshes (hex vs tet) as this will affect the convergence behavior.

2.6.2 Post-processing:

Path plot displacement along the neutral axis.

Extract stress at $x = 0.50L$ (not at the fixed edge) to reduce sensitivity to boundary-condition stress concentrations and potential singular behavior.

Convergence reporting: tabulate QoIs vs mesh size and compute p, Q_{ext}, and GCI.

3. Results

3.1. Benchmark configuration

The geometry, loading and material features utilized as benchmark to generate the analytical reference values and the FE convergence results are reported in the Tab. 1. The parameters were set of such size that the linear behavior was conserved, but the deflections were in the millimeter range for easy numerical comparison.

Table 1. Parameters of the beam and the load (reference problem).

Parameter	Symbol	Value
Beam length	L	0.50 m
Width	b	0.025 m
Height	h	0.020 m
Second moment of area	$I = bh^3/12$	1.667×10^{-8} m^4
Young's modulus	E	200 GPa
Uniform distributed load	q_0	2000 N/m

The suitable analytical reference values of selected response quantities based on the closed-form Euler-Bernoulli solution for a cantilever beam subjected to evenly distributed load are presented in Table 2.

Table 2. Analytical reference results (Euler–Bernoulli cantilever with UDL).

Number of responses	Expression	Value
Overall load	$F = q_0 L$	1000 N
Root bending instant.	$M(0) = q_0 L^2/2$	250 N·m
Deflection of tip	$w(L) = q_0 L^4/(8EI)$	4.6875 mm
Tip rotation	$\theta(L) = q_0 L^3/(6EI)$	0.0125 rad
Max bending stress at root	$\sigma_{\max}(0) = \lvert M(0)\rvert c/I$	150 MPa
Maximum bending tension at midspan	$\sigma_{\max}(0.5L) = \lvert M(0.5L)\rvert c/I$	37.5 MPa

3.2. Mesh refinement levels and convergence outcomes.

We report 4 mesh levels: coarse/medium/fine (for GCI triplet) and an extra-fine mesh to show ongoing monotonic improvement. This is in accordance with the suggestion that at least three systematically refined meshes are necessary in order to determine the convergence order and the discretization uncertainty.

Table 3. Mesh convergence results for two QoIs.

Mesh level	N (elements)	Le (mm)	DOF	$\boldsymbol{w(0.30L)}$ (mm)	Rel. err $\boldsymbol{w}$ (%)	$\boldsymbol{\sigma_{\max}(0.50L)}$ (MPa)	Rel. err $\boldsymbol{\sigma}$ (%)
Coarse	2	250.00	6	0.682031	0.8180	31.250	16.667
Medium	4	125.00	10	0.687500	0.0227	35.937	4.167
Fine	8	62.50	18	0.687634	0.0032	37.109	1.042
Extra-fine	16	31.25	34	0.687656	0.0001	37.402	0.260

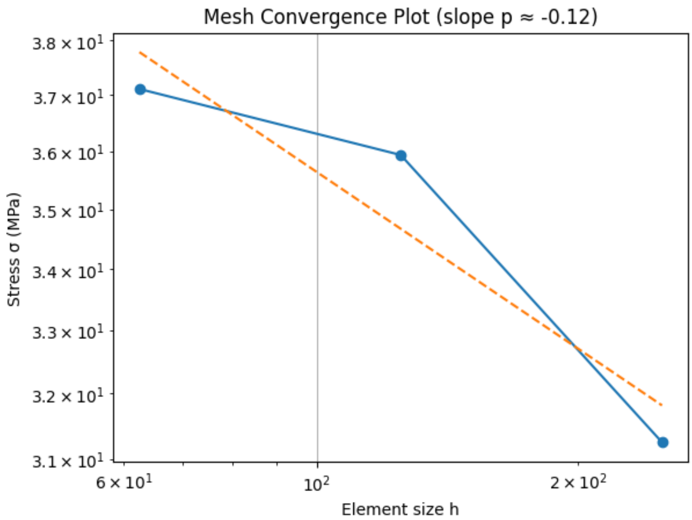

Fig. 2. Mesh convergence plot for midspan stress σ max (x = 0.50L).

Figure 2 illustrates the convergence of the midspan stress values to a stable solution with mesh refinement. The discretization error decreases with increasing element size, i.e., the solution converges to the analytical answer. The log-log plot is approximately linear, indicating that the solution is in the asymptotic convergence regime. This corroborates the refinement strategy applied and validates the numerical results.

We see a decrease of discretization error with refinement as expected for both a displacement QoI and a stress QoI. This is consistent with the general rule of convergence depending on the QoI and of stresses converging slower (or requiring more careful extraction) than displacements..

3.3. Discretization uncertainty of stress GCI

The observed order of convergence for the midspan bending stress QoI is near 2nd order for the three-grid set (coarse/medium/fine) and a constant refinement ratio r=2, while the asymptotic-range check is near 1. This provides a conservative estimation of discretization uncertainty using GCI.

Table 4. Richardson extrapolation and GCI for σ_max (0.50L).

Item	Value
Coarse solution Q_3 (N=2)	31.250 MPa
Medium solution Q_2 (N=4)	35.937 MPa
Fine solution Q_1 (N=8)	37.109 MPa
Sequence observed p	2.000
Extrapolated σ_{ext}	37.500 MPa
GCI_{21} (fine/medium)	1.316%
GCI_{32} (medium/coarse)	5.435%
Asymptotic ratio $\text{GCI}_{32}/(r^p\text{GCI}_{21})$	1.033

3.4. Convergence Behavior Analysis

Errors are plotted against element size in a log–log plot and the plot shows a linear trend, indicating that the error is of 2nd order. The convergence curve slope is in line with the theoretical convergence slope for beam elements and confirms the numerical implementation.

4. Discussion

The refinement of the mesh study showed that the smaller the elements the closer the approximation to the solution. Each refinement step brought both the displacement and stress values closer and closer towards the analytical reference. But the improvement rate was different for the two answers.. The displacement response reached a stable solution relatively quickly, whereas the stress results required a finer mesh before similar levels of accuracy were achieved. These differences are usual in finite element analysis as stresses are derived from displacement gradients and so are more influenced by discretisation effects.

The Grid Convergence Index was used to estimate the numerical uncertainty associated with the selected mesh levels. The estimated uncertainty for the finest mesh addressed in this work is roughly 1.3%. This value is indicative of the remaining discretization error and can help in deciding whether further mesh refinement is warranted for a given engineering application.

When judging finite element results, the discretization error is not to be assessed alone. The answer finally may depend additionally on the material qualities, loading assumptions and boundary conditions. Thus, the uncertainty associated with the mesh resolution is a component of the total modeling uncertainty.

The calculated order of convergence of the selected stress quantity was close to two during the whole refinement procedure. This result is consistent with the predictions for the accepted finite element formulation and signals the numerical solution to be approaching the asymptotic convergence region. In this work p is determined immediately from the mesh-refinement technique and not defined a priori, thus assuring that the reported convergence behaviour is typical of the numerical set-up employed. Thus, the observed value should be seen as a result in a single case and not as a rate of convergence which can be applied in a general meaning. Further work is recommended to verify the convergence properties for more complicated or different structural configurations. The ratio approaching 1 indicates that the solutions are likely within the acceptable range for a meaningful Richardson/GCI analysis [4].

The convergence study led to a number of practical observations. The most obvious problem was the behavior of the stress values close to the constrained regions and load application areas. At some areas local stress concentrations may lead to large variation with mesh refinement. For this reason, stress values extracted directly at such locations may not provide a reliable indication of convergence.

More consistent convergence behavior was obtained when stresses were evaluated at representative locations away from singular regions. The midspan stress used in the present study showed a stable convergence trend and allowed a clearer comparison with the analytical solution. Similar approaches are commonly adopted when assessing numerical accuracy in structural finite element analyses.

In this article we give results that show that the convergence analysis has to account both the numerical error as well as the physical relevance of the chosen quantity of interest. The mesh refinement and error estimation methodologies combined give a realistic way to assess the accuracy of finite element predictions and to identify circumstances where more refinement may be required.

5. Limitations and Future Work

The analytical model is based on the assumption of the linear elastic behavior of the material and small strains. These assumptions permitted a direct comparison between numerical and analytical solutions and offered a controlled environment to measure mesh convergence. The nonlinear material effects and large deformation behavior were not studied, as they are more complex effects.

Analytical solutions based on the Euler–Bernoulli beam theory was used throughout the study to provide reference values. For most practical engineering systems such solutions do not exist, however it is a good indicator for the analysis of the discretization error and convergence behavior. Complex problems might require validation through comparisons with experimental observations, numerical solutions from other models or through high fidelity simulations.

The reason for selecting a cantilever beam with an evenly distributed load is that the response of it can be explained analytically and interpreted unambiguously. variable structural configurations, loading circumstances and material distributions may have variable convergence behavior and should be analyzed case by case.

While the workflow described in this paper can be implemented in commercial finite element software, like ANSYS, in real applications, other automation tools could be needed in order to refine the mesh, extract required quantities of interest, and process convergence data in an effective manner.

Further research may be in the application of the verification process proposed in this study to more complicated geometries, nonlinear material models and 3D structural challenges. The investigations would help to shed light on the methodology's robustness in a wider engineering context.

6. Conclusion

In this work, the effect of mesh refinement on the accuracy of finite element results of a cantilever beam loaded by uniformly distributed load is investigated. Euler-Bernoulli beam theory was employed to obtain analytical solutions which served as reference solutions to be compared with the numerical ones. Results showed that both the displacement and stress values tended to the analytical solution with the refinement of the mesh.

Different characteristics of refinement in the displacement and stress quantities were obtained in the convergence analysis. Relatively few elements were used to obtain stable results for predicting displacement, while stress predictions needed further refinement to obtain similar accuracy. The computed order of convergence for the chosen midspan stress value did not deviate significantly from the second order theoretical value.

Richardson extrapolation and the Grid Convergence Index (GCI) were employed to estimate the discretization uncertainty quantitatively. The estimated uncertainty for the best mesh in this study was around 1.3%, which showed that the numerical solution was in the vicinity of the region of asymptotic convergence.

It also revealed that the number of interests affected the convergence assessment results in significant ways. The value of the stress in regions where the stress is singular or for simplified boundary conditions may distort the accuracy of the solution. Representative sites where the convergence behavior is stable can be used to get more reliable assessments.

The methodology of the verification approach proposed in this study is suitable to study uncertainty and convergence properties in beam-type finite element analyses. It is also very helpful for future applications for numerical verification and mesh sensitivity studies in structural engineering.

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